NEURO-FUZZY MODELLING AND CONTROL OF MULTISTAGE DYNAMIC PROCESSES THAT DEPEND ON INPUTS WITH UNCERTAINTY ELEMENTS

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ABSTRACT

In practice, we meet with a frequent necessity for modelling dynamic multistage processes that depend on several time-varying factors, which can be measured with accuracy. This requires using a model that combines properties of the difference, switched, and neuro-fuzzy models with a large number of inputs and a big memory depth of each input. Such model will take into account all uncertainties, converting input actions into fuzzy processes by means of fuzzification, and will precisely reflect the multistage nature and dynamics of the object being studied. However, a large number of parameters can significantly complicate its configuration. The purpose of this work is to improve efficiency of structural and parametric modelling of multistage dynamic processes due to the development of a class of difference neuro-fuzzy switched models, as well as to the research and development of approaches to fuzzification of discrete processes on model inputs in order to simplify the process of its configuration. The authors have introduced a new class of difference neuro-fuzzy switched models that are characterized by a combination of structures of difference fuzzy models, neural networks, and systems with switchings enabling to model complex multistage processes, which are characterized by abrupt changes in structure or parameters. The authors have proposed a mechanism for fuzzification of input actions of a difference neuro-fuzzy switched model, which is characterized by the ability to convert input actions into discrete fuzzy processes using two-dimensional fuzzy sets, and allows reducing the number of configurable parameters of the model.

Keywords: Modelling, Multistage Dynamic Processes, Neural Networks, Random Input Parameters.

1. INTRODUCTION

Currently, the high-quality control of complex processes requires highly precise calculation of significant parameters. Methods of artificial intelligence based on fuzzy and neuro-fuzzy modelling are widely used in the problems of modelling processes that are characterised by complexity and presence of measurement accuracies. The advantages of fuzzy models include resistance to inaccurate input data. Imprecision in fuzzy models is accounted for by fuzzification – conversion of crisp input values into fuzzy ones. Neuro-fuzzy models are very popular, as they combine advantages of fuzzy models and neural networks, which are the simplicity and ability to automatically configure parameters (training).

In practice, we meet with a frequent necessity for modelling dynamic multistage processes that depend on several time-varying factors, which can be measured with accuracy. This requires using a model that combines properties of the difference, switched, and neuro-fuzzy models with a large number of inputs and a big memory depth of each input. Such model will take into account all uncertainties, converting input actions into fuzzy processes by means of fuzzification, and will precisely reflect the multistage nature and dynamics of the object being studied. However, a large number of parameters can significantly complicate its configuration.

In order to overcome the disadvantage described, it is proposed to solve the problem of developing a class of difference neuro-fuzzy switched models, identification of which will not be labour intensive
and time consuming. Such models should combine the ability to account for input processes of large length with relatively simple structure, which provides for easy configuration.

2. EXISTING RESULTS

Implementation of neuro-fuzzy models and methods within the control systems is currently considered to be an extensively investigated problem. Thus, Barabanov [4] investigated the use of neural networks in the problems of time series forecasting. Du and Fei considered a two-layer control system based on a multi-agent neural network [7]. Kravets discussed the effectiveness of monitoring systems management from the perspective of neuro-fuzzy software development [12]. Kumpati analysed the problem of adaptive control of nonlinear multivariable systems. Cai and Xiang investigated adaptive neural finite-time control for a class of switched nonlinear systems [5]. Zhbanova studied features of identifying a difference fuzzy model with switchings [21].

Theoretical aspects of control over multistage dynamic processes that depend on inputs with uncertainty elements were investigated by Hua [10], Kravets [11], Arefi [3], and Wu [19].

Perspective directions in fuzzy modelling include difference fuzzy models and fuzzy models with switchings. Analysis of the works dedicated to these approaches shows that the difference fuzzy models have a number of useful features and are successfully applied for describing dynamic processes. Fuzzy switched models are used to describe multistage process characterized by abrupt changes in structure or parameters.


The purpose of this work is to improve efficiency of structural and parametric modelling of multistage dynamic processes due to the development of a class of difference neuro-fuzzy switched models, as well as to the research and development of approaches to fuzzification of discrete processes on model inputs in order to simplify the process of its configuration.

3. METHODOLOGY

Technology of neuro-fuzzy switched models is used, which combines the advantages of difference fuzzy models, neuro-fuzzy systems of ANFIS type, and systems with switchings. At the same time, accounting for the more complete information on the object being modelled entails a sharp increase in the number of configurable parameters of models and results in difficulty of settings.

