

FORMAL ANALYSIS OF WEIGHTED LONGITUDINAL ROUTERS NETWORK VIA SCALE-FREE NETWORK. A CASE STUDY

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ABSTRACT

The process of identifying the central nodes in complex networks research has remained interesting and very important issue for network analysis. The identification of main nodes in the network can lead to many answers for the solution of security and other problems depending on the type of complex networks under analysis. Different topological metrics of the network can be used to locate the major nodes in the network but the degree and betweenness (Load) centralities perform very important role in evolution and communication of nodes in growing networks. Unfortunately, these metrics have been analyzed in different complex systems mainly either on the bases of the number of links to nodes in the network or with much focused from the perspective of weights of links. Therefore, locating the main nodes in the network not only depends on links but majorly on weight of links. Routers network of the internet is an example of scale-free nature which follow power-law distribution and causes inhomogeneous structure with some nodes with large number of links while many with a few. Further, in this type of distribution few nodes become very important. In this paper, we analyze the behavior of routers network by using two metrics of centralities with weighted and un-weighted links based on the dataset of PTCL routers network in Pakistan. Furthermore, by using centralities measures we try to show that weight of links is important as compare to number of links by following the concept from “rich get richer” to “fit get richer” in routers network. Moreover, we prove that weighted routers network is very close to scale-free networks as compared to un-weighted, and due to this phenomenon these networks sustain their robustness.

Keywords: *Scale-free networks; weighted networks analysis; load distribution; degree distribution; shortest distance; graph theory*

1. INTRODUCTION

The concept of scale-free nature of many artificial (man-made) or natural complex networks in the world is extensively studied during the last decade [1-3]. For example, in the category of technological networks we have Internet [10, 26], World Wide Web [13], and electrical power grid [14] networks. Similarly, in the category of transportation networks we have the networks of airways [15]. Also, in the category of natural networks the, ecological [16] and biological [17] networks are scale-free networks. The complex structure of these networks have introduced and created a great thrust and interest among the

researchers to investigate the internal or hidden organizing principles and rules behind the emergence of these complex networked systems, and their resilience towards breakdown.

Complex networks formation has been seen and observed in many fields of life due to the availability of vast amount of data gathered and analyzed with the help of high processing and storage capabilities of modern processing systems. Unfortunately, in spite of the availability of a huge amount of data, there are still many microscopic phenomenon to be needed to fully understand the complexity level and dynamic behavior of these networks.

The most convenient way to represent any network is through graph theory. In graph theory, vertices represent the nodes and edges represent the links in the network. For example, the complex network of internet is a network of domain or routers. The domain or routers are nodes and their physical connections are the links or edges in between them. Similarly, air transportation network is the network of airports, in which airports are nodes and routes are the links in this complex network. Two Hungarian mathematician Erdos and Renyi (ER) [4] in 1959 introduced the concept of random graph theory in the classical mathematical graph theory. According to them, complex networks topology can be best described by random graph. For example, if we have a large number of nodes in the network with the equal probability p and we connect pair of nodes with links. The ultimate outcome will be physical example of ER random graph. Almost forty years this theory was used as an appropriate means to understand complex networks [17]. In this way, the network will be homogeneously connected with high length paths and low clustering coefficient. These features show that the connectivity distribution is homogeneous in networks as shown in figure 1. In 1999, Barabasi with the help of World Wide Web as a network observed that it does not follow random graph connections rather, it is scale-free graph and its degree distributions follow power-law form as given in [5]

$$P(k) \sim (k)^{-\gamma} \quad (1)$$

It means, the node degree k and the number of links a node can have, follows the power-law distribution relation where the degree exponent γ has been measured as well as confirmed in a number of research studies to be approximately 2.1 [9]. The Figures 2(a) and (b) shows power-law distribution on linear and log-log scales. Also, Faloutsos et al. [10] has shown that, from the autonomous systems perspective of internet it is scale-free network. The power-law implies that few nodes in the network can have large number of links whereas majority nodes have very small number of connections.

