Journal of Theoretical and Applied Information Technology
10th November 2015. Vol.81. No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

DEVELOPMENT OF THE MATHEMATICAL MODEL FOR ANALYSIS OF LEVELING POINTS STABILITY OF REFERENCE LEVELING GRID

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ABSTRACT

The main drawback of the current methods used for analyzing stability of leveling points in the reference leveling grid have been defined. The problem of developing a mathematical model of leveling points in the reference leveling grid is being solved. Equal accuracy of measurement in different cycles is expected. Possible subsidences between the reference points are represented by a polynomial of the second degree. In solving this problem, the polynomial coefficients are calculated using the least squares method. Accuracy of the obtained mathematical model used to make a conclusion about the need to identify elevations between the reference points with high accuracy (up to tenths of a millimeter) has been estimated.

Keywords: *Hyperbolic Equation, Nonlocal Integral Condition, Bessel Operator, Mathematical Model, Reference Point, Leveling Grid, Elevation, Subsidence, Polynomial, Differentiation, Model Error*

1. INTRODUCTION

Establishing a stable altitudinal leveling grid is an important milestone in the organization of observation over deformations of buildings and structures, and is paid a special attention. Stability of reference points of initial altitude is described in works of a number of domestic and foreign authors [1]-[7], [15], [16].

As a rule, high-rise leveling grid is made of three or more reference points [6], [13], [14], [18], [20].

Reference points in the leveling grid should be placed:

- away from passages, underground utilities, storage and other areas where vibrations from traffic may occur;

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

- outside the area where pressure is distributed to foundations from a controlled erected building or structure;

- outside the area of influence of newly constructed buildings and structures. The distance between the initial altitudinal foundation and the structure should be more than 150 m. Reference points of the altitudinal grid should be stable throughout the life of the structure being monitored [8]. However, due to the influence of a number of anthropogenic and natural factors, reference points lose their stability [1], [5], [6], [10]-[12], [22], [23].

If stability of the altitudinal grid is compromised, we can get a distorted picture of the structures in deformed state.

Therefore, finding the most stable reference point (or a group of stable reference points) in the altitudinal grid is an important task for organization of structures deformations monitoring. To monitor the stability of reference points in the grid, precision level runs are regularly placed between them. Relative changes in elevations between the reference points do not provide information about their stability. Therefore, to identify the mobile reference point, the following condition is checked: the number of mobile reference points should be less than half of the total number of points in the altitudinal foundation. For the purpose of mathematical processing of repeated measurements results, the problem of choosing an initial reference plane is solved. With respect to such a plane, elevations and displacement of the reference points are defined [10], [11], and their stability is assessed. Measured values of vertical displacements contain both the component of actual vertical displacements and the component of measurement error. If the data obtained from measurement of vertical displacements do not exceed the measurement errors, a conclusion is made about absence of displacements and stability of reference points [11].

Stability of reference points in the initial leveling base was studied by domestic and foreign scientists; several methods for solving the problem have been proposed [1], [2], [4], [5], [7], [8], [10], [16], [18], [19], [20]. However, all proposed methods have one common drawback - they are poorly adapted for computer analysis.

Taking into account the urgency of the problem, we set the task to develop a mathematical model suitable for analyzing stability of the reference points in the leveling grid adapted for computer analysis.

2. METHODS OF SOLVING THE PROBLEM

Possible subsidence between the base reference points can be expressed as a difference of equalized values of elevations h_i in the i-th and the zero-th cycle:

$$s_i = h_i - h_0. \tag{1}$$

Let us present the possible subsidence in the form of a second-degree polynomial:

(2)

where $i = 1, 2, 3, \ldots;$

a, b and are the polynomial coefficients; c

T_i is time of measurements in cycle with number i;

T_0 is time of measurements in the zero cycle;

Δ_i is the mathematical model error.

