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HYSTERETIC SYNCHRONIZATION MODELS OF BIOLOGICAL NEURONS

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ABSTRACT

The article provides a synchronization model of biological neurons based on the hysteretic properties of metabotropic receptive clusters. We study the properties of the proposed model and various variants of connections between individual neurons. The advantage of the proposed model is its ‘ordered’ response to external environment, which is determined by the hysteretic connection between individual neurons. The described biological neural network model may find application in the segmentation tasks and classification with efficiency exceeding standard numerical algorithms of data handling.

Keywords: *Neural Networks, Hysteresis, Simulation, Attractor, Delay Differential Equations.*

1. INTRODUCTION

Currently, the line in research of the oscillatory aspects of the brain functioning is actively developing in the theory of neural networks. There are a number of models [1], [2], [3], [4], [5], [6] which in virtue of their varying degrees of their biological substantiation are able to describe the interaction of neurons in the cerebral cortex. Hodgkin-Huxley model should be particularly noted as the closest to the biological data [4], [5] though it is scarcely applied for neural network simulation due to its complexity. The tasks that can provide answers to important questions of psychology and neuroscience are at the heart of the models. Given that to date, the computational power is growing steadily, to study neural networks it is possible to use models most closely approximating the biological data, as the human brain remains a benchmark in solving a variety of tasks.

2. LITERATURE REVIEW

In this paper we study the behavior of biological neural networks, based on Kashchenko-Mayorov model [1]. However, unlike that model in the present paper arrangement of connection between individual neurons is simulated with regard to biological peculiarities considered in detail in the work by A.N. Radchenko [7]. According to the present-day views, neuron is surrounded by cell formations (clusters), which under certain conditions can initiate the endogenous (internal) processes in the neuron, which may be followed by the spike. Initiation of endogenous processes has a hysteretic nature. To simulate the interneuronal connections the operator interpretation of hysteresis nonlinearities is used, which was first introduced by M.A. Krasnosel’skii and A.V. Pokrovskii [8], later this approach was developed in respect to the systems containing links of the hysteretic nature [9]. The proposed model was chosen because of its biological validity. The regularities in the dynamics of neurons depending on the neural network structure and connections between neurons are also revealed.

3. MATERIALS AND METHODS

3.1. Basic model

Phenomenological Kashchenko-Mayorov model is based on othe flow of potassium and sodium currents through the neural membranes. It has been successfully used for simulation of annular neural structures in which excitation of neural progenitors was transmitted wavelike to neural emulators [1]. An important feature of this model is that the biological fact of delayed potassium currents against sodium ones is formalized.

In this model ф neuron activation function is described by delay differential equation:

$$\dot{u} = \lambda(-1 + f\mathrm{K}(u(t-1)) - f\mathrm{Na}(u))u, \qquad (1)$$

with the appropriate initial condition:

$$u(t)\big|_{-1 \le t \le 0} = \varphi(t), \varphi(t) \le Ce^{-1-\varepsilon} \qquad (2)$$

where λ – coefficient and C – constant, which are determined by the modeling views about biological properties of neural membrane, functions $f\mathrm{K}$ (u) and $f\mathrm{Na}$ (u) characterize potassium and sodium currents. As a rule, the following restrictions are applied to these functions:

$$\begin{aligned} & f\mathrm{K}(u) > 0, f\mathrm{Na}(u) > 0, \\ & -1 - f\mathrm{Na}(0) + f\mathrm{K}(0) > 0, \\ & f\mathrm{K}(u) < Cu^{-1-\varepsilon}, \\ & f\mathrm{Na}(u) < Cu^{-1-\varepsilon}, \end{aligned} \qquad (2')$$

where $\varepsilon > 0$, is a parameter determined by the modeling views.

Neuron can perceive both electric and chemical excitation. In case of electric excitation of the neuron activation function will satisfy the equation:

$$\dot{u} = \lambda(-1 + f\mathrm{K}(u(t-1)) - f\mathrm{Na}(u))u + g(t), \qquad (3)$$

where $g(t)$ – intensity of electric action.

In case of chemical excitation:

$$\dot{u} = \lambda(-1 + f\mathrm{K}(u(t-1)) - f\mathrm{Na}(u) + v(t))u. \qquad (4)$$

where $v(t)$ – intensity of chemical action.

