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A FRAMEWORK OF METHODS CLASSIFICATION FOR MULTIPARTITE ENTANGLEMENT

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ABSTRACT

Multipartite entanglement study carried out after bipartite entanglement as multipartite has a structure that is more useful for processing data in quantum information. In entanglement, two major studies were often focus is the measurement and classification of entanglement. Since a lots of measurements have been used to detect and quantify the entanglement in multipartite but it cannot distinguish the state specifically hence the classification is used to discriminate the state easily. Therefore, this study investigated the methods of classification of selected published works. This paper aims to recognize and describe the type of methods used by those works. As a result, a conceptual framework for classifying the multipartite entanglement was proposed based on previous works. This paper is expected to provide other alternative methods for classification.

Keywords: *Multipartite Entanglement, Entanglement Measurement, Classification of Multipartite Entanglement, Local Unitary (LU), Stochastics Local Operations And Classical Communication (SLOCC)*

1. INTRODUCTION

Entanglement nowadays becomes more important since fast development in quantum sciences especially in quantum information. Entanglement is a quantum correlation that exhibits the characteristics of nonlocal between two or more particles without considering the distance between them[1]. It plays crucial role in many fields in quantum information such as quantum teleportation[2], quantum cryptography[3, 4] and quantum computation[5].

Therefore one of the main objectives in the quantum theory of information to understand the non-local (entanglement) properties of the quantum states. This can done through the measurement and classification of entanglement. These both play a role complementary to each other. However, in bipartite entanglement especially pure states, measurement[6] and classification already complete[4] since it just involves with two qubit and only consider entangled states and separable states. Despite in multipartite entanglement there are still issues that cannot be resolved due to it complex structure that very restrict[7, 8].

Measurement of entanglement used to identify either the state is separable or entangled. In addition, the measurement can also determine the strength of entanglement in each state. Mostly measurement in multipartite is extended from bipartite entanglement since the measurement in multipartite quite complicated to resolve[9].

Although there are plenty of methods of measurement propose in previous works[10] but it cannot be implemented exactly in multipartite entanglement since the states divided into three types i.e. fully separable state, biseparable state and genuine entangled states. Hence, classification is used to facilitate the states is distinguished by its type. These classification divided into three concepts include local unitary (LU), local operations and classical communication (LOCC) and stochastic local

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operations and classical communication (SLOCC).

Several research[11, 12] have classified tripartite entanglement based on LU concept. In another way, other research[13] have classified the states under SLOCC concept. The LU concepts is the optimum classification[14] compare other classification. This classification become more complicated because the number of parameter local invariants increased rapidly due to the increment number of qubit. Hence, it is difficult to classify the states exactly since the richer structure of multipartite entanglement[9]. Besides, LU concepts not preserve the characteristics of entanglement for all condition[15] and some of the states exist in multipartite is not in LU equivalence classes[16]. According to LU issues, most of research shift to SLOCC concept to make the classification become simpler[14] since an invertible local operator (ILO) exist in SLOCC. The dissimilarity between LU (LOCC) and SLOCC can be described by mathematics notation as follows[17]: the two state are SLOCC equivalent if it can be transform by LOCC with non-zero probability. The two pure states $|\psi\rangle$ and $|\phi\rangle$ are in the n-partite of Hilbert space $\mathrm{H}^n = \mathrm{H}_1 \otimes \mathrm{H}_2 \otimes \cdots \otimes \mathrm{H}_n$ are LOCC equivalent if and only if (iff) it can be transform by LU[18],

$$|\psi\rangle = U_1 \otimes U_2 \otimes \cdots \otimes U_n |\phi\rangle \qquad (1)$$

with $U_1 \otimes U_2 \otimes \cdots \otimes U_n$ are LU operators while if $|\psi\rangle$ and $|\phi\rangle$ are SLOCC equivalent, then there exist ILO relating them[19],

$$|\psi\rangle = F_{(1)} \otimes F_{(2)} \otimes \cdots \otimes F_{(n)} |\phi\rangle \qquad (2)$$

where $F_{(1)} \otimes F_{(2)} \otimes \cdots \otimes F_{(n)}$ are ILOs respectively represented by matrix vector.