Therefore, for explaining the power-law distribution Barabasi and Albert (BA) [5] proposed the model that is known as BA model in short. According to them, there are two main features of scale-free behavior of any complex network. These features are continuous growth and preferential node attachment. Later they noticed this behavior in many real world networks. Due to these two features, the node with many connections tends to

have more chances to acquire links in future like rich get richer phenomenon. Therefore, this is the reason for creation of giant nodes in the network with high node degree distribution. Both these factors influence the creativity of inhomogeneous or heterogeneous structure of network topology with potential hubs and make networks more robust under random node failures.

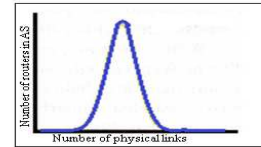


Figure:1 Bell shape curve distribution of routers linkages

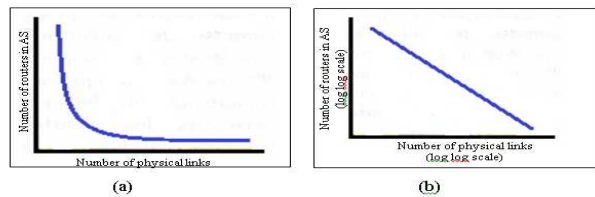


Figure: 2 Power law distributions of routers linkages

Links or edges in between vertices of networks are very fundamental, but all edges/connections are not equal in importance. For example for two nodes with equal number of links, the node with more powerful links should be more important in network as compared to the one with relatively weak links. Whereas, this phenomenon has not been considered in traditional centralities in case of un-weighted ties among nodes. Moreover, on the other hand very few generalizations are there which extends the Freeman's centrality measures for weighted networks [4, 5, 6]. Also, there is a major limitation in all these generalization- all of them depends majorly on the weights/strength of ties, and don't consider the number of ties, that was the basis for the actual node centrality measures [7].

In this paper we analyze the degree, and betweenness centralities in weighted routers network which must include the strength/weight of links as an important parameter for determining the overall centrality of node, with assumption that strong links are as much as important as number of links. Further, we try to show that degree and betweenness centrality in complex routers networks follow power-law trend and both parameters shows the scale-free behavior in the formation and evolution of the this networks. Also, we follow the generalized vertex centrality measures given by Opsahl et al. in [21] for calculating the major features/metrics in the network for node centrality. We have used "r-project" open source software package for the analysis of node centralities and the resulting behavior is based on the data set of PTCL

(Pakistan Telecommunication Limited) routers network.

The rest of this paper is structured as follows. Section II, discuss and reviews the metrics degree, betweenness and distance based on [7] and [21] approaches. In Section III, we discuss the features of PTCL routers network. In section IV, we discuss the outcomes of the analysis. Section V concludes the paper with future work.

2. NODE CENTRALITY MEASUREMENT IN NETWORKS WITH LINKS WEIGHT

The analysis of the issue of node centrality has remained very important problem in the research of network analysis [20, 21, 22]; as the identification of more prominent or central nodes in the network. Because, if node is more central as compared to others can have three main advantages. 1) Placement or location can potentially control the traffic/flow among other nodes 2) it can have more direct links/ties 3) From that node we can reach to all others nodes very quickly. Further, these characteristics actually are the basis of Freeman's [24] node centrality measurements namely degree, betweenness and closeness. Here, in this paper we try to see behavior of node centrality measures: degree, and betweenness from the perspective of scale-free networks. Moreover, Opsahl et al. [22] has also extended these centrality measurements in case of weighted networks in which they combined both number of links and links weights, by controlling the balance in between these two parameters of major node in the networks with the tuning parameter as alpha. Here, alpha is the tuning parameter which is positive quantity and its value can be adjusted according to the context of research setting. Freeman's and Opsahl et al generalizations give the same results when alpha = 0.0. Whereas on the other hands when alpha =1.0, the obtained values for node centrality are based on purely links weights as in [4, 19].

