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ARTIFICIAL NEURAL NETWORKS MODELS OF TETRAHEDRAL FINITE ELEMENTS FOR SOLVING ANTENNA RADIATION PROBLEMS

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ABSTRACT

Abstract. In this work, an Artificial Neural Network (ANN) model of Finite Element Method (FEM) is developed and presented. The proposed method uses Hopfield NN that is able to efficiently represent the tetrahedral Finite Element (FE) functional that is formulated to interpolate the field radiation of a microstrip patch antenna. This study provide a robust method to combine NN with tetrahedral FE to provide a highly-reconfigurable memory-saving method, that help in reducing computational complexity of antenna radiation problems, and build a real-time analyzer of a reconfigurable antenna system implemented in a remote area. The proposed method shows a good match with the measured data and the analytical solutions.

Keywords: *Artificial Neural Network, Finite Element Method, Microstrip Patch Antenna.*

1. INTRODUCTION

Microstrip antennas became popular since 1970s, due to their ease of fabrication, small size, and their broadside radiation characteristics [1-3]. One Electromagnetic problem resulting from the analysis of microstrip antennas is to estimate the radiation pattern and input impedance of the antenna. Its solution requires solving Maxwell's equations where the fields should satisfy the Sommerfeld radiation condition [4]. Exact solutions to Sommerfeld integral formulations are available [5], however, they are too complicated to most practical applications, and occasionally it is necessary to resort to a numerical method such as the finite element method to get an approximate solution.

The FEM is a numerical technique for obtaining approximate solutions to boundary-value problems. The FEM has been applied to the analysis of electromagnetic problems for decades [6-8]. FEM is a powerful and efficient numerical technique to be used to analyze microstrip antennas; this is due to the fact that the FEM can handle arbitrary boundaries with extreme irregularities. Also, the domain of the problem can include media with varying material characteristics [9-11].

Finite elements can be developed to analyze not more than a single antenna and small arrays, where the resulting matrix equation can be solved without resorting to special solution techniques. However, for larger systems, the matrix equation requires more size and time to solve, therefore, the use of NN can sharply reduce the computational complexity of the problem in terms of memory usage and time consumption. Besides, the incoherent non-linearities connected with antenna radiation characteristics make NNs a good candidate for solving antenna radiation related problems [12].

The close topography structures between the FEM functional and the NN energy minimization formulation resulted in models that combine the two methods [12-14]. Modeling FE using ANN by minimizing energy functional was used for solving electrostatic problems [15,16]. Using feed-forward multilayer NN in the nodal finite element discretisation, the network is solved using a gradient algorithm [17]. Also, a locally connected Hopfield Network was constructed by using the energy functional of two dimensional FEM as the computing energy of the NN [18] for solving nonlinear magnetic field problems. A nodal rectangular element finite element model was embedded in NN along with steepest descent algorithm [14]. ANN was used not only in FEM formulations but also as a mesh auto-generator for nodal two dimensional FE [19-21] as well as for tetrahedral FE mesh [22]; these approaches start the

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solver process by using an initial mesh and refine it later by means of an adaptive meshing using NNs.

In this paper, a NN model for FEM is developed to analyse microstrip patch antennas. A nonuniform tetrahedral mesh for the antenna geometry is generated, and then a FEM functional is formulated to represent the mesh. Finally, a neural network is built to solve the functional using a memory saving technique. Hopfield Neural Networks (HNN) were used because of their ability to minimize the stored network energy [23], and therefore to minimize the energy functional in the finite element problems.

To show the validity and accuracy of the proposed method, several examples are considered with results compared to analytical solution or measured data.

2. PROBLEM FORMULATION

An Electromagnetic problem resulting from analysis of microstrip antennas is to predict the pattern of the radiated fields and input impedance of the antenna. This requires solving Maxwell equations along the boundary condition $\hat{n}\times\bar{E}=0$ on the cavity surface, where $\hat{n}$ is the normal unit vector, and $\bar{E}$ is the electric field intensity.

To ensure the uniqueness of the required solution, the fields must satisfy the Sommerfeld radiation condition at infinity [4]:

$$\lim_{r\to\infty} r\left[\nabla\times\bar{E}+jk_0\hat{r}\times\bar{E}\right]=0 \quad (1)$$

The electromagnetic problem resulting from solving Maxwell equations along with boundary conditions and Sommerfeld radiation condition is very complicated to be solved analytically. So, the FEM is to be used to find a numerical solution. To use FEM, this unbound space must first be truncated to some finite limit. Therefore, an approximate, first-order absorbing boundary condition is applied [8]:

$$\hat{n}\times\nabla\times\bar{E}+jk_0\hat{r}\times\hat{n}\times\bar{E}\approx 0 \quad (2)$$

where, k_0 is the free space wavenumber.

To use this condition, we multiply the vector wave equation for the electric field (the magnetic fields can be solved in similar steps) by a testing function $\vec{T}$ and then integrate over the volume V that is enclosed by the truncation surface S:

$$\iiint_V \vec{T}.\left[\nabla\times\left(\vec{\mu}_r^{-1}.\nabla\times\vec{E}\right)-k_0^2\vec{\varepsilon}_r.\vec{E}\right]dV$$
$$=-\iiint_V \vec{T}.\left[jk_0Z_0\vec{J}_{ex}+\nabla\times\left(\vec{\mu}_r^{-1}.\vec{M}_{ex}\right)\right]dV \quad (3)$$

where, Z_0 is the intrinsic impedance, $\vec{\mu}_r$ and $\vec{\varepsilon}_r$ are the relative permeability tensor and the relative permittivity tensor, respectively, $\vec{J}_{ex}$ is the electric current density for the excitation current, and $\vec{M}_{ex}$ is the magnetic current density.

Applying Gauss's theorem, equation (2), and some mathematical identities to equation (3), yields:

$$\iiint_V \left[\left(\nabla\times\vec{T}\right).\,\vec{\mu}_r^{-1}.\left(\nabla\times\vec{E}\right)-k_0^2\,\vec{T}\,.\,\vec{\varepsilon}_r.\vec{E}\right]dV$$
$$=\oiint_S\left[\left(\hat{n}\times\vec{T}\right).\,\vec{\mu}_r^{-1}.\left(\nabla\times\vec{E}\right)\right]dS-j\,k_0\oiint_S\left[\left(\hat{n}\times\vec{T}\right).\left(\hat{n}\times\vec{E}\right)\right]dS$$
$$=\iiint_V \vec{T}.\left[jk_0Z_0\vec{J}_{imp}+\nabla\times\left(\vec{\mu}_r^{-1}.\vec{M}_{imp}\right)\right]dV \quad (4)$$

To solve this equation using FEM, we divide the solution volume V enclosed by the truncation surface S into a mesh of N small subdivisions and then we associate a function v_i with one subdivision such that v_i is nonzero only inside its domain of definition. If we make the size of the subdivisions small enough, very simple interpolating functions can be given to the trial formulas since the part of the solution is represented. Trial functions v_i are not allowed to vary outside their domain of definition.

Here, nonuniform tetrahedral meshing is used. The longest side of any of the elements is set to not exceed one tenth of the wavelength.

The field on the six edges of each tetrahedral element is [8]:

$$\bar{E}^e(x,y,z)=\sum_{i=1}^{6}\bar{N}_i^e\,E_i^e \quad (5)$$

where E_i^e is the electric field at edge number *i* of the tetrahedral element *e*, and $\bar{N}_i^e$ is the vector basis function of edge number *i* within element *e*.

A tetrahedral element has four nodes and six edges; we label all the edges with a set of integers, and label each node locally (within the element) and globally (within the whole mesh) with another two sets. In this proposed method, degrees of freedom are assigned to the edges of the elements (rather than on the nodes); Using vector FEM

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decreases the occurrence of spurious solutions, besides, it easily deals with conducting and electric edges and corners. Therefore, another two sets are needed to number the edges locally and globally.

Table 1. Nodes numbering within mesh elements

Edge	Start node	End node
1	1	2
2	1	3
3	1	4
4	4	2
5	2	3
6	3	4

Nodes and edges are numbered from top to bottom and Counter Clock Wise (CCW) as shown in Figure 1. Edge numbers and its associated nodes numbers are defined in Table 1.

The basis function defined on edge i that connects nodes j and k is [24]

$$\vec{N}_i^e = \left(L_j \nabla L_k - L_k \nabla L_j\right) l_i^e \qquad (6)$$

where, l_i^e is the length of the edge, L_l and L_k are the coordinates of the nodes of the element with $i < k$ (to ensure CCW numbering of the elements).

The matrix equation that results from formulating the FEM problem requires large memory size and time to solve. The use of NN can sharply reduce the computational complexity of the problem.

Neural network architecture has at least three layers, one input layer, one output layer, and one or several hidden layers. The input layer is the only layer exposed to external signals; this layer transmits signals to the neurons in the next layer (or layers), which is the hidden layer (or layers). The hidden layer (or layers).) extracts connected features or patterns from the received signals. The features or patterns that are interpolated as important are then directed to the final layer of the network, the output layer.

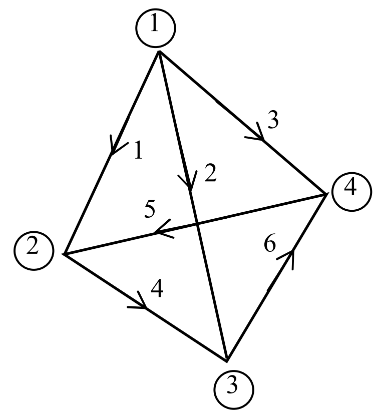
Degrees of freedom are assigned to the edges of the mesh elements. Each element is represented by three matrices, one connects its edges numbers locally and globally with its start and end nodes, one that connects elements vertices with their coordinates, and one that defines known and unknown edges.

For edge-based FEM problem, neural units are placed on all the FEM edges, and then connect them accordingly to the Hopfield NN energy function [23]

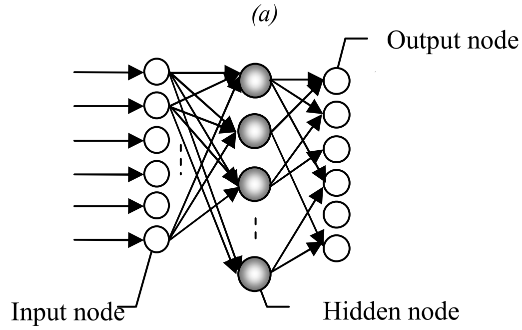
$$P = -\frac{1}{2}\sum\sum w_{ij} S_i S_j + \sum S_i T_i \qquad (7)$$

where, w_{ij} is the weight of the connection between neuron i to neuron j, S_i is the state of neuron i, and T_i is the threshold.

Figure 1b illustrates a neural network model for the tetrahedral element presented in Figure 1a.



(a)



(b)

Figure 1. Tetrahedral Element. A) Element Edge And Node Numbering. B) NN Model Of The Element.

3. RESULTS AND DISCUSSIONS

The First example is a slotted rectangular microstrip antenna [25]. The patch has 2 side slots. The antenna structure and dimensions are shown in Figure 2.

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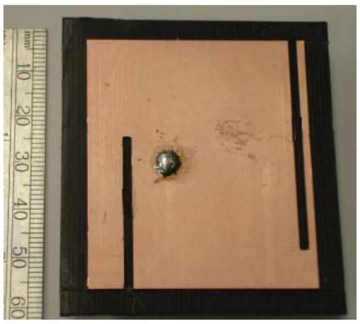
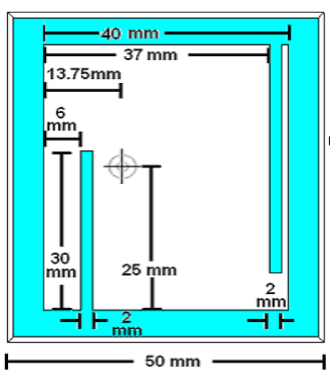
The antenna model is built on SolidMesh platform, with the dimensions and the truncation boundary determined. This software is capable of generating a nonuniform mesh. A Matlab code is written to translate the mesh into a Matlab data file. The Matlab translator works in two main steps; first it reads the information from SolidMesh output file and stores these data, then write a data Matlab file.



(a)



(b)

Figure 2. Slotted Rectangular Microstrip Antenna. A) Fabricated Model. B) Antenna Structure And Dimensions.

In the “read” step, the translator stores the general variables information about the mesh, and so, it reads: number of nodes, number of volume elements, and number of boundary faces. Some other variables that are not stored directly in SolidMesh file, such as type of tetrahedron element, number of nodes per element, will be extracted from the mesh itself.