Table 1 below summarized the criteria between LU concept and SLOCC concept.

Table 1: Criteria of LU concept and SLOCC concept.

Ref	LU concept	Ref	SLOCC concept
[20]	•Difficult to classify in higher order. •Certain methods harder to been applied in LU.	[21]	• Monotone properties if entanglement under LOCC. • Preserve properties of entanglement.
[15]	Not preserve the criteria of entanglement		
[4]	Interconvertability.		
[14]	Finest classification for quantum entanglement.	[22]	More easy to classify compare to LU.
[19]	Particular case of SLOCC case.	[23]	Used ILO to perform the transformation.

This research aims to determine the types of methods of classification used in several previous works based on LU concept and SLOCC concept and to propose a new framework of method classification on operational method. This study is expected to provide an alternative method of classification for multipartite entanglement. The new classification based on SLOCC is extended from LU concept with same basis provide a new approach to classify the states in multipartite entanglement.

The structure of the paper is as follow. In section 2, we describe the methodology used by using several search engines and specific keywords to determine the types of methods used in previous work. Section 3 discusses in detail a list of method classifications based on the LU concept and SLOCC concept and basis used for each. As a result, we have developed a framework to summarize the results from previous works and proposed a new framework for classification of multipartite entanglement. Lastly, in section 4, the conclusion for final findings for future research objectives.

2. METHODOLOGY

The classification for multipartite entanglement is intended to distinguish the state to the exact type either it fully separable state, biseparable state or genuine entangled state. In this research, all literature is secondary resources that indexed in Scopus, ProQuest, Springer, Sage journals and Association for computing Machinery (ACM) by using several keyword namely *classification*, *multipartite* and *entanglement* from year 2000 until 2014. There are twenty nine (29) literatures chosen that related to this classification. All this literatures are consisting of LU concept and SLOCC

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concept as the base of classification but with different method that used.

The facts from the previous work in all literatures were scrutinized in order to determine the particular method that used to classify the state in multipartite entanglement. Document analysis has been done for these methods. The analysis is organized in order from year 2000 till year 2014 by using Microsoft Word 2013. The result from this analysis is explained in the following sections. Henceforth, the conceptual framework for classification of multipartite entanglement was later designed and proposed results of the analysis from the previous works.

3. RESULTS AND DISCUSSION

This section discusses the result from document analysis that comprises methods that has been used. Following that, conceptual framework is proposed.

3.1 Methods Used in Previous Works

Table 2 shows the methods of classification used in previous works base on LU concept.

Table 2: Methods of Classification by LU concept.

Source	Brief of Method
[24]	Generalized Schmidt decomposition (GSD).
[25]	Local bases product states (LBPS).
[26]	Reduced of two qubit from three qubit.
[4]	Trace decomposition form.
[15]	Cayley hyper-determinant and 2×2 subdeterminants.
[27]	High order singular value decomposition (HOSVD) and symmetry local state.
[16]	Operational method
[12]	Singular values of coefficient matrices.
[9]	Expectation values of Pauli operators.
[11]	Generalized high order singular value decomposition (HOSVD).
[28]	Classical tensor isomorphism.
[1]	Local unitary invariants polynomial (LUIPs).

Based on Table 2, there was determined twelve methods of classification used under the LU concept. Some of the method is generalized from other method with the addition of variable or modification of the constraint that has been used to make the existence method become easier to compute. Classification by LU concept is limited to tripartite entanglement (three qubits) since LU too restrict and not appropriate to extend in multipartite with $n>4$ [29]. Whereas, Table 3 depicts the methods using SLOCC. Same as LU concept, the methods used to classify the states by SLOCC concept also generalized from existence method to a new methods that has been done with modification.