2.1 The Node Degree of Weighted Networks

It can be defined as the number of links directly connecting the node with other nodes in the network. According to Freeman's node centrality can be mathematically defined as:

$$C_D(k) = deg_k = \sum_i^N a_{ij} \tag{2}$$

Where, k is the main node, j represents all other nodes, a is the adjacency matrix where, the entry a_{kj} represent the connectivity if the value is 1, otherwise it is 0, and N is the total number of nodes in the network. In case of weighted networks, we use the concept of node strength [6], which can be defined as the sum of node direct links weights:

$$C_D^W(k) = strength_k = \sum_j^N w_{kj} \tag{3}$$

Here, w represents the weighted adjacency matrix. The value of the entry w_{kj} in matrix is > 0 , if the node k is connected with j, whereas the values show the weight of link.

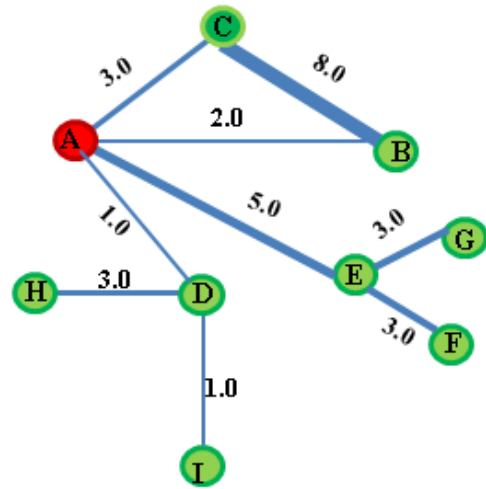


Figure 3: Example showing the weighted network, with 9 nodes and their links weight

Moreover, as node strength do not consider the number of links with particular node is connected, therefore it gives only rough idea or measure for the nodes actual contribution and involvement in the whole network. By taking simple scenario as given in figure 3, the three different nodes A, C, and E are equal in strength, whereas we can clearly see that all three are not equally central or important. From these given nodes, the node A has more connections or links with other nodes in the network; therefore we can say that is more central as compare to C and E. Further, Opsahl et al. [22] has proposed a very realistic approach for degree centrality in weighted networks by considering the both number of links and the total weight of links (strength), the reason for this is very much obvious as both these parameters can clearly indicate the involvement of major/focal nodes in overall network [22], thus

$$C_D^{W\alpha}(k) = \text{deg}_k \times \left(\frac{\text{strength}_k}{\text{deg}_k} \right)^\alpha \quad (4)$$

Here, alpha (α) in this equation works as tuning parameter. If the value of alpha is in the range of 0 and 1, then it will measure both the weights and degree favorably. If the value is more than 1 in that case high valued weights and low degrees are favorably measured [22].

2.2 The Betweenness Centrality In Weighted Networks

In simple words, it can be defined as a measure to quantify the importance of a vertex in the network on the basis of focal node position on the shortest paths in between the remaining other pairs of nodes. According to Freeman [7] betweenness centrality can be formalized as:

$$C_B(k) = \sum_j^N \sum_i^N \frac{h_{ji}(k)}{h_{ji}}, i, j \neq k \quad (5)$$

Where h_{ji} is the total number of shortest paths or routes between two nodes, and $h_{ji}(k)$ is the number of those routes that passes through or goes through node k .

Whereas, Opsahl et al. [22] generalizations of betweenness centrality depends on their generalization of shortest route. The betweenness centrality is formalized according to Opsahl in [22] as:

$$C_B^{W\alpha}(k) = \sum_j^N \sum_i^N \frac{h_{ij}^{W\alpha}(i)}{h_{ij}^{W\alpha}}, i, j \neq k \quad (6)$$

Here, alpha (α) is tuning parameter and when its value is 0 it will calculate the binary shortest distance, whereas in case of 1 it will use Dijkstra's algorithm. When the value of alpha is greater than 1, the shortest paths will be based on strongest edges rather than fewest shortest links in between nodes.

2.3 The Shortest Routes in Weighted Longitudinal Network

From the perceptive of un-weighted network, like in case of distance vector routing the shortest path totally depends on minimum hop count that means less number of intermediary vertices from source to destination is found, and its route length is the minimum number of links between the source and

destination. The shortest route between vertices k and j can be defined formally as:

$$d(k, j) = \min (x_k h + \dots + x_h j) \quad (7)$$

In this equation h are the nodes that come in between route of nodes k and j .