From equation (2) we find the error of the mathematical model

$$\Delta_i = -a - b\left(T_i - T_0\right) + c\left(T_i - T_0\right)^2 + S_i. \tag{3}$$

Polynomial coefficients can be found from condition:

$$F = \sum_{i=1}^{n} \Delta_i^2 \to \min.$$

Then we get

$$\begin{cases} \dfrac{\partial F}{\partial a} = -\sum\limits_{i=1}^{n}\left[S_i - a - b\left(T_i - T_0\right) - c\left(T_i - T_0\right)^2\right] = 0; \\ \dfrac{\partial F}{\partial b} = -\sum\limits_{i=1}^{n}\left[S_i - a - b\left(T_i - T_0\right) - c\left(T_i - T_0\right)^2\left(T_i - T_0\right)\right] = 0; \\ \dfrac{\partial F}{\partial c} = -\sum\limits_{i=1}^{n}\left[S_i - a - b\left(T_i - T_0\right) - c\left(T_i - T_0\right)^2\left(T_i - T_0\right)^2\right] = 0. \end{cases} \tag{4}$$

The system of equations (4) can be written as:

$$\begin{cases} na + b\sum\limits_{i=1}^{n}\left(T_i - T_0\right) + c\sum\limits_{i=1}^{n}\left(T_i - T_0\right)^2 - \sum\limits_{i=1}^{n} S_i = 0; \\ a\sum\limits_{i=1}^{n}\left(T_i - T_0\right) + b\sum\limits_{i=1}^{n}\left(T_i - T_0\right)^2 + c\sum\limits_{i=1}^{n}\left(T_i - T_0\right)^3 - \sum\limits_{i=1}^{n} S_i\left(T_i - T_0\right) = 0; \\ a\sum\limits_{i=1}^{n}\left(T_i - T_0\right)^2 + b\sum\limits_{i=1}^{n}\left(T_i - T_0\right)^3 + c\sum\limits_{i=1}^{n}\left(T_i - T_0\right)^4 - \sum\limits_{i=1}^{n} S_i\left(T_i - T_0\right)^2 = 0. \end{cases} \tag{5}$$

From equation (5) we can determine the polynomial coefficients (2). Since measurement cycles repeat over a fixed period, it can be written as

Journal of Theoretical and Applied Information Technology
10th November 2015. Vol.81. No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\frac{T_i - T_0}{\Delta T} = i.$$

The system of equations (5) can be written as:

$$\sum_{i=1}^{n}\left(T_i - T_0\right) \to \sum_{i=1}^{n} i = \frac{n(n+1)}{2}; \quad (7)$$

$$\sum_{i=1}^{n}\left(T_i - T_0\right)^2 \to \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}; \quad (8)$$

$$\sum_{i=1}^{n}\left(T_i - T_0\right)^3 \to \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}; \quad (9)$$

$$\sum_{i=1}^{n}\left(T_i - T_0\right)^4 \to \sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)\left(3n^2+3n-1\right)}{30}. \quad (10)$$

With regard to equations (7)-(10), the system of equations (5) can be represented as:

$$\begin{cases} na + \frac{n(n+1)}{2}b + \frac{n(n+1)(2n+1)}{6}c - \sum_{i=1}^{n} S_i = 0; \\ \frac{n(n+1)}{2}a + \frac{n(n+1)(2n+1)}{6}b + \frac{n^2(n+1)^2}{2}c - \sum_{i=1}^{n} iS_i = 0; \\ \frac{n(n+1)(2n+1)}{6}a + \frac{n^2(n+1)^2}{4}b + \frac{n^2(n+1)(2n+1)\left(3n^2+3n-1\right)}{30}c - \sum_{i=1}^{n} i^2 S_i = 0. \end{cases} \quad (11)$$

After some transformations (11), we get

$$\begin{cases} a + \frac{n+1}{2}b + \frac{(n+1)(2n+1)}{6}c - \frac{1}{n}\sum_{i=1}^{n} S_i = 0; \\ a + \frac{2n+1}{3}b + \frac{n(n+1)}{2}c - \frac{2}{n(n+1)}\sum_{i=1}^{n} iS_i = 0; \\ a + \frac{3n(n+1)}{2(2n+1)}b + \frac{3n^2+3n-1}{5}c - \frac{6}{n(n+1)(2n+1)}\sum_{i=1}^{n} i^2 S_i = 0, \end{cases} \quad (12)$$

Let us subtract the first equation from the second and third:

$$\begin{cases} \frac{n-1}{6}b + \frac{n^2+1}{6}c + \frac{1}{n}\sum_{i=1}^{n} S_i - \frac{2}{n(n+1)}\sum_{i=1}^{n} iS_i = 0; \\ \frac{n^2}{2(2n+1)}b + \frac{(8n+11)(n-1)}{30}c + \frac{1}{n}\sum_{i=1}^{n} S_i - \frac{6}{n(n+1)(2n+1)}\sum_{i=1}^{n} i^2 S_i = 0. \end{cases} \quad (13)$$

or

$$\begin{cases} b + (n+1)c + \frac{6}{n(n-1)}\sum_{i=1}^{n} S_i - \frac{12}{n\left(n^2-1\right)}\sum_{i=1}^{n} iS_i = 0; \\ b + \frac{(8n+11)(2n+1)}{15(n+1)}c + \frac{2(2n+1)}{n\left(n^2-1\right)}\sum_{i=1}^{n} S_i - \frac{12}{n(n+1)\left(n^2-1\right)}\sum_{i=1}^{n} i^2 S_i = 0. \end{cases} \quad (14)$$

After subtracting the first equation of the system of equations (14) from the second one, we obtain:

$$\frac{n^2-4}{15}c - \frac{2(n+2)}{n(n-1)}\sum_{i=1}^{n} S_i + \frac{12}{n(n-1)}\sum_{i=1}^{n} iS_i - \frac{12}{n\left(n^2-1\right)}\sum_{i=1}^{n} i^2 S_i = 0.$$

Accordingly,

$$c = \frac{30}{n(n-1)(n-2)}\sum_{i=1}^{n} S_i - \frac{180}{n(n-1)\left(n^2-4\right)}\sum_{i=1}^{n} iS_i + \frac{180}{n\left(n^2-1\right)\left(n^2-4\right)}\sum_{i=1}^{n} i^2 S_i.$$

(6)

(15)

Then

$$b = -(n+1)c - \frac{6}{n(n-1)}\sum_{i=1}^{n} S_i + \frac{12}{n\left(n^2-1\right)}\sum_{i=1}^{n} iS_i.$$

With regard to (15), the coefficient b is determined as:

$$b = -\frac{18}{n(n-1)(n-2)}\sum_{i=1}^{n} S_i + \frac{12(2n+1)(8n+11)}{n\left(n^2-1\right)\left(n^2-4\right)}\sum_{i=1}^{n} iS_i - \frac{180}{n(n-1)\left(n^2-4\right)}\sum_{i=1}^{n} i^2 S_i. \quad (16)$$

The ratio α can be calculated using the following formula

$$\alpha = \frac{n+1}{2}b - \frac{(n+1)(2n+1)}{6}c + \frac{1}{n}\sum_{i=1}^{n} S_i, \quad (17)$$

or, taking into account (15) and (16), the value $\alpha$is equal to:

$$\alpha = \frac{3\left(3n^2+3n+2\right)}{n(n-1)(n-2)}\sum_{i=1}^{n} S_i - \frac{18(2n+1)}{n(n-1)(n-2)}\sum_{i=1}^{n} iS_i + \frac{30}{n(n-1)(n-2)}\sum_{i=1}^{n} i^2 S_i. \quad (18)$$

Let us estimate the accuracy of coefficients a, band c. To do so, we can accept:

$$n+1 \approx n; \quad 2n+1 \approx 2n,$$

Then system (12) is converted to the form:

$$\left.\begin{aligned} a + \frac{n}{2}b + \frac{n^2}{3}c - \frac{1}{n}\sum_{i=1}^{n} S_i = 0 \\ a + \frac{2n}{3}b + \frac{n^2}{2}c - \frac{2}{n^2}\sum_{i=1}^{n} iS_i = 0 \\ a + \frac{3n}{4}b + \frac{3n^2}{5}c - \frac{1}{n^3}\sum_{i=1}^{n} i^2 S_i = 0 \end{aligned}\right\}. \quad (19)$$

Subtracting the first equation from the second and third we get:

$$b + nc + \frac{6}{n^2}\sum_{i=1}^{n} S_i - \frac{12}{n^3}\sum_{i=1}^{n} iS_i = 0; \quad (20)$$

$$b + \frac{16n}{15}c + \frac{4}{n^2}\sum_{i=1}^{n} S_i - \frac{12}{n^4}\sum_{i=1}^{n} i^2 S_i = 0. \quad (21)$$

Journal of Theoretical and Applied Information Technology
10th November 2015. Vol.81. No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

After subtracting equation (20) from (21) we obtain:

$$c=\frac{30}{n^3}\sum_{i=1}^{n}S_i-\frac{180}{n^4}\sum_{i=1}^{n}iS_i+\frac{180}{n^5}\sum_{i=1}^{n}i^2S_i=0. \quad (22)$$

Equation (20) can be written as

$$b=-nc-\frac{6}{n^2}\sum_{i=1}^{n}S_i+\frac{12}{n^3}\sum_{i=1}^{n}iS_i. \quad (23)$$

Taking into account (22) and (23) let us write

$$b=-\frac{36}{n^2}\sum_{i=1}^{n}S_i-\frac{168}{n^3}\sum_{i=1}^{n}iS_i+\frac{180}{n^4}\sum_{i=1}^{n}i^2S_i. \quad (24)$$

From (19), (22) and (24) we define a as:

$$a=\frac{9}{n}\sum_{i=1}^{n}S_i-\frac{144}{n^2}\sum_{i=1}^{n}iS_i+\frac{150}{n^3}\sum_{i=1}^{n}i^2S_i. \quad (25)$$

Coefficient a can be represented as:

$$a=\sum_{i=1}^{n}\left(\frac{9}{n}-\frac{144i}{n^2}+\frac{150i^2}{n^3}\right)S_i. \quad (26)$$

Let us differentiate equation (26)

$$da=\sum_{i=1}^{n}\left(\frac{9}{n}-\frac{144i}{n^2}+\frac{150i^2}{n^3}\right)dS_i. \quad (27)$$

We suppose that measurements in the cycles are of equal accuracy and proceed from differentials to the mean square error:

$$m_a^2=m_S^2\sum_{i=1}^{n}\left(\frac{9}{n}-\frac{144i}{n^2}+\frac{150i^2}{n^3}\right)^2. \quad (28)$$

Or

$$m_a^2=m_S^2\sum_{i=1}^{n}\left(\frac{81}{n^2}-\frac{2592i}{n^3}+\frac{23436i^2}{n^4}-\frac{43200i^3}{n^5}+\frac{22500i^4}{n^6}\right). \quad (29)$$

From equation (29) we obtain:

$$m_a^2=m_S^2\left[\frac{81}{n}-\frac{1296(n+1)}{n^2}+\frac{3906(n+1)(2n+1)}{n^3}-\frac{10800(n+1)^2}{n^3}+\frac{750(n+1)(2n+1)(3n^2+3n-1)}{n^5}\right]. \quad (30)$$

From equation (30), suggesting that $n\approx 1;\ n+1\approx n;\ 2n+1\approx 2n;$ $3n^2+3n-1\approx 3n^2$, we get

$$m_a=\frac{17,2}{\sqrt{n}}m_S. \quad (31)$$

From expression (24) we obtain:

$$b=-\sum_{i=1}^{n}\left(\frac{36}{n^2}+\frac{168i}{n^3}-\frac{180i^2}{n^4}\right)S_i.$$

Find the mean square error

$$m_b^2=m_S^2\sum_{i=1}^{n}\left(\frac{36}{n^2}+\frac{168i}{n^3}-\frac{180i^2}{n^4}\right)^2. \quad (32)$$

Or

$$m_b^2=m_S^2\sum_{i=1}^{n}\left(\frac{1296}{n^4}+\frac{12096i}{n^5}+\frac{15264i^2}{n^6}-\frac{30240i^3}{n^7}+\frac{32400i^4}{n^8}\right). \quad (33)$$