Table 3: Methods of Classification by SLOCC concept.

Source	Brief of Method
[19]	Invertible local transformation.
[30]	Measurement of concurrence in group theory.
[31]	Duality of set separable state and entangled state.
[32]	Hyperdeterminant and singular value.
[21]	Algebra equivalence and Legendre transformation.
[33]	Five-graded partially order structure.
[34]	Singular value decomposition of coefficient matrices.
[35]	Amplitudes of states for matrix equation.
[36]	Equivalence-SLOCC (SEB).
[37]	Freudenthal triple system (FTS).
[38]	Reduced bipartite operator density.
[39]	Rank of the coefficient matrices for n-qubit.
[8]	Locally ratio for homogen polynomial.
[40]	Asymptotic setting using the concepts of degeneration and border rank of tensor from algebraic complexity theory.
[17]	Rank of the coefficient matrix and permutation for qudit.

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[41]	Freudenthal triple system (FTS) specific to certain state.
[13]	Rank of the coefficient matrix for arbitrary dimension.

Based on Table 2 and Table 3, there are a lot of method used to classify the state in both concepts. Generalized Schmidt decomposition [24] method and method LBPS[25] slightly same since both method is a straight forward generalization of product states that can be extended from bipartite case to tripartite case. Methods[1, 4] used local polynomial invariants as a main tool to classify the state because absolute value used to measure genuine multipartite entanglement. Method[1] can be implement in SLOCC concept to form special linear group (SLIPs) [8] that is constitute subset to LUIPs. Different with LIUPs, SLIPs, implement Schur-Weyl duality formula and can classify all classes of SLOCC.

Concurrence measure be used in some methods that are operational method [16], Cayley hyperdeterminant and 2×2 subdeterminants[15], duality of set separable state and entangled state[31] and hyperdeterminant and singular value[32]. All these methods easily to compute in bipartite entanglement compare tripartite entanglement since it need additional variables to compute the measurement. Since local rank are not sufficient to discriminate the classes in higher order, concurrence measure used to simplify the classification of the state. Since method of Cayley hyperdeterminant does not describe exactly the properties of entanglement in the state, then classical tensor isomorphism method proposed to overcome the constraint. This method appliance Bourbaki approach of determining the rank of a vector in linear mapping[28].

The two methods slightly difference with other methods that reference in table as using separability criteria aid by positive partial transpose (PPT) as a tool to classify the state which expectation values of Pauli operators[9] and reduced bipartite operator density[38]. Even though the PPT criteria actually used in bipartite state but it can be generalized to multipartite pure state either theoretically or experimentally. Several methods[21, 36, 37, 41] used algebra concept such as algebra equivalence Jordan algebra and enumerative combinatoric to differentiate the states.

Methods using coefficient matrix approach whether in term of singular value[11, 12, 27, 34] or the rank of the coefficient[13, 17, 39] have their own capability to distinguish the state. However, rank of coefficient easier to computer compare singular value operationally. Besides, rank of coefficient methods often used in SLOCC concept meanwhile singular value in LU concept. These all methods can be extended to mixed states.

Other methods just used standard methods such as local transformation[19], matrix equation[35] and reduction of three qubits[26] to classify the state.

Graph in Figure 1 depicted frequency of basis used in the selected previous works based on LU concept and SLOCC concept. For LU concept, basis with the lowest frequency is linear group (1 frequency) meanwhile the highest frequency is coefficient of canonical form (6 frequencies). Also for SLOCC concept, the lowest with one frequency are Schmidt decomposition, rank of coefficient matrix and asymptotic measure. Basis of special linear group is the highest frequency with value of six.