When the links in the network or graph are weighted, then the binary shortest routes are not necessary to be shortest routes, the reason is that the connections or links are different and not all connections can be equally important for the flow of information like in many routing protocols scenario. For example, different routing protocols find the shortest path based on different weight/metric of the links like bandwidth, delay, speed and congestion etc. Therefore, if weight represents strength of links then route composed of high value or strength are shorter as compared to those routes of weaker links. For example, in the network of figure 4, we have three routes in between two nodes A and B which are connected with different number of intermediary vertices and edges with different weights. The binary shortest route in between these two nodes can be A to B. But as we use the concept of bandwidth in different routing protocols scenario, so route with high bandwidth will be selected as compare to direct low bandwidth route. Although the route from (A to D and B) and the route from (A to C to B) has one intermediary route but it can be used as much quicker since they have high value weight. For example, the information can be passed through longer route of strong links more quickly as compare to weak direct link

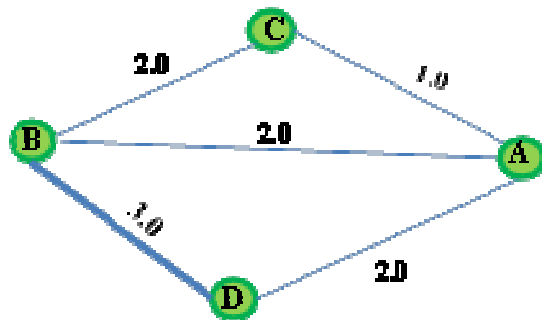


Figure 4: The weighted network with multiple routes between A & B

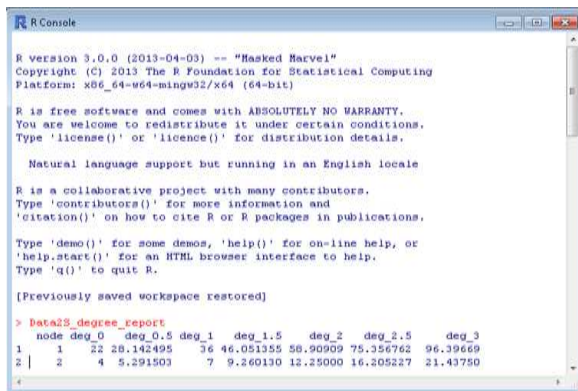
Many attempts have been made to find shortest paths in case of weighted networks as given in [23, 24]. The Dijkstra's in [25] proposed the algorithm for finding shortest path, which was used to find shortest path by considering the weight as costs for transferring the messages. Also well-known link

state routing protocols are based on this algorithm which is widely used in the field of computer networks today.

3. A LOOK AT PTCL (PAKISTAN TELECOMMUNICATION (LIMITED) ROUTERS NETWORK

In this network routers are nodes and physical links are connection in between them. There are 134 routers and 216 physical links. The density of this sample network is not high and there exists 5% possible connections/links. There are 6 core routers each having 3 (2*10 G) and 1(1*10 G) bandwidth links. Further, there are three other types of physical links in the network like one (Proposed 10 GE links over DWDM), second Existing (2*GE) bandwidth links and third (proposed 10 GE link in DF). For our analysis purpose we have assigned scales based on bandwidth of actual links from 1 to 4 according to increasing bandwidth of links. As we are dealing with longitudinal or growing networks it is worth mentioning here that initially there were 75 nodes in whole network but now currently due to growing nature the routers are increased up to 134. The metric diameter of the network is 12. This diameter represents the largest degree separation is 12 between any two routers in the network without consideration of shortest routes based on bandwidth (weight) of links.

For measuring the values of centralities and network analysis we have used open source software package the r- Project.



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R Console
R version 3.0.0 (2013-04-03) -- "Rasked Marvel"
Copyright (C) 2013 The R Foundation for Statistical Computing
Platform: x86_64-mingw32/x64 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> Data25_degree_report
  node deg_0 deg_0.5 deg_1 deg_1.5 deg_2 deg_2.5 deg_3
1 | 1 22 28.142495 36 46.051355 58.90909 75.356762 96.39669
2 | 2 4 5.291503 7 9.260130 12.25000 16.205227 21.43750
    
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Figure 5: A snapshot of 'r-project' used for network analysis.