Mechanism for fuzzification of the input values of a model is also applied and is based on their conversion into discrete fuzzy processes with the subsequent implementation analysis.

4. RESULTS

New class of difference neuro-fuzzy switched models

A new class of difference neuro-fuzzy switched models (DNFSM) is proposed. Also, a mechanism for fuzzification of input actions of models from the class suggested is presented, wherein its implementation leads to a significant reduction in the number of configurable parameters in the prerequisites of the DNFSM rules. Algorithms for determining DNFSM prerequisite parameters are developed.

DNFSM class is a combination of structures of difference fuzzy models, neuro-fuzzy systems, and systems with switchings (Fig. 1).
The DNFSM consists of several difference neuro-fuzzy ANFIS-submodels, which are supplemented by the switching law, and is described by the rule database:

\[
R^1: \text{if } u_{\sigma_i}[t] \text{ is } A_{\sigma_i}^1[t], \ldots, \text{and } u_{\sigma_m}[t - n + 1] \text{ is } A_{\sigma_m}^1[t], \text{ then } y_{\alpha_i}[t+1] = a_{\alpha_i}^1 u_{\alpha_i}[t] + b_{\alpha_i}^1,
\]

(1)

where \( y_{\alpha_i}[t+1] \in \mathbb{R}^1 \) is an output of each DNFSM rule; \( a_{\alpha_i}^1 \in \mathbb{R}^{1 \times \alpha_m} \), \( b_{\alpha_i}^1 \in \mathbb{R}^1 \) are parameters of difference equations in conclusions. Vector

\[
U_{\alpha_i}[t] = [u_{\alpha_1}[t], u_{\alpha_2}[t], \ldots, u_{\alpha_m}[t]] \in \mathbb{R}^{1 \times \alpha_m}
\]

comprises multistage input processes of DNFSM \( i = 1, \ldots, m \) is a number of input processes; \( n \) is the DNFSM memory depth.

The base (1) also includes a switching signal \( \sigma \in S = \{1, 2, \ldots, |S|\} \), which determines active submodel at each time point. Parameter \( l = 1, \ldots, L_\sigma \) determines the number of rules in each submodel, \( A_{\alpha_{ij}}^1[t] \) are the output (prerequisite) fuzzy sets.

Output of the DNFSM submodel (1) at the fixed value \( \sigma = S \):

\[
y_{\alpha_i}[t+1] = \sum_{l=1}^{L_\sigma} \alpha_{ij}^1 (a_{ij}^1 U_{\alpha_i}[t] + b_{ij}^1) / \sum_{l=1}^{L_\sigma} \alpha_{ij}^1,
\]

(2)

where \( U_{\alpha_i}[t] = [u_{\alpha_i1}[t], \ldots, u_{\alpha_im}[t]] \), \( u_{\alpha_i}[t] = [u_{\alpha_i1}[t], \ldots, u_{\alpha_i1}[t - n + 1]] \) are the submodel’s output actions, values of the \( i \)-th input subprocess at \( \sigma = S \) or stage of input multistage process \( u_{\alpha_i}[t] \). In (2)

\[
\alpha_{ij}^1 = \prod_{i=1}^{m} \prod_{j=1}^{n} \mu_{ij}^1(u_{\alpha_i}[t - j + 1])
\]

is the level of rule \( R^1 \) activation; \( \mu_{ij}^1(u_{\alpha_i}[t - j + 1]) \) is the degree of input \( u_{\alpha_i}[t - j + 1] \) membership in the fuzzy set \( A_{\alpha_{ij}}^1[t] \). Output values of DNFSM (1) at each time point are determined by the output values of the active submodel (2).

In work, the multistage input processes of DNFSM, functioning stepwise, are characterized by the set of \( \mathbb{M} \) (according to the number of DNFSM inputs) parameters and possess \( |S| \) (according to the number of DNFSM submodels) of working steps; multistage nature is reflected by the changes in structural links between parameters during the transition from one step to another.

The DNFSM proposed combines the advantages of difference fuzzy models, neuro-fuzzy systems of ANFIS type, and systems with switchings – precise reflection of reality due to considering for the large amount of information, possibility of neurolfuzzy parameter configuration on a training set, possibility of modelling multistage processes characterized by abrupt changes in structure or parameters. However, allowance for more complete information on the object being modelled entails a sharp increase in the number of configurable parameters of DNFSM (1) and results in difficulty of settings.

In order to eliminate this drawback, a mechanism for fuzzification of the input values of the model (1) is proposed, which is based on their conversion into discrete fuzzy processes with the subsequent implementation analysis.