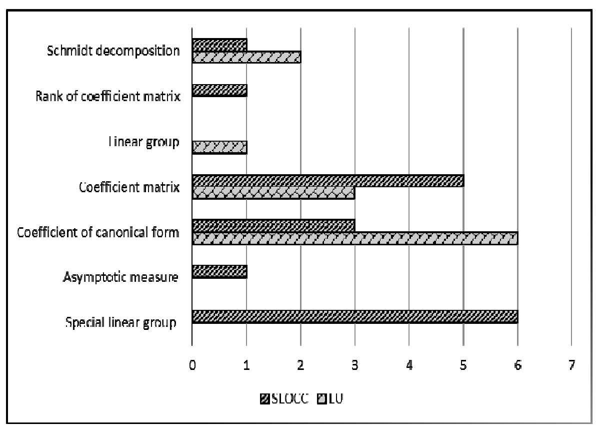

Figure 1: The Frequency of Methods Used based on basis for Classification in Multipartite Entanglement

Based on the Figure 1, the most common basis used in LU concept is coefficient of canonical form but restrict to tripartite entanglement (3 qubits). The canonical form is[25]

$$|\psi\rangle = \sum_{i,j,k} t_{ijk} |ijk\rangle \quad (3)$$

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where $\left|ijk\right\rangle \equiv \left|i\right\rangle_A \otimes \left|j\right\rangle_B \otimes \left|k\right\rangle_c$ with A, B and C are generic state shared by three parties. Differ in SLOCC concept, basis of special linear group with algebra concept represented by set of *SL*-invariant polynomial (SLIP)[8] in Hilbert space that shown in equation below

$$f : \mathrm{H}_n \to \mathbb{C}$$

that satisfies $f(\mathrm{g}\left|\psi\right\rangle) = f(\left|\psi\right\rangle)$ (4)

with $\forall g \in G$ and $\forall \left|\psi\right\rangle \in \mathrm{H}_n$ where $\left|\psi\right\rangle$ represent the state.

Each basis used has their own criteria that valid to certain classification to make the classification easier. Some of them cannot be used in higher dimension that involve more than 3 qubits since the computation become more complex in rich structure. Although the method can be implement in higher dimension, the states is partitioning to two qubits.

3.2 The Framework for Classification of Multipartite Entanglement

In designing a framework, the requirement in each concept has been identified to differentiate the feature between it. It is important since two concepts relate to each other.