As the network is still expanding so we assume here for our analysis that the network follows the preferential attachment concept like “When choosing the nodes to which the new node connects, we assume that the probability that a new

node will be connected to node i depends on the degree K_i of node i,” such that 2

$$\pi_i = \frac{K_i}{\sum_j K_j} \tag{8}$$

After t time steps, there are

$$N = t + m_0 \text{ nodes} \tag{9}$$

and

$$L = mt + E_0 \text{ edges} \tag{10}$$

This growth of the network can be based on number of links already attached with routers as suggested by BA scale-free network model as rich get richer phenomenon. On the other hands, it can be based on the fitness of routers as weight of attached links as fit-get-rich phenomenon. Here, we show that weighted network centrality measures are closer to scale-free network behavior. Therefore, fit nodes from the perspective of links bandwidth have more chances to get new links when the network grows.

4. OUTCOMES AND ANALYSIS

For observation of robustness we have obtained the observed network, by extracting the samples from the original or true network by randomly removing 3% and 5% links from the given network. Here, our main focus is to see the generalized centrality of node rankings in actual network with obtained rankings by changing the links percentage in all cases by taking different values of alpha. More specifically, our aim is to see the correlation in between true node centrality and observed networks centrality by changing alpha when it is 0.0, $0 < \alpha < 1$, $\alpha = 1$ and finally when $\alpha > 1$.



Table1: The effect of random links removal from the original network on degree and betweenness centralities using Spearman's rank correlation as compared to true network.

# of Node	The effect on degree centrality when (alpha=0.0)		The effect on degree centrality when (alpha=2.0)	
	Name of Node	Degree	Name of Node	Degree
1	Router 48	0.097744361	Router 05*	0.53588516
2	Router 102	0.097744361	Router 132	0.38775510
3	Router 05*	0.082706767	Router 69	0.36842105
4	Router 47*	0.082706767	Router 63	0.36842105
5	Router 121	0.082706767	Router 64	0.36842105
6	Router 23	0.07518797	Router 65	0.36842105
7	Router 79	0.07518797	Router 66	0.36842105
8	Router 125	0.07518797	Router 67	0.36842105
9	Router 52	0.067669173	Router 68	0.36842105
10	Router 69	0.067669173	Router 102	0.36148062
11	Router 70	0.067669173	Router 70	0.33416875
12	Router 62	0.060150376	Router 48	0.33314054
13	Router 19	0.060150376	Router 23	0.30075188
14	Router 32	0.052631579	Router 47*	0.27341079
15	Router 03	0.045112782	Router 79	0.27142857

Table2. The degree centrality of the Top fifteen nodes in the network when alpha (α) = 0.0 and 2.0.

Alph a	(Net1) 3 % connections Removal		(Net2) 5 % connections Removal	
	Degree	Betweenness	Degree	Betweenness
0.0	0.58	0.44	0.46	0.53
0.5	0.64	0.50	0.48	0.57
1.0	0.72	0.57	0.53	0.54
1.5	0.67	0.60	0.55	0.55
2.0	0.70	0.62	0.56	0.56
2.5	0.73	0.65	0.61	0.59
3.0	0.76	0.70	0.66	0.64

From table 1 we can compare the similarity in between actual and two observed networks when we remove 3% and 5% links from the networks. This is done by observing the ranking of node centrality when the values of α changed from 0.0 to 3.0. The values in case of 3% (Net)1 are higher than 5% (Net)2 links removed which clearly shows that (Net) 1 is closer to original network. The above observation clearly shows that if we increase the weight in analysis, the network show the higher similarity with true network. This implies that PTCL network is robust under random failure of links in network. It means in this network few nodes and links are very important which holds this sparse network. The experiment of removing the links is performed 5 times and average values are used in the table. We have measured the degree and betweenness centralities of all the nodes present in the network from weighted and un-weighted links point of view. Only top fifteen nodes are shown in table 2 and 3.