Let us note that the vectors of values

\[
u_{\alpha_i}[t] = [u_{\alpha_i1}[t], \ldots, u_{\alpha_i1}[t - n + 1]],
\]

arriving at the input of DNFSM (1) submodel at \( \sigma = S \), after fuzzification can be considered as fuzzy
processes. A fuzzy process is a process, each value of which is given by a certain fuzzy set (Fig. 2). Let us call the sequence of fuzzy process values a realisation, in a similar way to random processes. Centre and width of the fuzzy process are the time-dependent functions.

Parametric identification of DNFSM (1) includes the problem of identifying multistage fuzzy input processes $u_{\sigma_i}(t)$, as the fuzzy sets $A_{\sigma_{ij}}(t)$ represent their sections. It can be said that the parameter identification of the DNFSM rule prerequisites is an equivalent to identification of fuzzy process parameters.

At a standard approach to the identification of prerequisites, DNFSM parameters of each section of a fuzzy process are set individually. The number of sections increases simultaneously with the growth of memory depth $n$. Thus, accounting for the large number of factors turns into sharp increase in the number of difference model parameters and time-consuming identification.

To reduce the number of DNFSM parameters, a fuzzification mechanism is suggested, which resides in describing the steps of multistage fuzzy input processes not by the kits of one-dimensional membership function, but by some certain two-dimensional unified functions.

In the elementary case, such function is a mathematical description of $u_{\sigma_i}(t)$ step of a multistage fuzzy process with a linear centre and constant width. One can also say that it is a membership function of a two-dimensional fuzzy set.

The two-dimensional Gaussian fuzzy set $B_{\sigma_i}(t)$ with a linear centre and constant width is shown in Fig. 3. The sections represent one-dimensional fuzzy sets $A_{\sigma_{ij}}(t)$ of the DNFSM submodel at $\sigma = S$.

For an arbitrary DNFSM submodel, the degree of process membership $u_{\sigma_i}(t)$ in the two-dimensional fuzzy set $B_{\sigma_i}(t)$ is calculated by the formula

$$
\mu_{\sigma_i}(u_m(t)) = \frac{1}{\pi \cdot w_{\sigma_i}} \sum_{j=1}^{m} \exp \left[ -\frac{(c_{\sigma_{ij}} - u_m(t) - u_m(t-j))^2}{w_{\sigma_i}^2} \right],
$$

where $c_{\sigma_{ij}}$ and $c_{\sigma_{ij}}$ are the linear centre parameters of a two-dimensional fuzzy set; $w_{\sigma_i}$ is a parameter defining the set width.

DNFSM rules with two-dimensional fuzzy sets $B_{\sigma_i}(t)$ in prerequisites will be as follows:

$$
R^l_{\sigma_i} : \text{If } u_{\sigma_i}(t) \text{ and } u_{\sigma_i}(t+j) \text{ then } y_i(t) = a_i u_{\sigma_i}(t) + b_i.
$$

DNFSM (4) output will be determined by the formula (2) with account for (3).

From the base (4) it can be seen that for each DNFSM submodel, it is required to already set not $m \cdot n$, but $m$ of fuzzy set kits. The number of DNFSM prerequisite parameters will be reduced consequently. Two-dimensional fuzzy sets (3) are especially convenient in cases of big memory depth of the DNFSM input process.

**Algorithmic presentation of parameter settings**

For DNFSM, which multistage fuzzy input processes are described by the sets (3), we will provide algorithms of parameter configuration.

Method of identification, which is traditional for neuro-fuzzy models, – when parameter values of prerequisites and conclusions are selected from the condition of error function minimization – often
provides for very good results, especially at a large number of model inputs.

In such cases, to improve the accuracy of configuration, a less common method comprising individual configuration of conclusions and prerequisites is typically used. Parameters of rule conclusions are configured through minimizing the sum-squared error function of the model. Parameters of fuzzy sets in the prerequisites are determined based on the training sample analysis. Advantage of a separate approach is to reduce the dimension of the minimization problem.

When using the proposed fuzzification mechanism of DNFSM input actions, the rule prerequisite parameters include the centre and width parameters of fuzzy processes that are described by the Gaussian two-dimensional fuzzy sets of $B_{si}^l[t]$ type (3). The following algorithms are developed for their identification.

Algorithm 1. Numerical algorithm for configuring linear centres of two-dimensional fuzzy sets.