With mesh data available, the FEM functional is formulated. Within each tetrahedral FEM element face (four faces for each element) six nodes, pointed on its edges, three of them are defined at the corners for the axial components $\phi_z(E_z \, or \, H_z)$, and three are defined in the middle of each edge for tangential components. To unify the edge numbering, y-components of the tangential fields are to be considered all positive. This numbering will be adopted during the FEM formulation.

Finally, the NN model is built, the convergence criteria are predetermined, and each output unit in the NN is activated. Using the back propagation algorithm, each error is then fed back to its corresponding NN input neuron unit through the feedback path. Each unknown edge value is updated using the training algorithm until the minimum set error is achieved

Figure 3. shows the S_{11} vs frequency of the antenna for the proposed Neuromodel of the FEM solver compared to the measured data of the fabricated antenna.

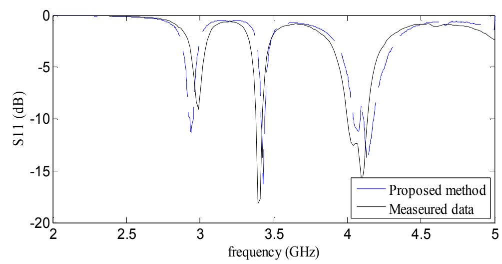


Figure 3. Frequency Response Of The Slotted Rectangular Microstrip Antenna. Proposed Method (Dashed Line), Measured Data (Solid Line)

Another example is a simple rectangular microstrip antenna with inset feeding; this one is chosen since its response can be calculated analytically.

The antenna has a length $L = 9.06\,mm$, a width $W = 11.86\,mm$, and a substrate height of $h = 1.588\,mm$ with relative permittivity $\varepsilon_r = 2.2$. The feed point is placed at distance χ from one end of the patch. Figure 4. shows the antenna structure and dimensions

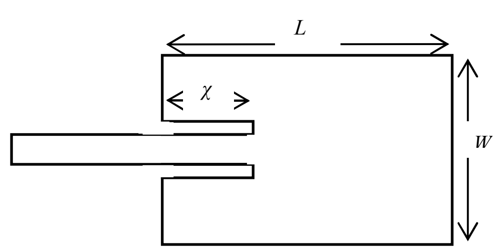


Figure 4. Antenna Structure And Dimensions

The FEM mesh has 1847 tetrahedral elements; using the NN reduces requirements on storing the sparse matrix of FEM functional. The NN model

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solution converges in 2214 iterations with a stop error of 10^{-5}.

To compare the proposed method to analytical solution, the input resistance can be calculated using the network model from [26]:

$$Z_{in}(\chi)=\frac{1}{2G}\left[\cos^2\beta\chi+\frac{G^2+B^2}{Y_c^2}\sin^2\beta\chi-\frac{B}{Y_0}\sin 2\beta\chi\right] \quad 8)$$

where, G and B are the conductance and the susceptance of each antenna radiating slots respectively, β is the propagation constant.

Figure 5. shows the input impedance versus the feed point distance from the antenna edge, for both the proposed method and the analytical solution.

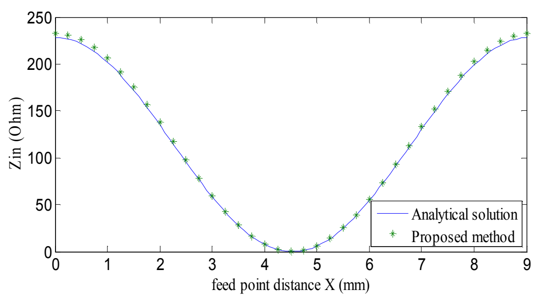

Figure 5. Input resistance vs feed point distance. proposed method (stars), analytical solution (solid line).

3. CONCLUSIONS

A new method to analyse microstrip patch antennas by using neural network models of edge FEM with tetrahedral elements was proposed. The method is based on building the FEM functional from a nonuniform tetrahedral mesh. The functional is then modeled using a HNN. Minimizing the energy of the NN is equivalent to solving FEM functional. The proposed method showed close agreement with the analytical solutions and measured data.

The need for this work is to build a "smart antenna controller". NN-Tetrahedral-FE scheme allows a real-time analysis of an antenna system in a remote area; where a fault is hard to reach and fix. The NN Tetrahedral FE, if implemented on a reconfigurable board, can become a "brain" that detect faults and re-program itself to re-control the switches and overcome the faults.

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