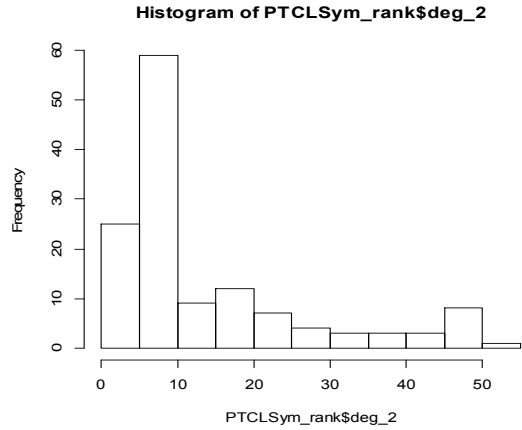
Table3. The betweenness centrality of the Top fifteen nodes in the network when alpha (α) = 0.0 and 2.0.

# of Node	The effect on degree centrality when (alpha=0.0)		The effect on degree centrality when (alpha=2.0)	
	Node	Betweenness	Node	Betweenness
1	Router 70	0.38617376	Router 70	0.535885
2	Router 47	0.36955551	Router 47	0.387755
3	Router 65	0.34054834	Router 65	0.368421
4	Router 68	0.33264031	Router 68	0.368421
5	Router 102	0.2144945	Router 102	0.368421
6	Router 48	0.21080632	Router 48	0.368421
7	Router 67	0.20299802	Router 66*	0.368421
8	Router 66*	0.19042872	Router 67*	0.368421
9	Router 69*	0.18844459	Router 52*	0.368421
10	Router 52*	0.18839712	Router 69*	0.361481
11	Router 64	0.16683754	Router 64	0.334169
12	Router 05	0.14848674	Router 05	0.333141
13	Router 63	0.135309866	Router 63	0.300752
14	Router 62	0.116655275	Router 62	0.273411
15	Router 23	0.105714096	Router 23	0.535885

The above table shows the effect on ranks of routers (with their names and degree centrality) when we are including the links only ($\alpha=0.0$), and when we are including weight of the links in analysis ($\alpha=2.0$) neglecting the number of links. The columns in the table represent the node numbers, names and their degree centralities rank wise.

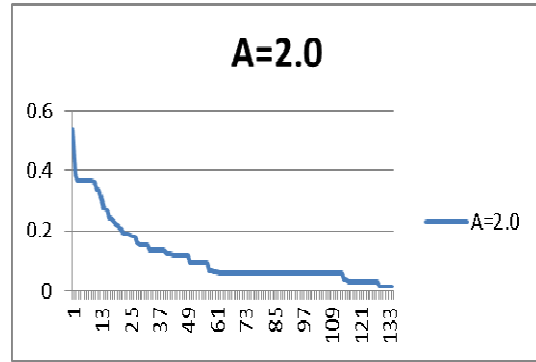
It is quite clear that when we are including weights of the links by adjusting (α) tuning parameter as 2.0 the ranks of routers centralities changes and it shows that including weight in analysis is more appropriate as compared to the numbers of links. The Faisalabad (5) router has 11 links and Lahore (47) router has 13 links so from the number of links point of view Lahore router is important but when we analyzed only from the weighted links point of view then Faisalabad (5) router is more important as compared to Lahore (47) router. As the location of Faisalabad router is at periphery of the network and Lahore in is the middle of the network therefore, from the betweenness perspective Lahore router is very important as compared to Faisalabad router.

This clearly shows that many shortest routes are passing through Lahore router. Similarly, if we compare Hyderabad (102) router with Mardaan (132) router then from the number of links point of view it has 13 links whereas Mardaan (132) router has 7 links so Hyderabad router is important from the Mardaan router. But from the weighted links point of view Mardaan router has high degree centrality. This shows it has less but powerful links in the network. Further, the core routers of 63 (Islamabad) and 64 (Karachi) has only 4 links but they have fourth and fifth highest centrality in the network. As for as betweenness centrality is concerned, the main Karachi router has highest betweenness centrality which clearly indicates that if this router is disconnected from the network the results will be creation of very longest routes in the network. Similarly, if we disconnect Lahore (47) router then Sialkot, Gujarat and Gujranwala will be the longest routes from other routers in the network. Further, the router (3) and router (47) both have 6 links. The betweenness of R3 is very less as compared to R42 which is the indication that R42 location and its connections are with highly connected routers in the network.