Step 1. Suppose the $k$-th element of the DNFSM submodel’s training set at a fixed value $\sigma = s$ has the form of $\{U_s^k[t], y_s^k[t + 1]\}$, $k = 1, ..., K$. Here, $U_s^k[t] = [u_{si}^k[t], ..., u_{sm}^k[t]]$ is the implementation of the step number $S$ of the $i$-th multistage input process.

Step 2. On the implementations of $u_{si}^k[t]$ (for $k = 1, ..., K$) using the least module method, regression lines of the form $u = c_1 t + c_0$ are built. The result is a set of parameters $\{(c_1^1, c_0^1), ..., (c_1^K, c_0^K)\}$. It should be noted that the choice of the method of least modules is explained by the lower sensitivity to outliers if compared to the method of least squares.

Step 3. Suppose we are given $t_0 = t - n + 1$. Let us calculate $u_0^k = c_1^k t_0 + c_0^k$. Let us choose $u_0^{\text{max}}$ and $u_0^{\text{min}}$.

Step 4. Let us divide segment $[u_0^{\text{min}}, u_0^{\text{max}}]$ into $Q$ parts. Here, $Q$ is the number of two-dimensional fuzzy sets $B_{si}^1[t], ..., B_{si}^Q[t]$ required for the process of fuzzification $u_{si}[t]$. As a result, domain of the process $u_{si}[t]$ values will be broken into $Q$ fragments by horizontal lines.

Step 5. Coefficients $c_0^k, c_1^k$ of the straight lines entering into fragments are averaged over each fragment. Averaged values $(c_1^1, c_0^1), ..., (c_1^Q, c_0^Q)$ are taken as the parameters of centres of input fuzzy sets $B_{si}^l[t], ..., B_{si}^Q[t]$ of the input number $i$ of DNFSM model at $\sigma = s$.

Step 6. Steps 1-5 are repeated for each input of each DNFSM submodel, $i = 1, ..., m$, $\sigma = 1, ..., s$.

In order to identify the width parameters of fuzzy processes, which are described by two-dimensional fuzzy sets (3), an algorithm that provides for fulfillment of the unity partition condition is proposed. Condition of the unity partition is fulfilled, when for each input value $u \in U$, the sum of degrees of membership in all fuzzy sets $A_i, \ 1 = 1, ..., L$ equals to 1:

$$\sum_{l=1}^{L} \mu_{A_i}(u) = 1$$

Fuzzy and neuro-fuzzy models show the best results, when the condition of unity partition for input fuzzy sets is fulfilled.

Algorithm 2. Numerical algorithm for configuring width of two-dimensional fuzzy sets based on the conditions of unity partition.
Step 1. Suppose we are given the adjacent two-dimensional sets $B^1[t]$ and $B^2[t]$ with the centres configured, which surfaces are determined by the following expressions

$$
\exp \left[ -\frac{1}{w_1} \left( c_1^1 t + c_0^1 - u \right)^2 \right],
$$

$$
\exp \left[ -\frac{1}{w_2} \left( c_1^2 t + c_0^2 - u \right)^2 \right],
$$

and level $z = 0.5$, at which the sets must intersect. Requirement $z = 0.5$ expresses fulfilment of the unity partition condition.

Step 2. Suppose we require that the line of fuzzy sets intersection (critical line) is lying on the level desired. Let us equate

$$
\exp \left[ -\frac{1}{w_1} \left( c_1^1 t + c_0^1 - u \right)^2 \right] = 0.5.
$$

Step 3. Let us determine type of the function that assigns the width value. It is obvious that for the critical line to be straight, widths $w^1$ must be assigned in the form $1 = 1.2$. The equation of critical line will be as follows

$$
u = (c_1^1 - w^1 \sqrt{\ln 2}) t + c_0^1 - w^1_0 \sqrt{\ln 2},
$$

$1 = 1.2$.

Step 4. Let us require that the critical line in OTU plane projection shall divide distance between the projections of fuzzy sets’ centres in half. For the two adjacent two-dimensional fuzzy sets $B^1[t]$ and $B^2[t]$, the line extending between the centres is defined by the formula

$$
u = c_1^{cr} t + c_0^{cr},
$$

where $c_1^{cr} = \frac{1}{2} (c_1^1 + c_1^2)$, $c_0^{cr} = \frac{1}{2} (c_0^1 + c_0^2)$.