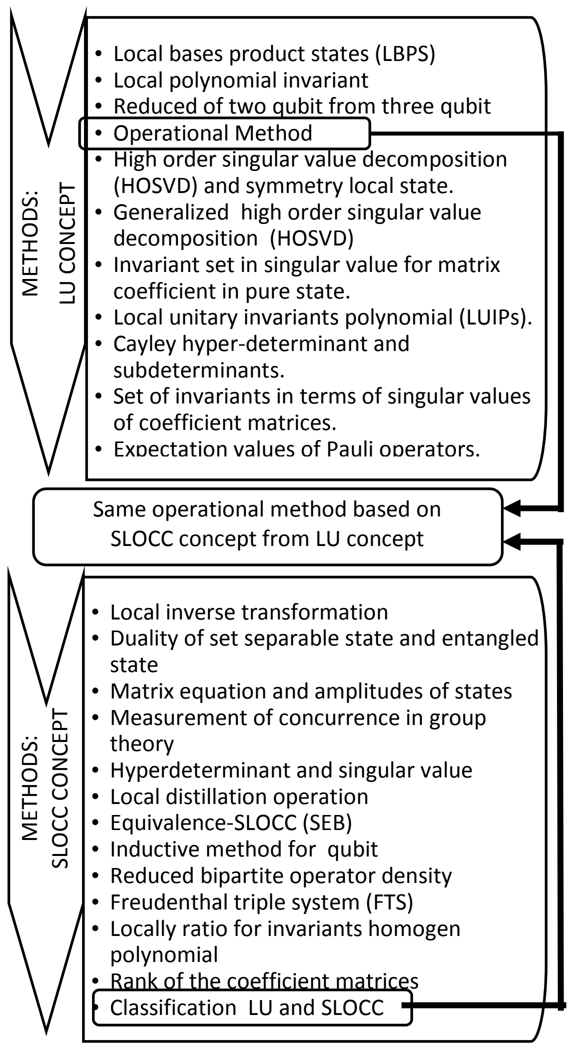

Figure 2: The Framework of Classification of Multipartite Entanglement

Figure 2 above illustrate the framework of a classification for multipartite entanglement that summarizes all the methods used according to LU concept and SLOCC concept.

The domain of this research is quantum entanglement with a focus on the classification of multipartite entanglement (focus on tripartite entanglement). Classification of tripartite

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entanglement concepts divided to LU and SLOCC. Each method has been reviewed to understand the way of classification with respect to both LU and SLOCC concepts. Since measurement is one way to detect entanglement in a given state, therefore we analyzed methods that are related to measurement.

Resulted from Figure 2, Figure 3 indicates the proposed framework of new classification for multipartite entanglement.

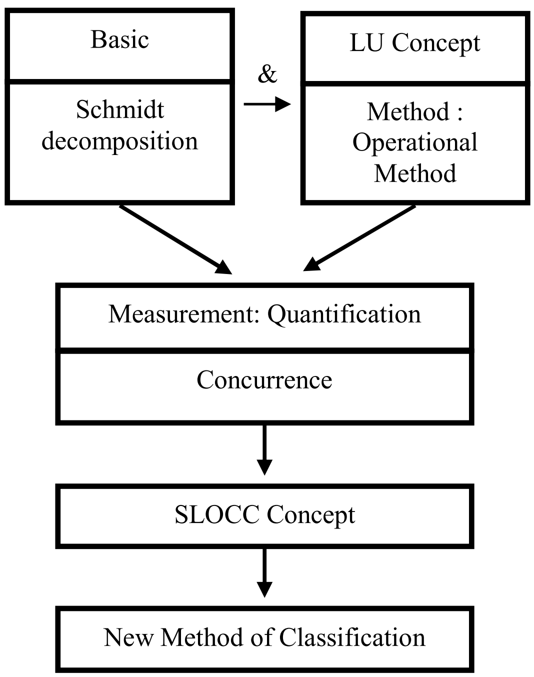

Figure 3: The Propose Framework of Classification of Multipartite Entanglement

Based on Figure 3, the research focuses deeper into tripartite entanglement. From the seven basis shown in Figure 1, the basis of Schmidt decomposition has been selected in this research because it is a very useful tool to detect entanglement with various application[23]. Alternatively, Schmidt decomposition will quantifies the level of degree of entanglement in quantum systems[7]. Schmidt decomposition basis for tripartite level can take two states orthogonal as a product in one parameter of the entanglements of the local basis. Since LU concept is a special case for SLOCC concept, the basis Schmidt decomposition is extended to SLOCC concept[19]. Operational method has been chosen to classify the state of tripartite entanglement since classification of LU classes using operational method is not widely studied[16]. In this method, the measurement of concurrence used to classify the state as concurrence can be extended from bipartite entanglement to tripartite entanglement[42]. The combination between Schmidt decomposition and operational method by LU concept propose new classifications to identify whether the basis and methods used in the LU can be extended to SLOCC easily. This is because SLOCC concept has ILO criteria that facilitate the calculation and make it simpler. It is therefore expected that the new classification is given in a different way to classify the state of tripartite entanglement whereas in previous work, this classification has not been done by SLOCC concept.

4. CONCLUSION

The proposed framework propose a new method of classification in multipartite entanglement into a specific class of the state in multipartite entanglement based on SLOCC concept.

There is further research that needs to be done to create a model of classification based on the proposed framework and to test the effectiveness of the model compared to existing models to determine the capability of the new model to classify the state in multipartite entanglement.

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