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CONSTRUCTION OF AN OPTIMAL MATHEMATICAL MODEL OF FUNCTIONING OF THE MANUFACTURING INDUSTRY OF THE REPUBLIC OF KAZAKHSTAN

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ABSTRACT

The article is devoted to the selection of an optimal model of the production function to simulate the functioning of the manufacturing industry of the Republic of Kazakhstan. An optimal model of the production function is selected from several models constructed and defined on the first stage of the study. The constructed optimal production function can be used to predict the value of gross domestic product based on the known or anticipated levels of capital and wage costs.

Keywords: *model, method, simulation, forecasting, production function, capital.*

1. INTRODUCTION

The condition and development of the manufacturing industry is one of the most important key points that determine the rate of economic development of the state.

As noted in the long-term strategy of development of Kazakhstan "Kazakhstan-2030", "the world experience shows the need for a specific progression, which consists in steady decline in the gross national product of the share of agriculture, mining, and, on the contrary, increase of the share of manufacturing industries and, above all, the science-intensive ones, with a high added value" [1].

Currently, the base materials sector is the basis of the Kazakhstan industry, which makes the state's economy dependent on the external factors such as demand and the level of prices for the exported raw materials. The raw material orientation dooms the state's economy to unequal foreign trade exchange and increasing technological inferiority.

In connection with this, the development of the manufacturing sector is of particular importance for the economic development of the Republic of Kazakhstan in modern conditions, and it was put in the foundation of the state Strategy of industrial-innovative development.

The production of competitive and export-oriented production in the manufacturing industry is the main concern of the state's industrial-innovation policy.

Due to the importance of manufacturing industry to the economy of Kazakhstan, it is necessary to use mathematical methods to forecast the industry's development. This has defined the purpose and scope of our research. In the present paper, we use production functions to address this goal. Several production functions are constructed, and then an optimal model of the production function is chosen to forecast the development of the manufacturing industry of Kazakhstan.

For the mathematical simulation of the functioning of the manufacturing industry of the Republic of Kazakhstan, we use the data on the gross domestic product of this industry for 16 years (1998-2013) with respect to the labor force (L) and capital (K). The data are listed in Table 1 [2].

Table 1. The Economic Performance Of The Manufacturing Industry Of The Republic Of Kazakhstan In The Years 1998-2013

Years	K is the capital costs (millions of KZT)	L is the wage costs (millions of KZT)	Y is the gross domestic product of the industry (millions of KZT)
1998	40 618.00	102 893.40	208 336.60
1999	52 907.28	130 240.20	284 152.00
2000	74 794.93	149 259.00	428 932.70
2001	102 421.81	167 483.10	534 563.00
2002	102 550.03	172 655.70	547 414.10
2003	119 870.48	213 417.00	655 719.00
2004	191 366.17	272 891.30	781 558.70
2005	258 886.78	325 058.20	914 013.20

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Years	K is the capital costs (millions of KZT)	L is the wage costs (millions of KZT)	Y is the gross domestic product of the industry (millions of KZT)
2006	293 475.13	423 004.30	1 188 108.00
2007	316 339.43	551 380.20	1 476 647.60
2008	370 062.97	669 651.40	1 890 053.00
2009	396 261.47	643 251.10	1 849 097.50
2010	404 925.35	821 158.50	2 469 804.10
2011	455 466.43	989 957.40	3 131 187.00
2012	595 214.22	1 066 127.50	3 436 730.50
2013	636 886.41	1 130 987.00	3 651 704.60

2. TECHNIQUE

The methodological basis of the study is the works by the Russian and foreign scientists, the applied and theoretical works on economic-mathematical methods by Berezhnaya E.V and Berezhnoi V.I. [4], Granberg A.G. [5], Ivanilov Yu.P. [6], Malykhin V.I. [7], Rayatskas R.L. and Plakunov M.K. [8], Bagrinovsky K. A. and Matyushok V.M. [9], Solow R.M. [10], Gale D. [11], Dorfman R. [12], Cantor D.G. and Lipman S.A. [13], [14], Sonin I.M. [15], Presman E.L. and Sonin I.M. [16], on forecasting the economic data by Demidenko E.Z. [17], Dubrova T.A. [18], on the construction of production functions by Barkalov N.B. [19], Kleiner G.B. [20], and Nazarova N.V. [21].

The study is based on using the methods of mathematical simulation, system analysis, regression analysis, and others. The information base for the study includes the legislative and regulatory acts of the Republic of Kazakhstan, the materials of the Statistics Agency of the Republic of Kazakhstan and scientific publications.

3. RESULTS

3.1 Construction of a linear production function

Let us carry out mathematical simulation of the functioning of the manufacturing industry of the Republic of Kazakhstan. To do this, we construct several production functions, and then perform a comparative analysis.

We use the following model of a linear production function:

$$F = a_0 + a_1 K + a_2 L \quad (1)$$

where K is the capital costs; L is the wage costs.

The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (a_0 + a_1 K_i + a_2 L_i) \right]^2 \to \min_{a_0, a_1, a_2} \quad (2)$$

We perform the calculations using the data of Table 1. As a result, we find that the residual function attains its minimum at a0 = -0.00003; a1 = -0.857; a2 = 3.542.

With respect to our data, the model of a linear production function will have the form:

$$F = -0.00003 - 0.857K + 3.542L \quad (3)$$

Table 2. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2013 With The Calculations Using The Linear Production Function

Years	K	L	Y	F	$(Y-F)^2$
1998	40 618.00	102 893.40	208 336.60	329 572.937	14 698 249 288.010
1999	52 907.28	130 240.20	284 152.00	415 885.287	17 353 658 889.240
2000	74 794.93	149 259.00	428 932.70	464 474.241	1 263 201 126.582
2001	102 421.81	167 483.10	534 563.00	505 327.933	854 689 137.359
2002	102 550.03	172 655.70	547 414.10	523 536.849	570 123 105.254
2003	119 870.48	213 417.00	655 719.00	653 043.174	7 160 045.851
2004	191 366.17	272 891.30	781 558.70	802 372.072	433 196 461.944
2005	258 886.78	325 058.20	914 013.20	929 229.922	231 548 618.604
2006	293 475.13	423 004.30	1 188 108.00	1 246 451.724	3 403 990 175.921
2007	316 339.43	551 380.20	1 476 647.60	1 681 493.409	41 961 805 501.088
2008	370 062.97	669 651.40	1 890 053.00	2 054 290.411	26 973 927 261.342
2009	396 261.47	643 251.10	1 849 097.50	1 938 330.692	7 962 562 494.886
2010	404 925.35	821 158.50	2 469 804.10	2 560 964.491	8 310 216 969.393
2011	455 466.43	989 957.40	3 131 187.00	3 115 434.875	248 129 452.271
2012	595 214.22	1 066 127.50	3 436 730.50	3 265 372.922	29 363 419 450.599
2013	636 886.41	1 130 987.00	3 651 704.60	3 459 344.262	37 002 499 625.557

We perform calculations using the constructed production function. The calculation results are shown in Table 2. Figure 1 gives a graphical representation of the calculation results.

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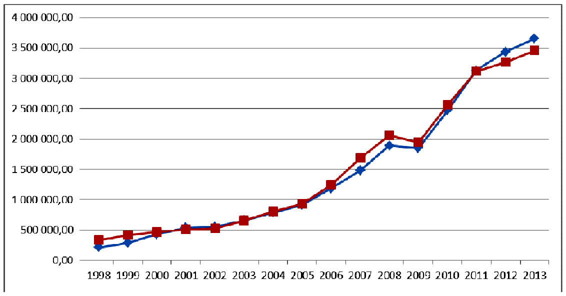

Figure 1. Graphical Representation Of The Calculation Results While Using The Linear Production Function. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

The analysis of the data in Table 2 and Figure 1 shows that the constructed model of the linear production function sufficiently accurately reflects the development trend of the real indicators in its first half of the time interval, whereas in the second half there are some discrepancies.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9928;
- standard error – 101 796.94;
- sum of the squared deviations – 190 638 377 603.9.

3.2 Construction of a Cobb-Douglas production function with α+β=1

Let us construct a Cobb-Douglas production function of the form:

$$F = AK^{\alpha}L^{\beta},$$

(4)

where α+β=1, K is the capital costs, L is the wage costs.

The residual function has the form:

$$\sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\left[Y_i - (a_0 K_i^{a_1} L_i^{(1-a_1)})\right]^2 \to \min_{a_0, a_1}$$

(5)

We perform calculations using the data of Table 1. As a result, we find that the residual function reaches minimum at a0 = 2.822; a1 = -0.139.

With respect to our data, the model of the Cobb—Douglas production function with α+β=1 will have the form:

$$F = 2.822K^{-0.139}L^{1.139}$$

(6)

We perform calculations on the basis of the constructed production function. The calculation results are given in Table 3. Figure 2 provides a graphical representation of the results of calculations.

The analysis of the data of Table 3 and Figure 2 shows that the constructed model of the Cobb-Douglas production function with α + β = 1 behaves like the linear production function.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9928;
- standard error – 101 445.767;
- sum of the squared deviations – 194 256 523 640.535.

Table 3. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2013 With The Calculations Using The Cobb-Douglas Production Function With A+B=1

Years	K	L	Y	F	$(Y-F)^2$
1998	40 618.00	102 893.40	208 336.60	330 501.335	14 924 222 528.507
1999	52 907.28	130 240.20	284 152.00	416 675.887	17 562 580 615.467
2000	74 794.93	149 259.00	428 932.70	463 765.190	1 213 302 371.735
2001	102 421.81	167 483.10	534 563.00	506 154.948	807 017 432.710
2002	102 550.03	172 655.70	547 414.10	523 910.912	552 399 866.266
2003	119 870.48	213 417.00	655 719.00	652 658.144	9 368 841.680
2004	191 366.17	272 891.30	781 558.70	809 137.337	760 581 195.879
2005	258 886.78	325 058.20	914 013.20	946 882.238	1 080 373 680.083
2006	293 475.13	423 004.30	1 188 108.00	1 256 101.620	4 623 132 358.792
2007	316 339.43	551 380.20	1 476 647.60	1 681 217.991	41 849 045 075.867
2008	370 062.97	669 651.40	1 890 053.00	2 052 524.015	26 396 830 565.006
2009	396 261.47	643 251.10	1 849 097.50	1 942 004.301	8 631 673 689.999
2010	404 925.35	821 158.50	2 469 804.10	2 557 155.871	7 630 331 821.296
2011	455 466.43	989 957.40	3 131 187.00	3 112 717.883	341 108 265.066
2012	595 214.22	1 066 127.50	3 436 730.50	3 263 101.961	30 146 869 605.843
2013	636 886.41	1 130 987.00	3 651 704.60	3 457 468.440	37 727 685 726.341

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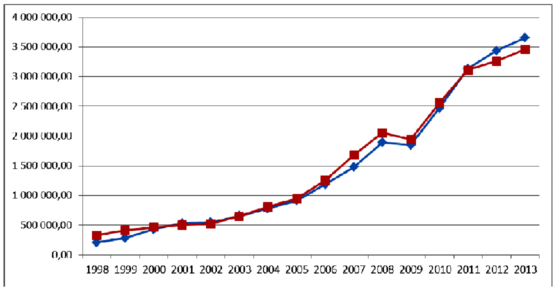

Figure 2. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function With A+B=1.
The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

3.3 Construction of a Cobb-Douglas production function with α+β≠1

Let us construct a Cobb-Douglas production function of the form:

$$F = AK^{\alpha} L^{\beta},$$

(7)

where α+β≠1, K is the capital costs, L is the wage costs.

The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (\mathrm{a}_0 K^{a_1} L^{a_2})\right]^2 \to \min_{a_0, a_1, a_2}$$

(8)

We perform calculations using the data of Table 1. As a result, we find that the residual function reaches minimum at a0 = 0.56; a1 = 0.004; a2 = 1.121.

With respect to our data, the model of the Cobb-Douglas production function with α+β≠1 will have the form:

$$F = 0.56K^{0.004} L^{1.121}$$

(9)

We perform calculations on the basis of the constructed production function. The calculation results are given in Table 4. Figure 2 provides a graphical representation of the results of calculations.

Table 4. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2013 With The Calculations Using The Cobb-Douglas Production Function With A+B≠1

Years	K	L	Y	F	$(Y-F)^2$
1998	40 618.00	102 893.40	208 336.60	242 870.986	1 192 623 841.205
1999	52 907.28	130 240.20	284 152.00	316 647.262	1 055 942 050.802
2000	74 794.93	149 259.00	428 932.70	369 432.548	3 540 268 146.566
2001	102 421.81	167 483.10	534 563.00	420 886.871	12 922 262 282.622
2002	102 550.03	172 655.70	547 414.10	435 487.283	12 527 612 263.155
2003	119 870.48	213 417.00	655 719.00	552 624.829	10 628 408 073.131
2004	191 366.17	272 891.30	781 558.70	729 324.344	2 728 427 932.668
2005	258 886.78	325 058.20	914 013.20	888 400.498	656 010 499.931
2006	293 475.13	423 004.30	1 188 108.00	1 194 119.445	36 137 474.474
2007	316 339.43	551 380.20	1 476 647.60	1 607 715.271	17 178 734 420.987
2008	370 062.97	669 651.40	1 890 053.00	2 000 274.026	12 148 674 668.573
2009	396 261.47	643 251.10	1 849 097.50	1 912 613.876	4 034 330 059.932
2010	404 925.35	821 158.50	2 469 804.10	2 515 010.202	2 043 591 669.680
2011	455 466.43	989 957.40	3 131 187.00	3 102 812.777	805 096 509.970
2012	595 214.22	1 066 127.50	3 436 730.50	3 375 271.455	3 777 214 208.153
2013	636 886.41	1 130 987.00	3 651 704.60	3 607 262.340	1 975 114 500.264

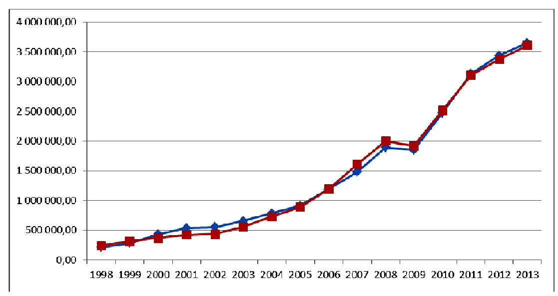

Figure 3. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function For A+B≠1. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Analysis of the data of Table 4 and Figure3 demonstrates that the constructed model of the Cobb-Douglas production function for α+β≠1 is sufficiently precise over the entire time interval under consideration.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9959;
- standard error – 76 549.895;
- sum of the squared deviations – 87 250 448 602.112.

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3.4 Construction of a *Cobb*-Douglas production function with α+β=1 taking into account scientific-technological progress (STP)

We will construct a Cobb-Douglas production function taking into account STP of the following form:

$$F = Ae^{p_0 t} K^{\alpha} L^{(1-\alpha)}, \tag{10}$$

where α+β=1, K is the capital costs, L is the wage costs, $e^{p_0 t}$ is a special factor of scientific progress, p0 is a parameter of neutral STP (p0 > 0). The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (a_0 e^{p_0 t} K^{a_1} L^{(1-a_1)})\right]^2 \rightarrow \min_{a_0, a_1, p_0} \tag{11}$$

We perform calculations using the data of Table 1. As a result, we find that the residual function reaches minimum at p0 = 0.003; a0 = 2.833; a1 = -0.129.

With respect to our data, the model of the Cobb-Douglas production function with α+β=1 and taking into account STP will have the form:

$$F = 2.833 e^{0.003t} K^{-0.129} L^{1.129} \tag{12}$$

We perform calculations on the basis of the constructed production functions. The calculation results are given in Table 5. Figure 4 provides a graphical representation of the results of calculations.

Table 5. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan For The Years 1998-2013 With The Calculations Using The Cobb-Douglas Production Function Taking Into Account STP With A+B=1

Years	K	L	Y	F	(Y-F)^2
1998	40 618.00	102 893.40	208 336.60	329 491.108	14 678 414 848.254
1999	52 907.28	130 240.20	284 152.00	415 523.433	17 258 453 384.606
2000	74 794.93	149 259.00	428 932.70	463 472.106	1 192 970 592.786
2001	102 421.81	167 483.10	534 563.00	506 861.930	767 349 253.270
2002	102 550.03	172 655.70	547 414.10	524 486.900	525 656 478.339
2003	119 870.48	213 417.00	655 719.00	653 003.942	7 371 539.268
2004	191 366.17	272 891.30	781 558.70	811 397.852	890 374 963.193
2005	258 886.78	325 058.20	914 013.20	950 758.976	1 350 252 030.283
2006	293 475.13	423 004.30	1 188 108.00	1 259 473.537	5 093 039 881.424
2007	316 339.43	551 380.20	1 476 647.60	1 682 472.397	42 363 847 137.688
2008	370 062.97	669 651.40	1 890 053.00	2 053 271.756	26 640 362 273.849
2009	396 261.47	643 251.10	1 849 097.50	1 944 861.839	9 170 808 612.473
2010	404 925.35	821 158.50	2 469 804.10	2 555 121.403	7 279 042 247.669
2011	455 466.43	989 957.40	3 131 187.00	3 108 046.491	535 483 178.716
2012	595 214.22	1 066 127.50	3 436 730.50	3 264 630.365	29 618 456 411.584
2013	636 886.41	1 130 987.00	3 651 704.60	3 459 391.254	36 984 423 065.707

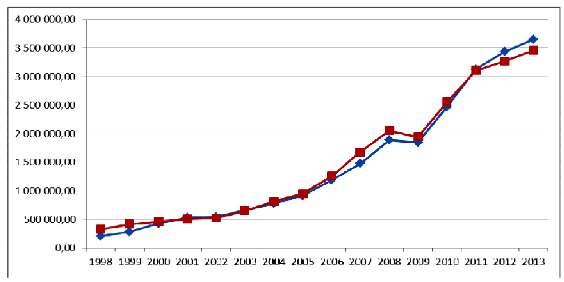

Figure 4. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function With Taking Into Account STP And With A+B=1. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Analysis of the data of Table 5 and Figure 4 shows that the constructed model of the Cobb-Douglas production function with α+β=1 and taking into account STP behaves analogously to the linear production function and the Cobb-Douglas production function without taking into account STP and with α+β=1.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9929;
- standard error – 101 087.447;
- sum of the squared deviations – 194 356 305 899.109.

3.5 Construction of a Cobb-Douglas production function taking into account scientific-technological progress (STP) with α+β≠1

We will construct a Cobb-Douglas production function taking into account STP of the following form:

$$F = Ae^{p_0 t} K^{\alpha} L^{\beta}, \tag{13}$$

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where α+β≠1, K is the capital costs, L is the wage costs, $e^{p_0 t}$ is a special factor of scientific progress, p0 is a parameter of neutral STP (p0 > 0). The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (\mathrm{a}_0 e^{p_0 t} K^{a_1} L^{a_2}) \right]^2 \rightarrow \min_{a_0, a_1, p_0, a_2} \quad (14)$$

We perform calculations using the data of Table 1. As a result, we find that the residual function reaches minimum at p0 = 0.001; a0 = 29.985; a1 = -0.647; a2 = 1.45.

With respect to our data, the model of the Cobb-Douglas production function with α+β≠1 and taking into account STP will have the form:

$$F = 29.985 e^{0.001t} K^{-0.647} L^{1.45} \quad (15)$$

We perform calculations on the basis of the constructed production function. The calculation results are given in Table 6. Figure 5 provides a graphical representation of the results of calculations.

Table 6. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan For The Years 1998-2013 With The Calculations Using The Cobb-Douglas Production Function Taking Into Account STP And With A+B≠1

Years	K	L	Y	F	$(Y-F)^2$
1998	40 618.00	102 893.40	208 336.60	578 223.304	136 816 173 970.454
1999	52 907.28	130 240.20	284 152.00	685 808.539	161 327 975 681.466
2000	74 794.93	149 259.00	428 932.70	667 874.495	57 093 181 185.422
2001	102 421.81	167 483.10	534 563.00	643 953.838	11 966 355 466.617
2002	102 550.03	172 655.70	547 414.10	672 449.015	15 633 729 885.903
2003	119 870.48	213 417.00	655 719.00	826 533.490	29 177 590 044.496
2004	191 366.17	272 891.30	781 558.70	872 058.521	8 190 217 590.292
2005	258 886.78	325 058.20	914 013.20	924 144.032	102 633 754.371
2006	293 475.13	423 004.30	1 188 108.00	1 248 394.182	3 634 423 737.521
2007	316 339.43	551 380.20	1 476 647.60	1 746 537.908	72 840 778 282.891
2008	370 062.97	669 651.40	1 890 053.00	2 091 514.096	40 586 573 089.904
2009	396 261.47	643 251.10	1 849 097.50	1 887 536.871	1 477 585 249.085
2010	404 925.35	821 158.50	2 469 804.10	2 652 145.890	33 248 528 211.903
2011	455 466.43	989 957.40	3 131 187.00	3 223 007.497	8 431 003 621.479
2012	595 214.22	1 066 127.50	3 436 730.50	3 017 861.545	175 451 201 296.266
2013	636 886.41	1 130 987.00	3 651 704.60	3 146 772.993	254 955 927 334.286

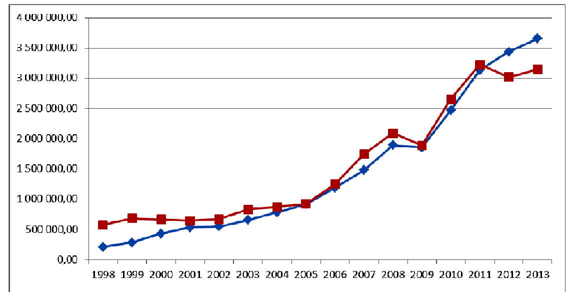

Figure 5. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function With Taking Into Account Stp And With A+B≠1. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Analysis of the data of Table 6 and Figure 5 shows that the obtained model of the Cobb-Douglas production function with α+β≠1 and taking into account STP does not reflect with sufficient precision the development tendency of the real indicators over the entire time interval under consideration.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9729;
- standard error – 196 752.076;
- sum of the squared deviations – 1 010 933 878 402.36.

3.6 Construction of a quadratic production function (of the type 1)

Let us construct a production function of the form:

$$F = a_0 + a_1 K + a_2 L + a_3 K^2 + a_4 L^2 \quad (16)$$

where K is the capital costs, L is the wage costs. The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (\mathrm{a}_0 + a_1 \mathrm{K}_i + a_2 \mathrm{L}_i + \mathrm{a}_3 \mathrm{K}_i^2 + a_4 \mathrm{L}_i^2) \right]^2 \rightarrow \min_{a_0, a_1, a_2, a_3, a_4} \quad (17)$$

We carry out calculations using the data from Table 1. As a result, we get that the residual function attains minimum for a0 = 0.000000001; a1 = 0.00024; a2 = 0.0007; a3 = 0.000004; a4 = 0.000002.

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As applied to our data, the model of a quadratic production function will have the form:

$$F = 0.000000001 + 0.00024K + 0.0007L + \\ + 0.000004K^2 + 0.000002L^2 \quad (18)$$

We perform calculations on the basis of the constructed production function. The calculation results are given in Table 7. Figure 6 gives a graphical representation of the calculation results.

Analysis of the data of Table 7 and Figure 6 shows that the obtained model of the quadratic production function (of the type 1) has substantial deviations from the real data over the entire time interval under consideration.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9711;
- standard error – 203 357.327;
- sum of the squared deviations – 2 740 703 153 148.11.

Table 7. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2013 With The Calculations Using The Quadratic Production Function (Of The Type 1)

Years	K	L	Y	F	(Y-F)^2
1998	40 618.00	102 893.40	208 336.60	28 574.908	32 314 266 075.165
1999	52 907.28	130 240.20	284 152.00	46 395.079	56 528 353 328.788
2000	74 794.93	149 259.00	428 932.70	68 791.645	129 701 579 478.189
2001	102 421.81	167 483.10	534 563.00	100 747.906	188 195 535 358.469
2002	102 550.03	172 655.70	547 414.10	104 469.337	196 200 062 995.551
2003	119 870.48	213 417.00	655 719.00	152 601.222	253 127 498 566.086
2004	191 366.17	272 891.30	781 558.70	303 327.690	228 704 899 146.879
2005	258 886.78	325 058.20	914 013.20	492 151.669	177 967 151 154.925
2006	293 475.13	423 004.30	1 188 108.00	720 972.195	218 215 860 213.761
2007	316 339.43	551 380.20	1 476 647.60	1 034 943.568	195 102 451 769.359
2008	370 062.97	669 651.40	1 890 053.00	1 482 685.732	165 948 090 711.226
2009	396 261.47	643 251.10	1 849 097.50	1 493 952.311	126 128 105 578.480
2010	404 925.35	821 158.50	2 469 804.10	2 057 117.740	170 310 031 680.475
2011	455 466.43	989 957.40	3 131 187.00	2 862 975.593	71 937 358 794.193
2012	595 214.22	1 066 127.50	3 436 730.50	3 787 001.194	122 689 559 324.952
2013	636 886.41	1 130 987.00	3 651 704.60	4 290 165.522	407 632 348 971.615

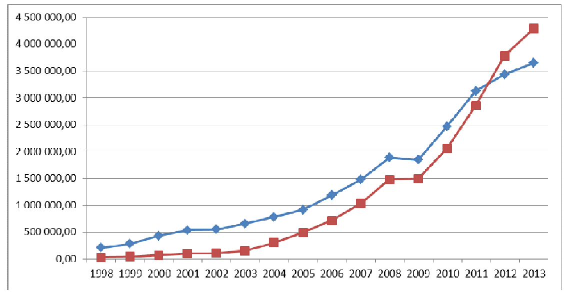

Figure 6. Graphical Representation Of The Calculation Results By The Quadratic Production Function. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

3.7 Construction of a quadratic production function (of the type 2)

Let us construct a production function of the form:

$$F = a_0 + a_1K + a_2L + a_3K^2 + a_4L^2 + a_5KL \quad (19)$$

where K is the capital costs, L is the wage costs. The residual function will have the following form:

$$\sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\left[Y_i - (a_0 + a_1K_i + a_2L_i + a_3K_i^2 + a_4L_i^2 + a_5K_iL_i)\right]^2 \to \min_{a_0,a_1,a_2,a_3,a_4,a_5} \quad (20)$$

We carry out calculations using the data from Table 1. As a result, we get that the residual function attains minimum for a0 = 0; a1 = 0.000000000005; a2 = 0.000000000009; a3 = 0.0000015; a4 = 0.0000017; a5 = 0.0000021.

As applied to our data, the model of a quadratic production function will have the form:

$$F = 0 + 0.000000000005K + 0.000000000009L + \\ + 0.0000015K^2 + 0.0000017L^2 + 0.0000021KL \quad (21)$$

We perform calculations according to the constructed production functions. The calculation results are given in Table 8. Figure 7 gives a graphical representation of the calculation results.

Table 8. The Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2013

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With The Calculations Using The Quadratic Production Function (Of The Type 2)

Years	K	L	Y	F	$(Y-F)^2$
1998	40 618.00	102 893.40	208 336.60	28 989.956	32 165 218 547.263
1999	52 907.28	130 240.20	284 152.00	47 102.781	56 192 332 056.349
2000	74 794.93	149 259.00	428 932.70	69 328.773	129 314 984 013.043
2001	102 421.81	167 483.10	534 563.00	99 210.739	189 531 590 954.387
2002	102 550.03	172 655.70	547 414.10	103 342.241	197 199 816 163.244
2003	119 870.48	213 417.00	655 719.00	152 141.702	253 590 095 355.687
2004	191 366.17	272 891.30	781 558.70	291 156.503	240 494 315 181.738
2005	258 886.78	325 058.20	914 013.20	457 774.903	208 153 383 949.997
2006	293 475.13	423 004.30	1 188 108.00	693 856.927	244 284 122 833.111
2007	316 339.43	551 380.20	1 476 647.60	1 029 754.555	199 713 393 982.674
2008	370 062.97	669 651.40	1 890 053.00	1 482 287.359	166 272 818 298.918
2009	396 261.47	643 251.10	1 849 097.50	1 470 933.798	143 007 785 454.888
2010	404 925.35	821 158.50	2 469 804.10	2 078 634.309	153 013 805 014.998
2011	455 466.43	989 957.40	3 131 187.00	2 904 449.106	51 410 072 629.022
2012	595 214.22	1 066 127.50	3 436 730.50	3 781 906.838	119 146 704 508.309
2013	636 886.41	1 130 987.00	3 651 704.60	4 279 910.852	394 643 094 916.688

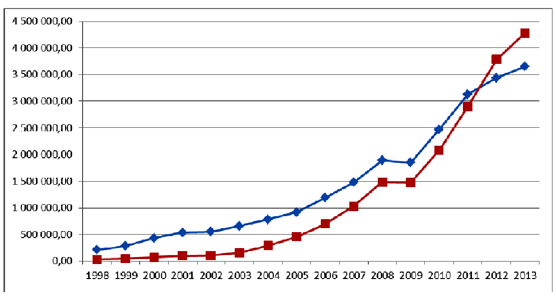

Figure 7. Graphical Representation Of The Calculation Results By The Quadratic Production Function. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Analysis of the data of Table 8 and Figure 7 yields the conclusion that the obtained model of the quadratic production function of the type 2, similar to the production function of the type 1, has substantial deviations from the real data over the entire considered interval of time.

As a result of the conducted regression analysis of the data, we obtain the following values:

- determination coefficient – 0.9722;
- standard error – 199 632.8;
- sum of the squared deviations – 2 778 133 533 860.31.

3.8 Regression analysis of data

Regression analysis of data was performed for each of the constructed production functions. The results of regression analysis of the data are shown in Table 9.

Table 9. The Results Of Regression Analysis Of The Data

Model of the production function	Determination coefficient	Standard error	Sum of the squared deviations
Linear	0.9927608	101 796.940	190 638 377 603.900
The Cobb-Douglas model with α+β=1	0.9928107	101 445.767	194 254 528 383.208
The Cobb-Douglas model with α+β≠1	0.9959064	76 549.895	87 250 448 602.112
The Cobb-Douglas model with α+β=1 and taking into account STP	0.9928614	101 087.447	194 356 305 899.109
The Cobb-Douglas model with α+β≠1 and taking into account STP	0.9729569	196 752.076	1 010 933 878 402.360
Quadratic (of the type 1)	0.9711106	203 357.327	2 740 703 153 148.110
Quadratic (of the type 2)	0.9721592	199 632.800	2 778 133 533 860.310

The criterion of selection is the following: maximum value of the determination coefficient, minimum error and minimum sum of the squared deviations.

As is seen from Table 9, the Cobb-Douglas production function with α+β≠1 fits the indicated selection criteria best of all. Besides, three models of the production function also satisfy the selection criteria:

- the linear one;
- the Cobb-Douglas one with α+β=1;
- the Cobb-Douglas model with α+β=1 and with taking into account STP.

It is exactly these 4 models that will be considered on the second stage of choosing an optimal model.

3.9 Construction of additional models

The second stage of selecting an optimal model consists in constructing additional models based on the models selected on the first stage.

The algorithm for selection of an optimal model on the basis of an additional model is as follows:

- as the initial data there are taken the economic indicators of the manufacturing industry of Kazakhstan in the years 1998-2010;
- on the basis of these data, a corresponding additional model is constructed;
- in the framework of the additional model, there are calculated the forecast data F for the years

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2011-2013 on the basis of the known values of economic indicators K and L in the years 1998-2010;

- the real GDP values (Y) and the obtained values (F) of the additional model for the years 2011-2013 are compared.

Additional model of a linear production function

We will use a model of a linear production function

$$F = a_0 + a_1 K + a_2 L \,, \tag{22}$$

where K is the capital costs, L is the wage costs.

The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (a_0 + a_1 K_i + a_2 L_i)\right]^2 \to \min_{a_0, a_1, a_2} \tag{23}$$

We carry out calculations using the data from Table 1. As a result, we get that the residual function attains minimum for a0 = -0.000009; a1 = -0.374; a2 = 3.087.

As applied to our data, the additional model of the production function will have the form:

$$F = -0.000009 - 0.374K + 3.087L \tag{24}$$

In Table 10 there are presented the economic indicators of the manufacturing industry of Kazakhstan in the years 1998-2010 with additional calculations using the linear production function.

Table 10. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2010 With Additional Calculations Using The Linear Production Function

Years	K	L	Y	F	$(Y-F)^2$
1998	40 618.00	102 893.40	208 336.60	302 422.628	8 852 180 695.121
1999	52 907.28	130 240.20	284 152.00	382 241.246	9 621 500 227.979
2000	74 794.93	149 259.00	428 932.70	432 763.575	14 675 604.602
2001	102 421.81	167 483.10	534 563.00	478 686.598	3 122 172 325.009
2002	102 550.03	172 655.70	547 414.10	494 605.461	2 788 752 317.633
2003	119 870.48	213 417.00	655 719.00	613 950.607	1 744 598 640.100
2004	191 366.17	272 891.30	781 558.70	770 799.990	115 749 846.853
2005	258 886.78	325 058.20	914 013.20	906 579.353	55 262 083.156
2006	293 475.13	423 004.30	1 188 108.00	1 195 985.448	62 054 186.578
2007	316 339.43	551 380.20	1 476 647.60	1 583 706.720	11 461 655 153.864
2008	370 062.97	669 651.40	1 890 053.00	1 928 696.739	1 493 338 569.457
2009	396 261.47	643 251.10	1 849 097.50	1 837 407.041	136 666 822.441
2010	404 925.35	821 158.50	2 469 804.10	2 383 332.782	7 477 288 861.870

On the basis of the data for K and L over the period 1998-2010, we calculate, using the additional model, the predicted values of F (Table 11). A graphical representation of the results of calculations by the linear production functions is given in Figure 8.

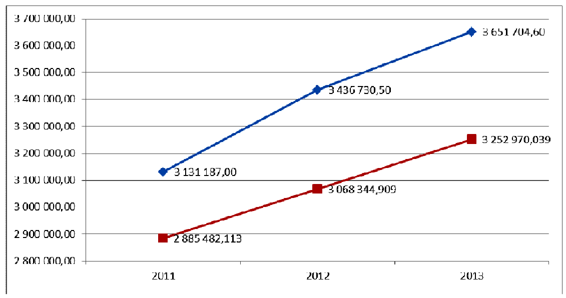

Figure 8. Graphical Representation Of The Calculation Results By The Linear Production Function. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Table 11. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 2011-2013 With Additional Calculations Using The Linear Production Function

Years	K	L	Y	F_{add}	F_{basic}	$\lvert Y-F_{add}\rvert$	$\lvert Y-F_{basic}\rvert$
2011	455 466.43	989 957.4	3 131 187.0	2 885 482.113	3 115 434.875	245 704.887	15 752.125
2012	595 214.22	1 066 127.5	3 436 730.5	3 068 344.909	3 265 372.922	368 385.591	171 357.578
2013	636 886.41	1 130 987.0	3 651 704.6	3 252 970.039	3 459 344.262	398 734.561	192 360.338

Additional model of a Cobb-Douglas production function with α+β=1

Let us construct a Cobb-Douglas production function of the form:

$$F = AK^{\alpha} L^{\beta} \,, \tag{25}$$

where α+β=1, K is the capital costs, L is the wage costs.

The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (\mathrm{a}_0 K_i^{a_1} L_i^{(1-a_1)})\right]^2 \to \min_{a_0, a_1} \tag{26}$$

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We carry out calculations using the data from Table 1. As a result, we get that the residual function attains minimum at a0 = 2.756; a1 = -0.072.

As applied to our data, the additional model of the linear production function will have the form:

$$F = 2.756 K^{-0.072} L^{1.072} \tag{27}$$

In Table 12 there are presented the economic indicators of the manufacturing industry of Kazakhstan in the years 1998-2010 with additional calculations using the Cobb-Douglas production function for α+β=1.

On the basis of the data for K and L over the period 1998-2010, we calculate, using the additional model, the predicted values of F (Table 13). A graphical representation of the results of calculations by the Cobb-Douglas production function with α+β=1 is given in Figure 9.

Table 12. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2010 With Additional Calculations Using The Cobb-Douglas Production Function With A+B=1.

Years	K	L	Y	F	$(Y\text{-}F)^2$
1998	40 618.00	102 893.40	208 336.60	303 235.738	9 005 846 471.126
1999	52 907.28	130 240.20	284 152.00	383 035.100	9 777 867 556.019
2000	74 794.93	149 259.00	428 932.70	432 357.733	11 730 849.702
2001	102 421.81	167 483.10	534 563.00	478 212.158	3 175 417 386.146
2002	102 550.03	172 655.70	547 414.10	494 022.031	2 850 713 046.856
2003	119 870.48	213 417.00	655 719.00	613 125.263	1 814 226 427.390
2004	191 366.17	272 891.30	781 558.70	771 508.793	101 000 637.589
2005	258 886.78	325 058.20	914 013.20	910 575.884	11 815 139.734
2006	293 475.13	423 004.30	1 188 108.00	1 196 828.944	76 054 868.487
2007	316 339.43	551 380.20	1 476 647.60	1 581 629.847	11 021 272 088.434
2008	370 062.97	669 651.40	1 890 053.00	1 926 101.397	1 299 486 916.572
2009	396 261.47	643 251.10	1 849 097.50	1 835 693.889	179 656 790.545
2010	404 925.35	821 158.50	2 469 804.10	2 381 412.360	7 813 099 722.069

Table 13. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 2011-2013 With Additional Calculations Using The Cobb-Douglas Production Function With A+B=1

Years	K	L	Y	$F_{add.}$	F_{basic}	$\lvert Y-F_{add.}\rvert$	$\lvert Y-F_{basic}\rvert$
2011	455 466.43	989 957.4	3 131 187.0	2 885 365.842	3 112 717.883	245 821.158	18 469.117
2012	595 214.22	1 066 127.5	3 436 730.5	3 064 211.235	3 263 101.961	372 519.265	173 628.539
2013	636 886.41	1 130 987.0	3 651 704.6	3 248 603.857	3 457 468.440	403 100.743	194 236.16

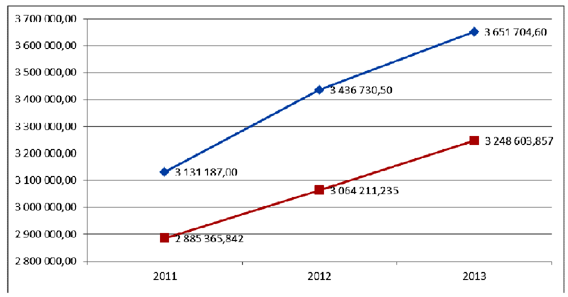

Figure 9. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function With A+B=1. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Additional model a Cobb-Douglas production function with α+β≠1.

Let us construct a Cobb-Douglas production function of the form:

$$F = AK^{\alpha} L^{\beta}, \tag{28}$$

where α+β≠1, K is the capital costs, L is the wage costs.

The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (\mathrm{a}_0 K^{a_1} L^{a_2}) \right]^2 \rightarrow \min_{a_0, a_1, a_2} \tag{29}$$

We carry out calculations using the data from Table 1. As a result, we get that the residual function attains minimum at a0 = 1.923; a1 = -0.028; a2 = 1.057.

As applied to our data, the additional model of the linear production function will have the form:

$$F = 1.923 K^{-0.028} L^{1.057} \tag{30}$$

In Table 14 there are presented the economic indicators of the manufacturing industry of Kazakhstan in the years 1998-2010 with additional calculations using the Cobb-Douglas production function with α+β≠1.

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Table 14. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2010 With Additional Calculations Using The Cobb-Douglas Production Function With A+B≠1.

Years	K	L	Y	F	$(Y\text{-}F)^2$
1998	40 618.00	102 893.40	208 336.60	283 976.384	5 721 376 885.953
1999	52 907.28	130 240.20	284 152.00	361 631.670	6 003 099 255.927
2000	74 794.93	149 259.00	428 932.70	413 645.831	233 688 360.711
2001	102 421.81	167 483.10	534 563.00	463 116.560	5 104 593 804.662
2002	102 550.03	172 655.70	547 414.10	478 232.474	4 786 097 365.138
2003	119 870.48	213 417.00	655 719.00	595 720.646	3 599 802 430.309
2004	191 366.17	272 891.30	781 558.70	762 440.689	365 498 362.247
2005	258 886.78	325 058.20	914 013.20	909 572.345	19 721 196.701
2006	293 475.13	423 004.30	1 188 108.00	1 197 358.976	85 580 554.741
2007	316 339.43	551 380.20	1 476 647.60	1 581 205.943	10 932 447 096.865
2008	370 062.97	669 651.40	1 890 053.00	1 933 281.785	1 868 727 848.512
2009	396 261.47	643 251.10	1 849 097.50	1 849 258.299	25 856.191
2010	404 925.35	821 158.50	2 469 804.10	2 392 402.203	5 991 053 678.765

On the basis of the data for K and L over the period 1998-2010 we calculate, using the additional model, the predicted values of F (Table 15). A graphical representation of the calculation results by the Cobb-Douglas production function with α+β≠1 is given in Figure 10.

Table 15. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 2011-2013 With Additional Calculations Using The Cobb-Douglas Production Function With A+B≠1

Years	K	L	Y	$F_{add.}$	F_{basic}	$\lvert Y\text{-}F_{add.}\rvert$	$\lvert Y\text{-}F_{basic.}\rvert$
2011	455 466.43	989 957.4	3 131 187.0	2 905 541.562	3 102 812.777	225 645.438	28 374.223
2012	595 214.22	1 066 127.5	3 436 730.5	3 118 900.463	3 375 271.455	317 830.037	61 459.045
2013	636 886.41	1 130 987.0	3 651 704.6	3 313 528.523	3 607 262.340	338 176.077	44 442.26

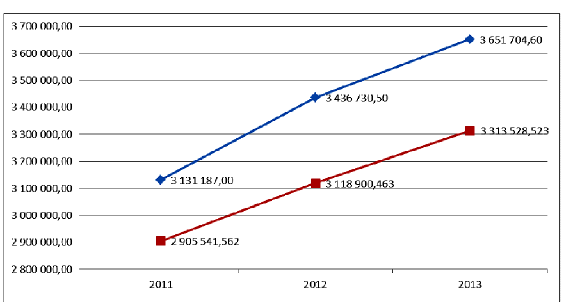

Figure 10. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function With A+B≠1.

The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Additional model of a Cobb-Douglas production function, taking into account STP, with α+β=1.

Let us construct a Cobb-Douglas production function, which takes into account STP, of the form:

$$F = Ae^{p_0 t} K^{\alpha} L^{(1-\alpha)} , \tag{31}$$

where α+β=1, K is the capital costs, L is the wage costs, $e^{p_0 t}$ is a special factor of scientific progress, p_0 is a parameter of neutral STP ($p_0 > 0$). The residual function has the form:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[Y_i - (a_0 e^{p_0 t} K^{a_1} L^{(1-a_1)})\right]^2 \to \min_{a_0, a_1, p_0} \tag{32}$$

We carry out calculations using the data from Table 1. As a result, we get that the residual function attains minimum at p0 = 0.0014; a0 = 2.752; a1 = -0.073.

As applied to our data, the additional model of the linear production function will have the form:

$$F = 2.752 e^{0.0014 t} K^{-0.073} L^{1.073} \tag{33}$$

In Table 16 there are presented the economic indicators of the manufacturing industry of Kazakhstan in the years 1998-2010 with additional calculations using the Cobb-Douglas production function with α+β =1 and with taking into account STP.

Table 16. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 1998-2010 With Additional Calculations Using The Cobb-Douglas Production Function With A+B=1 And With Taking Into Account STP.

Years	K	L	Y	F	$(Y\text{-}F)^2$
1998	40 618.00	102 893.40	208 336.60	303 484.036	9 053 034 597.986
1999	52 907.28	130 240.20	284 152.00	383 342.435	9 838 742 464.790
2000	74 794.93	149 259.00	428 932.70	432 652.493	13 836 859.582

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2001	102 421.81	167 483.10	534 563.00	478 483.461	3 144 914 669.093
2002	102 550.03	172 655.70	547 414.10	494 310.582	2 819 983 676.471
2003	119 870.48	213 417.00	655 719.00	613 503.067	1 782 184 973.543
2004	191 366.17	272 891.30	781 558.70	771 885.814	93 564 726.972
2005	258 886.78	325 058.20	914 013.20	910 954.291	9 356 924.460
2006	293 475.13	423 004.30	1 188 108.00	1 197 421.171	86 735 153.451
2007	316 339.43	551 380.20	1 476 647.60	1 582 585.148	11 222 764 075.593
2008	370 062.97	669 651.40	1 890 053.00	1 927 306.230	1 387 803 153.460
2009	396 261.47	643 251.10	1 849 097.50	1 836 727.608	153 014 240.135
2010	404 925.35	821 158.50	2 469 804.10	2 383 057.893	7 524 904 489.558

On the basis of the data for K and L over the period 1998-2010 we calculate, using the additional model, the predicted values of F (Table 17). A graphical representation of the calculation results by the Cobb-Douglas production function with α+β=1 and with taking into account STP is given in Figure 11.

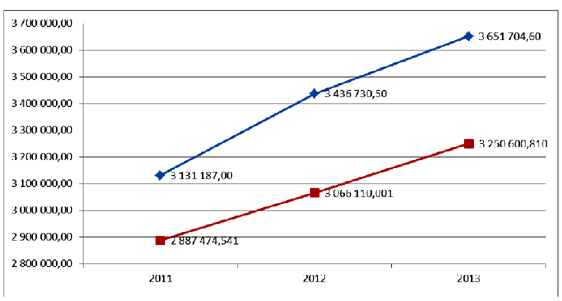

Figure 11. Graphical Representation Of The Calculation Results By The Cobb-Douglas Production Function With Α+Β=1 And With Taking Into Account STP. The Blue Line Represents The Real Values; The Red Line, The Values Of The Model.

Table 17. Economic Indicators Of The Manufacturing Industry Of Kazakhstan In The Years 2011-2013 With Additional Calculations Using The Cobb-Douglas Production Function With Α+Β=1 And With Taking Into Account STP

Years	K	L	Y	$F_{add.}$	F_{basic}	$\vert Y-F_{add.}\vert$	$\vert Y-F_{basic}\vert$
2011	455 466.43	989 957.4	3 131 187.0	2 887 474.541	3 108 046.491	243 712.459	23 140.509
2012	595 214.22	1 066 127.5	3 436 730.5	3 066 110.001	3 264 630.365	370 620.499	172 100.135
2013	636 886.41	1 130 987.0	3 651 704.6	3 250 600.810	3 459 391.254	401 103.79	192 313.346

4. DISCUSSION

The Cobb-Douglas function is often used to simulate manufacturing. For example, in the paper [22], by means of the Cobb-Douglas function, there is described the management of the production, which consumes a scarce natural resource and replaces outdated production assets. The model includes some resource-saving technological changes, various prices of resources and the ecological quotas of resource consumption. The model is reduced to nonlinear integral equations with unknown limits of integration. It is assumed in some models that the capital and resources are complementary factors of production [23, 25, 26, 27, 28]. Other models suggest that the capital and resource are interchangeable [24.31]. In the paper [30], it is demonstrated that the Cobb-Douglas function is the best for the problems with a non-renewable resource. A Cobb-Douglas function, involving capital and labor but without resource, is used in [29].

In our work we have carried out research and built some additional models of production functions in order to identify the optimal one for the manufacturing industry of Kazakhstan.

Let us bring together the results of constructing the additional models into a single table, Table 18.

Table 18. Indicators Of The Constructed Additional Models

Model of the production function	2011	2012	2013
Linear	2 885 482.113	3 068 344.909	3 252 970.039
The Cobb-Douglas model with α+β=1	2 885 365.842	3 064 211.235	3 248 603.857
The Cobb-Douglas model with α+β≠1	2 905 541.562	3 118 900.463	3 313 528.523
The Cobb-Douglas model with α+β=1 and with taking into account STP	2 887 474.541	3 066 110.001	3 250 600.810
Real values	3 131 187	3 436 730.5	3 651 704.6

As is seen from Table 18, the data obtained by the Cobb-Douglas model of the production function with α+β≠1 are the closest ones to the real data. Thus, we select this model as an optimal one for the given industry.

5. CONCLUSION

The purpose of this paper is to construct an optimal mathematical model of functioning of the manufacturing industry of the Republic of Kazakhstan. To solve this problem, there are created several models of production functions. Then, out of the constructed production functions, there is selected the optimal one for the manufacturing industry of Kazakhstan. Our studies have shown that the optimal production function for the Kazakhstan manufacturing industry is the Cobb-Douglas function with α+β≠1.

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The selected optimal model of the Cobb-Douglas production function with $\alpha + \beta \neq 1$ can be used to predict the values of the gross domestic product on the basis of known or anticipated levels of the capital and wage costs.

In the future, similar research can be carried out for other sectors of the economy of Kazakhstan. In this case, to automate the processing of large volume of information and to perform computing works, one can create a software package having graphical interface which constructs the production functions, defines their parameters, makes forecasts, draws the corresponding graphics, etc.

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