Step 5. By equating the corresponding coefficients of straight lines obtained in steps 3 and 4, let us calculate the unknown parameters of width $w^1_1$ and $w^0_0$ of two-dimensional fuzzy sets $B^1[t]$ and $B^2[t]$: $w^1_1 = \frac{1}{\sqrt{\ln 2}} (c_1^1 - c_1^{cr}) = \frac{1}{2\sqrt{\ln 2}} (c_1^1 - c_1^2)$, $w^0_0 = \frac{1}{\sqrt{\ln 2}} (c_0^1 - c_0^{cr}) = \frac{1}{2\sqrt{\ln 2}} (c_0^1 - c_0^2)$, $1 = 1.2$.

Two two-dimensional fuzzy sets with linear centres and selected according to the algorithm 2 width parameters are shown in Fig. 4, a. It can be seen that the critical line of fuzzy sets coincide at the level 0.5 marked by a horizontal plane. In this case, there will be no breach of the unity partition condition (in contrast to the case shown in Fig. 4, b).

![Figure 4. Two-Dimensional Fuzzy Sets With Satisfied (a) And Unsatisfied (b) Condition Of Unity Partition](image)

The DNFSM class proposed allows for description of complex multistage dynamic production processes. The new mechanism for fuzzification of input actions of the model class proposed allows for reduction in the number of parameters configured. Numerical algorithms for the identification of two-dimensional fuzzy sets parameters allow for DNFSM configuration with high accuracy.

**Software package. Results of computational experiments**

Structural features of the software package for implementation of the DNFSM class and numerical algorithms of parametric identification are discussed. Results of computational experiments...
are provides, confirming rationality of the proposed mechanism for fuzzification of DNFSM input actions, which is based on the conversion of inputs into discrete fuzzy processes using two-dimensional fuzzy sets (3).

The software package is developed in the MATLAB environment. Generalized structure of the software package is shown in Fig. 5.

**Figure 5. Structure Of Software Package For Working With The DNFSM Class**

Let us briefly describe the software package modules that implement the DNFSM class and algorithms for configuration of their parameters.

Module of definition for model structure – DNFSM_Struct. In this module, researcher assigns the structure of DNFSM – the number of submodels, inputs, and rules. This information is transmitted to the module of configuration for DNFSM parameters. Furthermore, a function for model output calculation will be formed on its base.

Modules of configuration for prerequisite parameters – WideFinding and CenterFinding implement the numerical algorithms for configuration of centre and width of input fuzzy processes, which, in accordance with the proposed mechanism for fuzzification, are assigned by two-dimensional fuzzy sets (3). The centre and width parameters obtained are transmitted to the module of calculation for output value.

Module of calculation for output value – DNFSM_Output receives data on the DNFSM structure and parameters configured. Based on these data, it generates a MATLAB file-script, which is a function of DNFSM output. 

**Computational Experiment**

Here is a brief description and results of one of the computational experiments carried out with the implementation of software package.

The first computational experiment. Let us conduct a comparative analysis of DNFSM work with the proposed mechanism for fuzzification of input actions, which is based on implementation of two-dimensional fuzzy sets (3), and DNFSM with standard approach to fuzzification.

Suppose that a certain response depends on a set of values of a single factor: 
\[ y_\sigma[t + 1] = f_\sigma(u_\sigma[t],...,u_\sigma[t - n + 1]) \]

Let the depth of model memory be \( n = 5 \). Both models will be configured on a set of data obtained from seven elements, \( \{ y_\sigma^k[t + 1], u_\sigma^k[t],...,u_\sigma^k[t - 4] \} \), \( k = 1,\ldots,7 \), configuration of parameters for prerequisites and conclusions is performed separately.

Let us consider the simplest case of DNFSM with a single submodel, \( \sigma = 1 \). The case of a standard approach to fuzzification of input process \( u_1[t] \) will require 10 one-dimensional fuzzy sets (assuming that each input value is described by two sets \( A_{11j}^1 \) and \( A_{11j}^2 \)). Base of DNFSM rules with one-dimensional Gaussian fuzzy sets is as follows:

\[ R_1: \text{If } u_1[t] \text{ is } A_1^1, \ldots, \text{ and } u_1[t-4] \text{ is } A_4^1, \text{ then } y[t+1] = a_1(u_1[t]) + b_1, l = 1,2.\]  

Parameters of 10 one-dimensional fuzzy sets \( A_{11j}^l \) were determined manually.