According to [1] electric excitation effectively imposes generation period of neuron spikes, while chemical excitation is more effective during connection simulation and ensemble synchronization in a neural network.

3.2. Radchenko's neural memory model

Ability of the neural network to perceive and store information is related to learning. In classical artificial neural networks this problem is solved by weight matching and adjusting for connections between neurons according to definite regularities. Radchenko [7] suggested a new model, in which the structure of connections between neurons was based on the biological data and was of hysteretic nature. The same work presented the results of biological experiments, in the course of which special formations were detected around neural synapses (the point where the neuron perceives action) – metabotropic receptive cluster (MRC). This formation when stimulating the neuron can trigger endogenous (internal) chemical processes, thereby triggering a spike (the burst of energy, a surge in membrane potential). Hysteretic nature of endogenous process initiation depends on the type of action on the neuron (chemical or electric).

3.2.1. Chemical action on MRC

Chemical excitation of the neuron will trigger internal processes by means of MRC according to Figure 1:

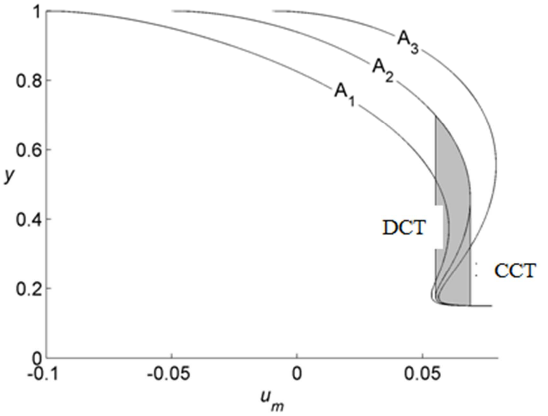

Figure 1: Reshaping of hysteresis curves characterizing the MRC dynamics at chemical exposure

Curves shown in Figure 1 are defined by the equation:

$$u = By\sqrt{\frac{1-k}{y-k}} - Ay^2, \qquad (5)$$

where y – a relative distance between charges in the MRC, $A = \frac{\lambda_0 \rho}{\varepsilon}, B = \lambda_0\sqrt{\frac{2Y}{\varepsilon}}$, ρ - charge concentration in the MRC. Experimentally calculated values of constants are determined by the equations:

$$\lambda_0 = 1.4 * 10^{-9}, \varepsilon = 8.85 * 10^{-11},$$
$$Y = 0.67 * 10^6.$$

The curve form (Figure 1) is defined by the coefficient $0 \le k \le 1$. From the biological viewpoint this parameter characterizes charge mobility in the MRC.

With increasing exposure of u_m on the MRC, a transition occurs from the upper part of the curve to its lower part (Figure 1). From the biological viewpoint chemical conformational transition (CCT) will occur, i.e. receptive cluster particles will consolidate, thereby accumulating energy. With the further growth of u_m the coefficient k approaches unity and, as a consequence, MRC consolidation will continue, and in this case the curve will vary from A_1 to A_3.

When decreasing u_m to the value sufficient for reverse transition from the lower part of the curve to the upper one, depolarization-induced conformational transition (DCT) occurs, and in this case increased by a factor of hundreds energy, accumulated in the MRC after CCT, will be directed on the neuron.

The described mechanism of the MRC operation makes it clear that intermittent chemical exposure is most efficient with increasing amplitude similar to the action on the part of neural ensemble, which attracts neurons, thereby increasing the gravity force.

3.2.2. Electric action

If the neuron is exposed to electric excitation, the equation (5) is transformed and takes the form:

$$u = By\sqrt{\frac{1-k}{y-k}}, \tag{6}$$

Reshaping of curves as a function of coefficient *k* is shown in the following figure [7]:

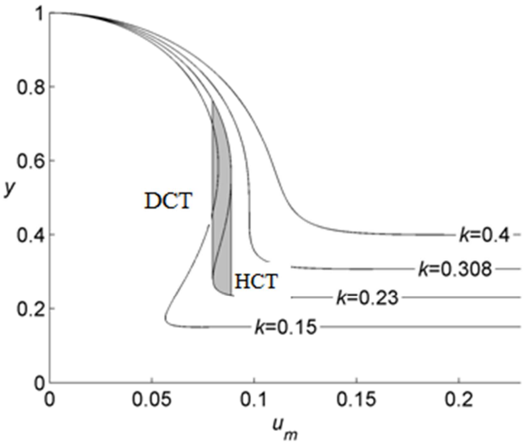

Figure 2: Reshaping of hysteresis curves characterizing MRC dynamics at electric exposure

When increasing the force of the electric action on the neuron, the relative distance between the charges in the MRC decreases and the hyperpolarization transition (HCP) occurs. When reducing the action force, similarly to the case with a chemical external action, DCT will occur.