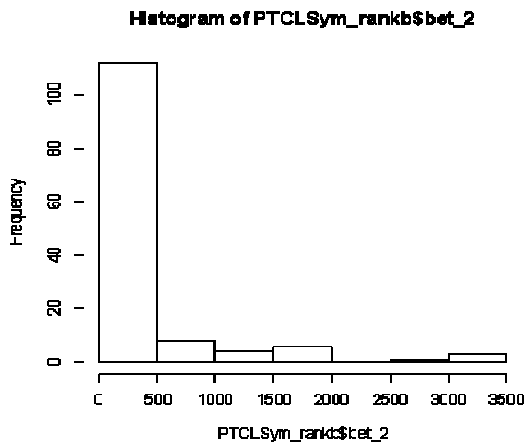


6 (a)

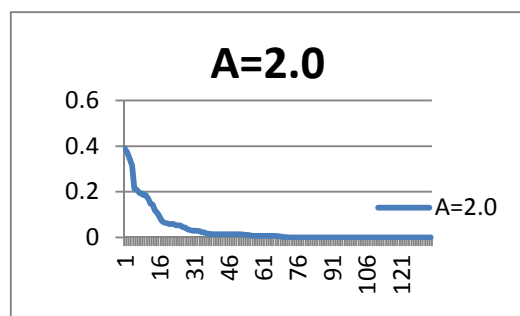
The histogram and line graph between of degrees of nodes and their links from the perspective of weighted links is shown in figure 6(a) and (b). Figure 7 (a) and (b) shows the histogram and line graph of weighted betweenness of nodes in the network.



6 (b)



7 (a)



7 (b)

The Figures 6 (a) and (b) shows that less than 10 nodes in this network have highest degree centralities and line graph clearly shows that this trend of degree distribution of centralities with weighted links is very close to power-law distribution. As discussed earlier, in this network we have only 134 nodes and for getting the value of power-law exponent gamma we need thousands nodes. We can assume that the linear graph if increased with fit get rich phenomenon then for more nodes in the network this behavior continues in the same way. Similarly, the Figure 7 showing the histogram and line graph of betweenness centralities also show power-law behavior in the flow of information in the network. Therefore, by removing or disrupting few highest betweenness centralities nodes in the network can badly affect the communication system of the network. Also, this distribution shows that few routers in the network have highest load of network traffic.

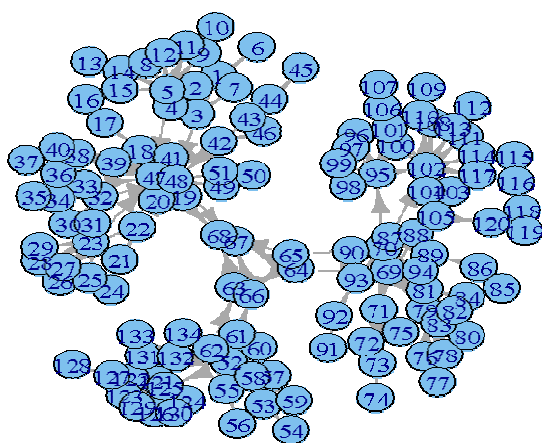


Figure 8: The network visualization of PTCL routers network in r-project.

5. CONCLUSIONS

The network analysis in different domain of life has gained very much importance due to many applications. In this research, we have analyzed the PTCL routers network as a case study to see the effects of links weight from scale-free network approach. We analyzed this network under random links removal and found that it is showing very robust behavior when analyzed from weighted point of view. Therefore, we can conclude that random routers or links failures cannot disturb its functionality. Further, the two network metrics degree and betweenness also showed that these measures have power-law behavior in their formation and functions. Although the nodes in this network are not in thousands which could give us power-law exponent gamma. However, if this network grows in this way then we believe it will show the power-law trend in its growth. According to our knowledge this approach of identifying important nodes in the network using scale-free network is novel idea. Further, there are many metrics that we can find in this network like closeness, clustering, assortativity and that can constitute future work.

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