When using the proposed mechanism for fuzzification of input process based on the implementation of two-dimensional fuzzy sets (2), the base of DNFSM rules will be as follows:
$\mathbf{R}_1^i : \text{If } \mathbf{u}_i[t] \text{ is } B_i^1[t] \text{ then } y_i^1[t+1] = a_i^1 \mathbf{u}_i[t]^T + b_i^1, \quad l = 1, 2.$ \hfill (6)

Here and in (5), $\mathbf{u}_i[t] = [u_i[t], ..., u_i[t-4]]$ is the input process. In (6), $\mathbf{u}_i[t]$ is described by two two-dimensional fuzzy sets with linear centres and constant widths $B_i^1[t]$ and $B_i^2[t]$. Parameters of fuzzy sets were selected using the algorithms described and software package developed. Parameters of rule conclusions of both models were determined through minimization of sum-squared error function.

In order to more fully illustrate the benefits of the proposed mechanism for fuzzification of DNFSM input actions, let us double the value of memory depth. Dependence being modelled will be as follows

$$y_i[t+1] = f_i \left( u_i[t], ..., u_i[t-9] \right).$$

In this case, base of DNFSM rules with one-dimensional Gaussian fuzzy sets will be as follows:

$$\mathbf{R}_1^i : \text{If } \mathbf{u}_i[t] \text{ is } A_i^l, ..., \text{ and } u_i[t-9] \text{ is } A_{im}, \text{ then } y_i^l[t+1] = a_i^l \mathbf{u}_i[t]^T + b_i^l, \quad l = 1, 2.$$

$\mathbf{R}_1^i : \text{If } \mathbf{u}_i[t] \text{ is } A_i^l, ..., \text{ and } u_i[t-9] \text{ is } A_{im}, \text{ then } y_i^l[t+1] = a_i^l \mathbf{u}_i[t]^T + b_i^l, \quad l = 1, 2.$ \hfill (7)

Standard approach to fuzzification of the input process $\mathbf{u}_i[t]$ will require determination of the parameters of 20 one-dimensional fuzzy sets.

When using the proposed mechanism for fuzzification of DNFSM input actions, the base of model rules will be as follows:

$$\mathbf{R}_1^i : \text{If } \mathbf{u}_i[t] \text{ is } B_i^1[t] \text{ then } y_i^1[t+1] = a_i^1 \mathbf{u}_i[t]^T + b_i^1, \quad l = 1, 2.$$

$\mathbf{R}_1^i : \text{If } \mathbf{u}_i[t] \text{ is } B_i^1[t] \text{ then } y_i^1[t+1] = a_i^1 \mathbf{u}_i[t]^T + b_i^1, \quad l = 1, 2.$ \hfill (8)

Here and in base (7), $\mathbf{u}_i[t] = [u_i[t], ..., u_i[t-9]]$ is the input process.

The number of two-dimensional fuzzy sets $B_i^l[t]$ in prerequisites of rules (8) will not change in comparison with (6). This happens due to the nature of a two-dimensional fuzzy set (3), which allows for fuzzification of input processes of any length. Parameters of prerequisites and conclusions of DNFSM (7), (8) rules were determined in a similar to the DNFSM (5), (6) parameters way.

After determining the DNFSM parameters on the test set, their outputs were calculated. Errors of the models (5)-*8) were of the same order. Meanwhile, the major advantage of DNFSM (6) and (8) with the proposed mechanism for fuzzification, which was based on the implementation of two-dimensional fuzzy sets, a smaller number of configurable parameters in the prerequisites of rules. This advantage is shown in the Table 1.

Table 1. Number of DNFSM Prerequisite Parameters

<table>
<thead>
<tr>
<th>Number of DNFSM prerequisite parameters at memory depth $n = 5$</th>
<th>Number of DNFSM prerequisite parameters at memory depth $n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard approach to fuzzification</td>
<td>20</td>
</tr>
<tr>
<td>Proposed mechanism for fuzzification</td>
<td>6</td>
</tr>
</tbody>
</table>

Implementation of the proposed mechanism for fuzzification of input actions leads to reduction in the number of DNFSM parameters and simplifies the process of its configuration.

Experiments discussed in Chapter 3 demonstrate advantages of applying DNFSM class and fuzzification mechanism, which is based on conversion of input processes into fuzzy ones using two-dimensional fuzzy sets (2). When using the fuzzification mechanism proposed, the number of model’s configurable parameters is significantly reduced.