As can be seen in Figure 2, with decrease in the mobility of charges in the MRC, which corresponds to an increase in *k*, MRC loses its hysteretic nature and the ability to activate the endogenous processes in the neuron. The charge mobility reduction can be caused by a long-term increase in u_m. Thus, a strong electric excitation attenuates the MRC impact on the neuron.

4. MODEL DESCRIPTION

Below we consider a neural network, individual elements of which are defined by the differential equations:

$$\dot{u}_i = \lambda\left(-1 + f\mathrm{K}\left(u_i\left(t-1\right)\right) - f\mathrm{Na}\left(u_i\right) + Y_i\right) + I_i, \tag{7}$$

where u_i – neural membrane potentials Y_i – excitation applied to the *i* neuron on the part of neural network, I_i – external action on the *i* neuron. Here Y_i is responsible for the chemical bond of the elements in the network, and I_i, in its turn, simulates electric external action on the neurons. Let us assume that

$$Y_i = \begin{cases} y_i, if\ \dot{y}_i > \gamma \\ 0, in\ other\ cases \end{cases}, \tag{8}$$

where γ – threshold rate of y_i growth. Here restriction is imposed to determine the CCT stage, when y_i is growing rapidly. The connection between neurons will be characterized in the quantity terms by the relation:

$$x_i\left(t\right) = \sum_{j=1}^{N} D_{ij} \frac{\int_{t-T}^{t} \left|u_i - u_j\right| ds}{\int_{t-T}^{t} u_i ds}, \tag{9}$$

where D_{ii} – coefficient of coupling force between *i* and HCT rons, *N* – total number of neurons. Parameter T is selected so that exactly one spike would fit in the appropriate interval. By definition, the value of the variable $x_i(t)$ increases with an integrated measure of neuron desynchronization.

In its turn $x_i(t)$ is an input of hysteresis quantizer $\Gamma\left[x_0, y_0\right]$, i.e. connection between y_i and x_i will be defined as follows:

$$\dot{y}_i = f\left(x_i, y_i\right), y_i\left(0\right) = y_0, \tag{10}$$

where function $f\left(x_i, y_i\right)$ is given by the relation:

$$f\left(x_i, y_i\right) = \chi\left(arctg\left(\frac{\xi\left(y_i-\rho\right)^3 + 4\left(1-k_i\right)\left(y_i-\rho\right)}{2}\right) + \frac{\pi}{2}\right) - x_i. \tag{11}$$

Here $\chi = 50, \xi = -4, \rho = 0.6$ – constants characterizing the form of the hysteresis loop. The contour line $f\left(x_i, y_i\right) = 0$ is shown in Figure 3.

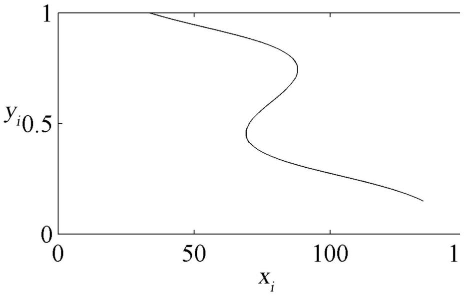

Figure 3: Contour line $f(x_i, y_i)=0$

Electric action from the MRC will be defined as follows:

$$I_i = \begin{cases} y_i, if \ \dot{y}_i > \gamma \\ 0, in\ other\ cases \end{cases}, \qquad (12)$$

$$\dot{y}_i = f\left(x_i, y_i\right), y_i\left(0\right) = y_0, \qquad (13)$$

$$x_i = Ae^{-\alpha t}\sin\left(t - g_i\right), \qquad (14)$$

where g_i – external action on the i neuron, A – a positive constant, α - coefficient determining the rate of losing MRC hysteresis properties. The quantizer hysteresis loop (10) with the intermittent input $x_i = 15sin\ (t) + 80$ is shown in Figure 4.