(33) we can obtain:

$$m_b^2=m_S^2\left[\frac{1296}{n^3}+\frac{6048(n+1)}{n^4}+\frac{2544(n+1)(2n+1)}{n^5}-\frac{7560(n+1)^2}{n^5}+\frac{1080(n+1)(2n+1)(3n^2+3n-1)}{n^7}\right]. \quad (34)$$

Expression (33) can be transformed to:

$$m_a=\frac{106,5}{n\sqrt{n}}m_S. \quad (35)$$

Likewise, we assess accuracy of coefficient c. Let us represent (22) in form:

$$c=\sum_{i=1}^{n}\left(\frac{30}{n^3}-\frac{180i}{n^4}+\frac{180i^2}{n^5}\right)S_i. \quad (36)$$

Find the mean square error

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$m_c^2 = m_S^2 \sum_{i=1}^{n} \left(\frac{30}{n^3} - \frac{180i}{n^4} + \frac{180i^2}{n^5} \right)^2 . \quad (37)$$

Or

$$m_c^2 = m_S^2 \sum_{i=1}^{n} \left(\frac{900}{n^6} - \frac{1080i}{n^7} + \frac{33480i^2}{n^8} - \frac{32400i^3}{n^9} + \frac{32400i^4}{n^{10}} \right). \quad (38)$$

From (38) we obtain:

$$m_c^2 = m_S^2 \left[\frac{900}{n^5} - \frac{540(n+1)}{n^6} + \frac{5580(n+1)(2n+1)}{n^7} - \frac{8100(n+1)^2}{n^7} + \frac{1080(n+1)(2n+1)(3n^2+3n-1)}{n^9} \right]. \quad (39)$$

Expression (39) can be transformed to:

$$m_c = \frac{99,5}{n^2 \sqrt{n}} m_S .$$

3. RESULTS

To eliminate the influence of the measurement (leveling) error, possible vertical displacement between the reference points are presented in the form of equalized elevations. In addition, it is assumed that measurements in different cycles were performed with equal accuracy. This condition can be satisfied even in the most tough loading conditions in the area of the studied object, for example, infrastructure elements such as overpasses, pipelines, fire suppression systems, and so on, if the method of high-precision leveling with a short reticle beam is used for the definition of the reference points in the leveling grid. For the study, we used the second degree polynomial (formula 2) as the initial model of possible vertical displacement. To calculate the coefficients of the polynomial we used the principle of least squares - condition of the minimum sum of squared errors of the mathematical model. Thus, as a result of our theoretical research, we completed our task - we developed a mathematical model for analyzing stability of reference points in the leveling grid adopted for computer analysis, which consists of formulas (2), (15), (16) and (18) that can be used for identifying the most stable reference points. To analyze the accuracy of the model coefficients, we obtained respective formula (31), (34) and (39). Assessment of accuracy shows that in order to achieve high accuracy of approximation, it is necessary to either define the vertical displacements with the accuracy of a few tenths of a millimeter, or perform many repeated measurements, desirably in more than 10 cycles.

4. DISCUSSION

Scientific literature shows many ways to analyze stability of reference points in an altitudinal base, which can be divided into two groups:

1) methods based on the principle of constant average level of all (or a group of) reference points;

2) methods based on the principle of constant elevation of one of the most stable reference points.

The methods based on the principle of constant elevation of one of the most stable reference points include methods of A.D. Soloviev [9], A. Costachel [18], G.J. Botyan [4], L.I. Serebryakova [8], I.V. Runov [7], and methods of mathematical statistics [10],[11].

The methods based on the principle of constant average elevation of the reference point in the leveling grid include methods such as those of P. Marcak [20], [22], V.F. Chernikov and B. Gotz [17], [19]. Let us briefly examine each of them. http://northumbria.summon.serialssolutions.com/search?s.dym=false&s.q=Author:

In the method of A.D. Soloviev [9] the initial elevations are compared to the elevations obtained in the current cycle. First, the elevation difference is calculated between the first reference point and all others, and then between the second reference point and all others (except for the first one), and so on. The disadvantage of this method is the impossibility to use it when the number of reference points with initial elevation is more than three.

In case of the method of A. Costachel, it is assumed that after adjusting the leveling grid, the equalized elevations will contain only the component of vertical displacement of reference points. In course of method implementation, the following works are performed:

The leveling grid is equalized;

The most stable reference points in the current cycle of leveling are determined;

Elevations of reference points are determined in relation to the initial height of the stable reference point and equalized elevations;

The level of instability is determined for each reference point, and all unstable reference points are excluded from the grid.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

The most stable reference point is considered to be the one for which the sum of elevations fluctuations in relation to other reference points is minimum. The altitude of such a reference point (from the first cycle) is taken as the initial one, in relation to which the altitude of other reference points in current measurement cycle is calculated. The disadvantage of the method of A. Costachel is that it cannot be used for large values of vertical displacement of the reference points.

Similar in the content to the method of A. Costachel is the method of G.K. Botyan. In this method, the most stable reference point is considered to be the one for which the sum of elevation changes squared relative to the other reference points in different cycles is minimal. The reference points for which this value is maximum are excluded from the number of items of the initial altitudinal base. This method can be used for analyzing stability of reference points exposed to vertical movement in the form of subsidence only.

The assumption that the reference points are experiencing vertical displacement only in the form of subsidence is the basis for the method of L.I. Serebryakova. In the areas where reference points are subject to vertical displacements in the form of lifting (bulging), this method is unacceptable [5].

In the method of I.V. Runov, the most stable reference point is determined from the relative displacement of the reference points. Results obtained using the method of I.V. Runov coincide with the results obtained using the methods of A. Costachel and G.K. Botyan.

The methods of mathematical statistics for analyzing stability of reference points can be used in case of many measurement cycles, which is rarely experienced in practice. With the use of such methods it is impossible to define stable reference points in each successive measurement cycle.

A common drawback of all methods based on the principle of a constant level of one of the most stable reference points is the impossibility to detect local displacement of the initial reference point. As a result, we get unreliable information about the magnitude and direction of displacement of all other reference points in the altitudinal grid. Judging by this criterion, more acceptable for practical use are the methods based on the principle of the constant average elevation of the reference points in the leveling grid. A typical method in this group is the method of G.P. Marchak. Stability of the reference points in this method is determined on the basis of analysis of elevations fluctuations between reference points in the first and the current measurement cycles according to a certain method. In the method of V.F. Chernikov, the medium plane of relevance is taken for the initial plane. The elevation of such a plane is defined as the arithmetic average of the elevations of stable reference points in the leveling grid. In the method of B. Gotz, the results of the first and the current measurement cycles are equalized first. Next, elevations of the reference points are determined in the current cycle in relation to the initial reference point, and the average fluctuations of reference points elevations. Next, the most probable elevations of reference points are determined in the current cycle, as well as adjustments to the elevations of the reference points.

The above methods that are based on the principle of constant average elevation of the reference points in the leveling grid give similar values of adjustments in the elevation of the reference points by cycles.

As was noted above, all the above methods of analyzing stability of reference points in the leveling grid have one common disadvantage, i.e., they are poorly suited for computer analysis. The proposed mathematical model of analyzing reference points stability in the reference leveling grid solves this problem.

5. CONCLUSION

As can be seen from the foregoing, the method of analyzing stability of reference points has been repeatedly studied by many scientists in Russia and other countries. In this regard, no additional practical research of the analyzed methods is required. The goal of creating a mathematical model for analyzing stability of a reference leveling network adapted for computer analysis has been achieved. The method of calculating polynomial coefficients has been developed with the use of a strict mathematical apparatus. The approximate methods of calculation were only used for determining the simple working formulas for analyzing the accuracy of calculating polynomial coefficients. The authors of the article have analyzed the accuracy of calculating the root-mean-square errors by formulas (31), (35) and (40), which showed that inaccuracy of these formulas for $n \geq 6$ does not exceed 10% of the calculated value. This is a good indicator for the root-mean-square value.

As the prospect of further research on this topic, the authors consider it practical to study the possibility of using abandoned wells in oil-and-gas

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

field as reference points for the initial leveling network, using the developed mathematical model. Such research may be necessary for exhausted (used) oil-and-gas fields in the Chechen Republic.

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