Application of DNFSM class and methods of identifying their parameters to the description of multi-stage process on the example of sugar syrup-making process

Sugar syrup-making process plays an important role in sugar production. The main parameter that determines the course of sugar-making process is the syrup density $D$, which is to be changed in a certain way. Density $D$ required is achieved through a combination of other three parameters, which are changed by the adjusting devices (valves): level of syrup $L$, vacuum $V$, and steam pressure $P$. Sugar-making parameters are the
processes are regulated in accordance with the trajectories – broken lines, the form of which is determined by the “reference points” (RP). Operator assigns the RP in such a way that implementations of the $L$, $V$, $P$, and $D$ processes shall coincide with the trajectories required. Even an experienced operator is not always able to assign RP values without mistakes. In this regard, it was tasked to develop a system that would recommend RP values to operator.

Modelling of operator’s reasoning based on the information about the $L$, $V$, $P$, and $D$ processes is possible with the help of a fuzzy or neuro-fuzzy model. A prerequisite for application of a neuro-fuzzy model is the availability of data on the sugar-making processes, on which basis it is possible to generate a training set.

The syrup-making process comprises several stages: thickening, formation of crystals, growing of crystals, and binding. On different stages, parameters of syrup-making process are connected in different dependences; therefore it is advisable to introduce switchings into the model of sugar-making process and put a submodel in correspondence with each stage.

It is known that the progress and result of syrup-making process as well as RP depend not only on the specific parameter values at each time point, but also on the speed of their changes. Dynamic of the $L$, $V$, $P$, and $D$ processes must be accounted for. This fact justifies application of a difference model, which inputs receive values for a series of consecutive time points.

In general, dependence of RP on the parameters of syrup-making process, which is reflected by DNFSM, can be written as follows:

\[ y_{\sigma}[t+\Delta] = f_{\sigma}(u_{\sigma 1}[t],...,u_{\sigma n}[t-n+1]) \]

Inputs $u_{\sigma i}$ are formed from the values of multistage $L$, $V$, $P$, and $D$ processes, output $y_{\sigma}[t+\Delta]$ is a value of any RP, which is determined with a certain advancing $\Delta$ with respect to current point in time.

In order to determine RP of the syrup-making process parameters, 3 DNFSMs were applied. The following describes two of them.

Difference neuro-fuzzy switched model for determining reference points of density. RP for syrup density $D$ can be determined through application of DNFSM, which implements the following function

\[ D_{\sigma}[t+\Delta] = f_{\sigma}(l_{\sigma}[t],v_{\sigma}[t],p_{\sigma}[t]) \]

Choosing as factors the model of values of level $L$, vacuum $V$, and steam pressure $P$ is justified physically; it is known that density depends on combination of these parameters. Factors are the multistage processes with the memory depth $n_{\sigma}$:

\[ l_{\sigma}[t] = [l_{\sigma}[t],...,l_{\sigma}[t-n+1]] \]

\[ v_{\sigma}[t] = [v_{\sigma}[t],...,v_{\sigma}[t-n+1]] \]

\[ p_{\sigma}[t] = [p_{\sigma}[t],...,p_{\sigma}[t-n+1]] \]

At its output, DNFSM (Fig. 6) generates the RP value of $D_{\sigma}[t+\Delta]$ for different stages of syrup-making process; these stages correspond to the values of switching signal $\sigma$.

\[ \text{Figure 6. Scheme Of The DNFSM That Defines Density of RP} \]

Base of DNFSM rules determining RP for density $D$ is as follows:

\[ R_{\sigma}^i : \text{If } \mathbf{u}_{\sigma i}[t] \text{ is } B_{\sigma i}[0],..., \text{ and } u_{\sigma i}[t] \text{ is } B_{\sigma i}[1] \text{ then } y_{\sigma}[t+\Delta] = a_{\sigma} \mathbf{U}_{\sigma}[t] + b_{\sigma} \]

(9)

where $\mathbf{U}_{\sigma}[t] = [u_{\sigma 1}[t], u_{\sigma 2}[t], u_{\sigma 3}[t]]$ is the vector of input actions,

\[ u_{\sigma i}[t] = [u_{\sigma i 1}[t],...,u_{\sigma i n}[t-n+1]] \]

is the values of the $i$-th multistage input process,

$\sigma \in S = \{1,\ldots,4\}$ is a switching signal that determines the DNFSM submodel, corresponding to a certain stage of syrup-making process (4 stages were allocated),

$y_{\sigma}$ is the reference point value.
Input processes in accordance with the proposed mechanism for fuzzification are described by the two-dimensional fuzzy sets $B^l_{\sigma}[t]$.

Value of a switching signal $\sigma$ is determined by the level $l[t]$ value: $\sigma = \sigma(l[t])$. Submodels are similar in structure and different in parameter values of rules $a^l_\sigma$, $b^l_\sigma$ conclusions and in parameters of fuzzy sets $B^l_{\sigma}[t]$.