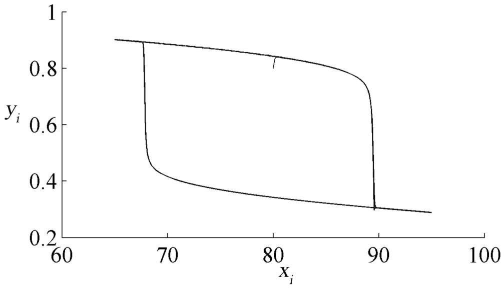

Figure 4: Dynamics of y_i with the intermittent input

As follows from the results [7] MRC is able to initiate a neural spike during DCT. In other words, while the action on the neuron in growing or remains constant, neuron will response to it without intermediaries. However, in case of reduction of this action the neural spike will be determined by MRC.

5. RESULTS

5.1. Research of the fully connected neural network with a 'weak' coupling

Let us consider a neural network consisting of three neurons with input actions (12) on each of its elements, assuming that external actions satisfy the condition:

$$0 \le g_i \le 1 \qquad (15)$$

In the example given below these parameters were supposed to be equal to 0.1, 0.5, 0.8, respectively.

At first let us consider the case when the coefficients D_{ij} in the relation (9) are sufficiently small, thereby neutralizing the influence of connection between individual neurons on the network dynamics in general. The simulation results are demonstrated in the following figure.

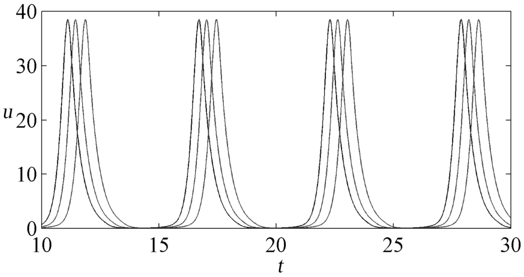

Figure 5: Dynamics of membrane potentials of three neurons with coupling coefficients

D_{ij}=0.01

The projections of the phase portrait on the relevant planes are given below:

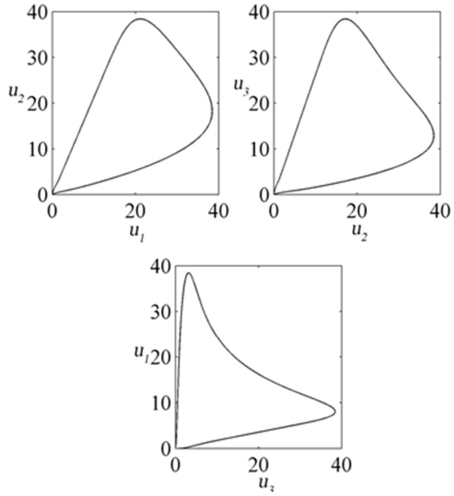

Figure 6: Phase portrait projections of (7) system at D_{ij}=0.01

As can be seen in the figures, the effect of synchronization is not available within a relatively long time period. In the absence of hysteresis in the input action when the external action was determined by the relation:

$$I_i = g_i, \qquad (16)$$

The network dynamics will have the form illustrated in Figures 7, 8.

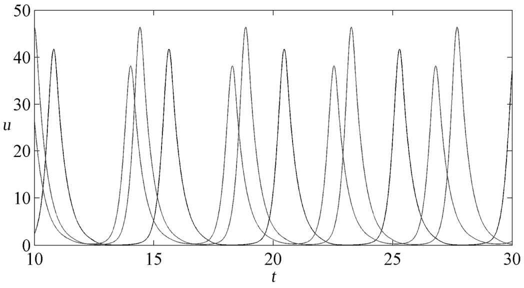

Figure 7: Dynamics of 3 neurons with the satisfied condition (16)

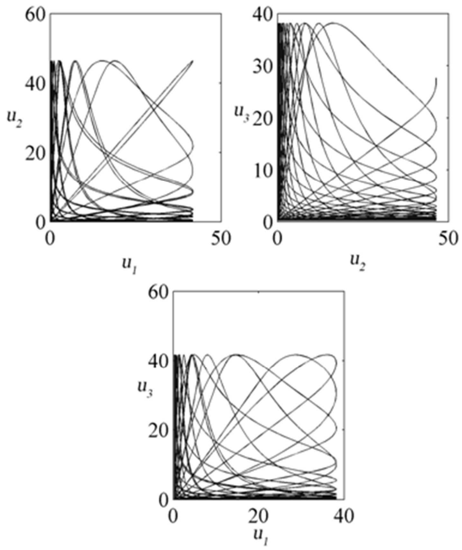

Figure 8: Phase portraits of the system (6) with the satisfied condition (16)

The comparison of the graphs 5-6 and 7-8 demonstrates that the hysteresis during the external action introduces the ordering effect in the neural network, but the synchronization is observed only for sufficiently large periods of the simulation.

5.2. Research of the fully connected neural network with a 'strong' constant hysteresis coupling

Let us consider a neural network consisting of three neurons defined by the system of differential equations (7). Assume the hysteretic connection between the network elements is active and invariable, i.e.

$$k_i = const, 0 < k_i < 1.$$

The results of numerical simulation of the described neural network are given below.

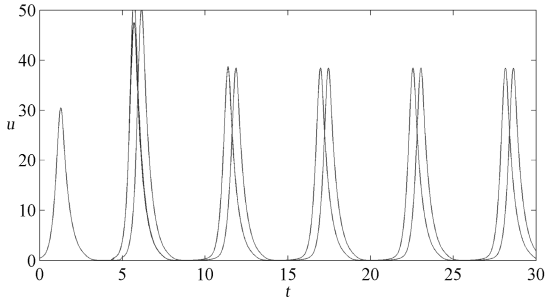

Figure 9: Dynamics neural network with input actions 0, 0.5, 1.

As can be seen in Figure 9, at constant hysteretic connection, one of the neurons is synchronized with the other, starting from the 10th second and full synchronization starts approximately from the 90th second. This fact complies with the biological data [10] – the neuron with the largest external action becomes the central element in the network, so the entire network is configured for perception of the strongest external action, and the other ones are ignored. According to [7] the return of the MRC hysteresis nature, and hence the possibility of storing information, occurs during sleep. Thus, simulation of the neural network (7) under the condition of $k_i = const, 0 < k_i < 1$ may characterize cerebration during sleep. According to [11] the main task of the brain during sleep is to consolidate the information received during wakefulness, and having a sufficiently intensive stimulus, it is this stimulus that will be remembered.

Dynamics of the neural network (7) consisting of three neurons at external actions

$$g_1=1, \quad 0.4 \leq g_2 \leq 1, \tag{17}$$

is illustrated by the Poincare section of phase trajectories with the plane u_1=0.4.

The result is shown in Figure 10 for t >10.

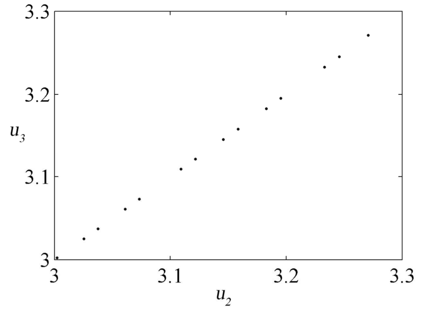

Figure 10: Poincare section for the system of 3 neurons with input actions

The Figure 10 demonstrates that $u_2(t) \approx u_3(t)$. Such behavior may be interpreted as occurrence of a central element in the network. It is notable that the detailed study of biological aspects of networks with a central element is presented in [12].

6. CONCLUSIONS

The neural network model considered in the paper shows that even with small number of neurons its dynamics complies with the biological data [13]. A weighty advantage of the proposed model is its 'ordered' response to external environment, which distinguishes it from the model described in [1]. This peculiarity is determined by the hysteretic connection between individual neurons, enabling to simulate a neural network with a set of elements, at the same time achieving neuron synchronization in the ensembles [14], [15]. Based on the fact that models of pulsed neural networks are effectively used in solving applied problems [16], it can be believed that the described biological neural network model may find application in the segmentation tasks and classification with efficiency exceeding standard numerical algorithms of data handling [15].

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