Each DNFSM submodel was trained on an individually generated set of data. Centre and width parameters of the two-dimensional sets $B^l_{\sigma}[t]$ were determined by using the earlier proposed algorithms. For training and subsequent monitoring of the submodels operation results, the above described software modules were implemented.

Difference neuro-fuzzy switched model for determination of level reference points. For RP of the levels $L_3$, $L_4$, and $L_5$, significant are their abscissas $T_7$, $T_8$, and $T_9$; RPs themselves vary very slightly (Fig. 7). It should be noted that the time moments $T_7$, $T_8$, and $T_9$ are also presented as RP abscissas of density $D_3$, $D_4$, and $D_5$. They must be determined for a complete description of sugar-making technology.

Abscissas $T_7$, $T_8$, and $T_9$ of the level reference points are determined by the DNFSM implementing the following function $T_\sigma(t + \Delta) = f_\sigma(l_\sigma(t), d_\sigma(t))$. Factors are the multistage processes $l_\sigma(t) = [l_\sigma(t),...,l_\sigma(t-n+1)]$, $d_\sigma(t) = [d_\sigma(t),...,d_\sigma(t-n+1)]$. Base of DNFSM rules that determines the values $T_7$, $T_8$, and $T_9$ is as follows:

$$l^1 \sigma l^2 \sigma R : \text{If } l_\sigma(t) \text{ is } B^l_{\sigma}[t] \text{ and } d_\sigma(t) \text{ is } B^d_{\sigma}[t] \text{ then } y_\sigma(t + \Delta) = a_\sigma[l^1_\sigma(t)] + b_\sigma.$$  \hspace{1cm} (10)

According to the fuzzification mechanism proposed, the multistage input processes $u_\sigma(t)$ are assigned by the two-dimensional fuzzy sets $B^u_{\sigma}[t]$. Outputs $y_\sigma[t + \Delta]$ are the abscissas of level reference points on three stages of syrup-making process. Switching signal $\sigma$ is determined by the level value: $\sigma = \sigma(l[t])$.

Training sets for configuration of each submodel were different. Parameters of two-dimensional fuzzy sets $B^l_{\sigma}[t]$ were configured with the help of numerical algorithms 1 and 2. Table 2 demonstrates a sample of DNFSM operation results.

Deviations of model values $T_7$, $T_8$, and $T_9$ from the real values on a test set did not exceed 2%. This means that the RP values calculated by the DNFSM are close to the points passed during the real process.

Figure 7. Density And Level Of The Two Sugar Syrup-Making Cycles

Deviations of model values $T_7$, $T_8$, and $T_9$ from the real values on a test set did not exceed 2%. This means that the RP values calculated by the DNFSM are close to the points passed during the real process.
5. DISCUSSION

The results provided support the possibility of using DNFSM for description of multistage processes – in particular, sugar syrup-making process [20].

It seems appropriate to apply the obtained scientific and practical results in the previously studied applied problems: continuous chemical production – Li and Li [14]; timber drying – Ge and Chen [8]; Internet-based e-commerce systems – Lee and Ahn [13], etc.

The proposed mechanism for fuzzification of input processes allows for reduction in the number of configurable parameters and simplification of the model training process. Numerical algorithms for identifying the parameters of two-dimensional fuzzy sets allows for configuration of model with high accuracy.

5. CONCLUSION

1. The authors have introduced a new class of difference neuro-fuzzy switched models that are characterized by a combination of structures of difference fuzzy models, neural networks, and systems with switchings enabling to model complex multistage processes, which are characterized by abrupt changes in structure or parameters.

2. The authors have proposed a mechanism for fuzzification of input actions of a difference neuro-fuzzy switched model, which is characterized by the ability to convert input actions into discrete fuzzy processes using two-dimensional fuzzy sets, and allows reducing the number of configurable parameters of the model.

3. The authors have developed numerical methods for identifying centre and width parameters of input discrete fuzzy processes that are distinguished by the analysis of fuzzy process implementation using the method of least modules, by the account for condition of unity partition, and allowing for enhancement of the modelling accuracy.

4. The authors have developed structure of the software package that enables to modelling complex multistage processes, which are distinguished by implementation of the proposed class of difference neuro-fuzzy switched models and numerical methods for identifying parameters of input discrete fuzzy processes.

5. The authors have created a difference neuro-fuzzy switched model for a multistage dynamic production process of sugar syrup-making, which allows for prevention of decrease in sugar quality level that can be observed in case of an operator’s mistake.

REFERENCES:


