OBJECT CLASSIFICATION VIA GEOMETRICAL, ZERNIKE AND LEGENDRE MOMENTS

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ABSTRACT

In many applications, different kinds of moments have been utilized to classify images and object shapes. Moments are important features used in recognition of different types of images. In this paper, three kinds of moments: Geometrical, Zernike and Legendre Moments have been evaluated for classifying 3D object images using Nearest Neighbor classifier. Experiments are conducted using ETH-80 database, which contains 80 objects.

Keywords: *3D Objects, Geometrical Moments, Zernike Moments, Legendre Moments, Nearest Neighbor Classifier*

1. INTRODUCTION

In image analysis, it is of utmost importance to look for pattern features that are invariant with respect to change of size, translation, and/or rotation [1]. There are two different moment approaches to this problem: (i) direct description by moment invariants and (ii) image normalization [2]. The direct description of moment invariants was first introduced by Hu, showing how they can be derived from algebraic invariants in his fundamental theorem of moment invariants [3]. He used geometric moments to generate a set of invariants that were then widely used in pattern recognition [4], ship identification [5], aircraft identification [6], pattern matching [7], scene matching [8], image analysis [9], object representation [10], edge detection [11], and texture analysis [12].

The Hu's invariants became classical and, despite of their drawbacks, they have found numerous successful applications in various areas. Major weakness of the Hu's theory is that it does not provide for a possibility of any generalization [13].

Examples of moment-based feature descriptors include Cartesian geometrical moments, rotational moments, orthogonal moments, and complex moments. Moments with an orthogonal basis set (e.g., Legendre and Zernike polynomials) can be used to represent the image with a minimum amount of information redundancy [14]. These orthogonal moments and their inverse transforms have been used in the field of pattern representation [15], image analysis [16], and image reconstruction [17] with some success. As is well known, the difficulty in the use of moments is due to their high computational complexity, especially when a higher order of moments is used.

Teague proposed Zernike moments based on the basis set of orthogonal Zernike polynomials [18]. Other orthogonal moments are Legendre and pseudo-Zernike moments which are derived from Legendre and pseudo-Zernike polynomials, respectively. Zernike moments have been proven to be more robust in the presence of noise. They are able to achieve a near-zero value of redundancy measure in a set of moment functions where the moments correspond to independent characteristics of the image [19]. Since their moment functions are defined using a polar coordinate representation of the image space, Zernike moments are commonly used in recognition tasks requiring rotation invariance. However, this coordinate representation does not easily yield translation invariant functions, which are also sought after in pattern recognition applications [1].

Since the Zernike and Legendre polynomials are defined only inside the unit circle, the computation of those moments requires a coordinate transformation and suitable approximation of the continuous moment integrals [20].

In various computer vision applications widely used is the process of retrieving desired images from a large collection on the basis of features that can be automatically extracted from the images themselves. These systems called CBIR (Content-Based Image Retrieval). The algorithms used in

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these systems are commonly divided into three tasks [21]:

- extraction,
- selection, and
- classification.

The extraction task transforms rich content of images into various content features. Feature extraction is the process of generating features to be used in the selection and classification tasks. Feature selection reduces the number of features provided to the classification task. Those features which are likely to assist in discrimination are selected and used in the classification task. Features which are not selected are discarded [22].

Image classification helps the selection of proper features and descriptors for the indexing and retrieval purpose. It enhances not only the retrieval accuracy but also the retrieval speed, since a large image database can be organized according to the classification rule and search can be performed within relevant classes [23].

In this paper three kinds of moments (Geometric, Zernike, and Legendre) are used for feature extraction. While Nearest Neighbor Classifier is used for classifying the contours of 3D images.

The rest of the paper is organized as follows. In Sec. 2 feature extraction methods based on different kinds of moments are presented. Classification method using nearest neighbor is intruduced in Sec. 3. Finally, experimental results and a comparative study are given in Sec. 4, followed by conclusions in Sec. 5.

2. FEATURE EXTRACTION

The feature is defined as a function of one or more measurements, each of which specifies some quantifiable property of an object, and is computed such that it quantifies some significant characteristics of the object. In pattern recognition and in image processing, feature extraction is a special form of dimensionality reduction. When the input data to an algorithm is too large to be processed and it is suspected to be notoriously redundant (much data, but not much information) then the input data will be transformed into a reduced representation set of features (also named features vector). Transforming the input data into the set of features is called features extraction. If the features extracted are carefully chosen it is expected that the features set will extract the relevant information from the input data in order to perform the desired task using this reduced representation instead of the full size input [21].

Feature extraction involves simplifying the amount of resources required to describe a large set of data accurately. When performing analysis of complex data one of the major problems stems from the number of variables involved. Analysis with a large number of variables generally requires a large amount of memory and computation power or a classification algorithm which overfits the training sample and generalizes poorly to new samples. Feature extraction is a general term for methods of constructing combinations of the variables to get around these problems while still describing the data with sufficient accuracy. There are various features currently employed [21]:

1) General features: Application independent features such as color, texture, and shape. According to the abstraction level, they can be further divided into:

- Pixel-level features: Features calculated at each pixel, e.g. color, location.
- Local features: Features calculated over the results of subdivision of the image band on image segmentation or edge detection.
- Global features: Features calculated over the entire image or just regular sub-area of an image.

2) Domain-specific features: Application dependent features such as human faces, fingerprints, and conceptual features. These features are often a synthesis of low-level features for a specific domain.

Moment functions of the two-dimensional image intensity distribution are used in a variety of applications, as descriptors of shape. Image moments that are invariant with respect to the transformations of scale, translation, and rotation find applications in areas such as pattern recognition [26], object identification [17] and template matching [27].

Orthogonal moments have additional properties of being more robust in the presence of image noise, and having a near-zero redundancy measure in a feature set. Zernike moments, which are proven to have very good image feature representation capabilities, are based on the orthogonal Zernike radial polynomials. They are effectively used in pattern recognition since their rotational invariants can be easily constructed. Legendre moments form another orthogonal set, defined on the Cartesian coordinate space. Orthogonal moments also permit the analytical reconstruction of an image intensity function from a finite set of moments, using the inverse moment transform. Both Legendre and Zernike moments are defined as continuous integrals over a domain of normalized coordinates [28].

In retrieval applications, a small set of lower order moments is used to discriminate among different images. The most common moments are:

1) Geometrical moments.
2) Zernike moments.
3) Legendre moments.

2.1 GEOMETRICAL MOMENTS

The shape of an object is a very important character in human's perception, recognition, and comprehension. Because geometric shape represents the essential characteristic of an object, and has invariance with respect to translation, scale, and orientation, the analysis and discernment like geometry is of important significance in computer vision [24].

Historically, Hu published the first significant paper on the use of image moment invariants for two-dimensional pattern recognition applications [3]. His approach is based on the work of the 19th century mathematicians Boole, Cayley and Sylvester, and on the theory of algebraic forms.

$$M_1 = \mu_{20} + \mu_{02}$$
$$M_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2$$
$$M_3 = (\mu_{30} - 3\mu_{12})^2 + 3(\mu_{21} + \mu_{03})^2$$
$$M_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2$$
$$M_5 = (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})[(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] + (3\mu_{21} - \mu_{03})(\mu_{21} + \mu_{03})[3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2]$$
$$M_6 = (\mu_{20} - \mu_{02})[(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] + 4\mu_{11}(\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03})$$
$$M_7 = (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12})[(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] + (3\mu_{12} - \mu_{30})(\mu_{21} + \mu_{03})[3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2]$$

Geometric moments of a 1D signal S(x) are defined by [25]:

$$M_n(x) = \int_{-\omega}^{\omega} S(x+t)t^n dt \quad n = 0, 1, 2, ...$$

where $M_n(x)$ is the moment of order n calculated from a window of size (2w + 1) pixels centered at the point x. Geometric moments of a 2D image I(x, y) are defined by [25]:

$$M_{m,n}(x,y) = \int_{-\omega 1}^{\omega 1} \int_{-\omega 2}^{\omega 2} I(x+u, y+v) u^m v^n du dv \quad m,n = 0,1,2,...$$

where $M_{m,n}(x)$ is the moment of order (m, n) calculated from a window of size $(2\omega_1 + 1) \times (2\omega_2 + 1)$ pixels centered at the pixel (x, y).

2.1 ZERNIKE MOMENTS

Teague first introduced the use of Zernike moments to overcome the shortcomings of information redundancy present in the popular geometric moments [18]. Zernike moments are a class of orthogonal moments and have been shown effective in terms of image representation.

Zernike moments, a type of moment function, are the mapping of an image onto a set of complex Zernike polynomials. As these Zernike polynomials are orthogonal to each other, Zernike moments can represent the properties of an image with no redundancy or overlap of information between the moments [29]. Due to these characteristics, Zernike moments have been utilized as feature sets in applications such as pattern recognition [30] and content-based image retrieval [31].

To calculate the Zernike moments, the image (or region of interest) is first mapped to the unit disc using polar coordinates, where the centre of the image is the origin of the unit disc. Those pixels falling outside the unit disc are not used in the calculation. The coordinates are then described by the length of the vector from the origin to the coordinate point. An important attribute of the geometric representations of Zernike polynomials is that lower order polynomials approximate the global features of the shape/surface, while the higher ordered polynomial terms capture local shape/surface features. Zernike moments have the following advantages [21, 32]:

1) Rotation invariance: the magnitude of Zernike moments has rotational invariant property.
2) Robustness: they are robust to noise and minor variations in shape.
3) Expressiveness: Since the basis is orthogonal, they have minimum information redundancy.
4) Effectiveness: an image can be better described by a small set of its Zernike moments than any other types of moments such as geometric moments.
5) Multilevel representation: a relatively small set of Zernike moments can characterize the global shape of pattern. Lower order moments represent the global shape of pattern and higher order moments represent the detail.
6) The ease of image reconstruction from them.

The computation of Zernike moments from an input image consists of three steps [29]:

1) Computation of radial polynomials.
2) Computation of Zernike basis functions.
3) Computation of Zernike moments by projecting the image on the basis functions.

The procedure for obtaining Zernike moments from an input image begins with the computation of Zernike radial polynomials. The real-valued 1-D radial polynomial $R_{nm}(\rho)$ is defined as [29]:

$$R_{nm}(\rho) = \sum_{s=0}^{(n-|m|)/2} c(n,m,s)\rho^{n-2s} \qquad (1)$$

where

$$c(n,m,s) = (-1)^s \frac{(n-s)!}{s!((n+|m|)/2-s)!((n-|m|)/2-s)!}$$

In equation (1), n and m are generally called order and repetition, respectively. The order n is a non-negative integer, and the repetition m is an integer satisfying n-|m|=even and |m| ≤ n. The radial polynomials satisfy the orthogonal properties for the same repetition [29]:

$$\int_0^{2\pi}\int_0^1 R_{nm}(\rho,\theta)R_{n'm}(\rho,\theta)\rho d\rho d\theta = \begin{cases}\frac{1}{2(n+1)} & if\ n=n' \\ 0 & otherwise\end{cases} \quad (2)$$

Using the radial polynomial, complex-valued 2-D Zernike basis functions, which are defined within a unit circle, are formed by [29]:

$$V_{nm}(\rho,\theta) = R_{nm}(\rho)exp(jm\theta), \quad |\rho| \le 1 \quad (3)$$

where j = √-1. Zernike basis functions are orthogonal and satisfy [29]:

$$\int_0^{2\pi}\int_0^1 V^*_{nm}(\rho,\theta)V_{pq}(\rho,\theta)\rho d\rho d\theta = \begin{cases}\frac{\pi}{n+1} & if\ n=p,\ m=q \\ 0 & otherwise\end{cases} \quad (4)$$

The orthogonality implies no redundancy or overlap of information between the moments with different orders and repetition. This property enables the contribution of each moment to be unique and independent of the information in an image.

Complex Zernike moments of order n with repetition m are finally defined as [29]:

$$Z_{nm} = \frac{n+1}{\pi}\int_0^{2\pi}\int_0^1 f(\rho,\theta)V^*_{nm}(\rho,\theta)\rho d\rho d\theta \quad (5)$$

where f(x, y) is the image function and * denotes the complex conjugate. As can be seen from the definition, the procedure for computing Zernike moments can be seen as an inner product between the image function and the Zernike basis function.

To compute Zernike moments from a digital image, the integrals in (5) are replaced by summations and the coordinates of the image must be normalized into [0, 1] by a mapping transform.

The discrete form of the Zernike moments of an image size N x N is expressed as follows [29]:

$$\begin{aligned} Z_{nm} &= \frac{n+1}{\lambda_N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)V^*_{nm}(x,y) \\ &= \frac{n+1}{\lambda_N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)R_{nm}(\rho_{xy})exp(-jm\theta_{xy}) \end{aligned} \quad (6)$$

where $0 \le \rho_{xy} \le 1$ and λ_N is a normalization factor. In the discrete implementation of Zernike moments, the normalization factor λ_N must be the number of pixels located in the unit circle by the mapping transform, which corresponds to the area of a unit circle π in the continuous domain.

2.2 LEGENDRE MOMENTS

Moments with Legendre polynomials as kernel function, denoted as Legendre moments, were first introduced by Teague [18]. Legendre moments belong to the class of orthogonal moments, and they were used in several pattern recognition applications [33]. They can be used to attain a near zero value of redundancy measure in a set of moment functions, so that the moments correspond to independent characteristics of the image [34].

By convention, the translation and scale invariant functions of Legendre moments are achieved by using a combination of the corresponding invariants of geometric moments. They can also be accomplished by normalizing the translated and/or scaled images using complex or geometric moments. However, the derivation of these functions is not based on Legendre polynomials. This is mainly due to the fact that it is difficult to extract a common displacement or scale factor from Legendre polynomials. The two-dimensional Legendre moments of order (p + q), with image intensity function f(x, y), are defined as [19]:

$$L_{pq} = \frac{(2p+1)(2q+1)}{4}\int_{-1}^{1}\int_{-1}^{1} P_p(x) X P_q(y) f(x,y)dxdy \quad x,y \in [-1,1] \quad (1)$$

Where Legendre polynomial, $P_p(x)$, of order p is given by [39]:

$$P_p(x) = \sum_{k=0}^{p}\left\{(-1)^{\frac{p-k}{2}}\frac{1}{2^p}\frac{(p+k)!x^k}{\left(\frac{p-k}{2}\right)!\left(\frac{p+k}{2}\right)!k!}\right\}_{p-k=even} \quad (2)$$

The recurrence relation of Legendre polynomials, $P_p(x)$, is given as follows [33]:

$$P_p(x) = \frac{(2p-1)xP_{p-1}(x) - (p-1)P_{p-2}(x)}{p} \quad (3)$$

Where $P_o(x) = 1$, $P_1(x) = x$ and p>1. Since the region of definition of Legendre polynomials is the interior of [-1, 1], a square image of N x N pixels with intensity function f(i, j), 0 ≤ i, j ≤ (N - 1), is scaled in the region of -1 < x, y < 1. In the result of this, equation (1) can now be expressed in discrete form as [33]:

$$L_{pq} = \lambda_{pq}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} P_p(x_i)P_q(y_j)f(i,j) \quad (4)$$

where the normalizing constant,

$$\lambda_{pq} = \frac{(2p+1)(2q+1)}{N^2}$$

x_i and y_i denote the normalized pixel coordinates in the range of [-1, 1], which are given by [33]:

$$x_i = \frac{2i}{N-1} - 1 \quad and \quad y_j = \frac{2j}{N-1} - 1 \quad (5)$$

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3. CLASSIFICATION

Image classification methods can be roughly divided into two broad families of approaches:
(i) Learning-based classifiers, which require an intensive learning/training phase of the classifier parameters (e.g., parameters of Support Vector Machines [35], Boosting [36], parametric generative models [37], decision trees [38], fragments and object parts [39]. These methods are also known as parametric methods.
(ii) Nonparametric classifiers, which base their classification decision directly on the data, and require no learning/training of parameters. The most common non-parametric methods rely on Nearest-Neighbor distance estimation [35].

Non-parametric classifiers have several very important advantages that are not shared by most learning-based approaches [35]:
(i) Can naturally handle a huge number of classes.
(ii) Avoid overfitting of parameters, which is a central issue in learning based approaches.
(iii) Require no learning/ training phase. Although training is often viewed as a one-time preprocessing step, retraining of parameters in large dynamic databases may take days, whereas changing classes/training-sets is instantaneous in non-parametric classifiers.

3.1 NEAREST NEIGHBOR CLASSIFIER

The nearest neighbor classifier relies on a metric or a distance function between points. For all points x, y and z, a metric D(·, ·) must satisfy the following properties:
1) Nonnegativity: $D(x, y) \geq 0$.
2) Reflexivity: $D(x, y) = 0$ if and only if $x = y$.
3) Symmetry: $D(x, y) = D(y, x)$.
4) Triangle inequality: $D(x, y) + D(y, z) \geq D(x, z)$.

The nearest neighbor classifier is used to compare the feature vector of the prototype image and feature vectors stored in the database. It is obtained by finding the distance between the prototype image and the database. Let $C_1, C_2, C_3, \ldots, C_k$ be the k clusters in the database. The class is found by measuring the distance $d(x(q),C_k)$ between x(q) and the k^{th} cluster C_k. The feature vector with minimum difference is found to be the closest matching vector. It is given by [40]:
$d(x(q), C_k) = \min\{\| x(q) - x \| : x \in C_k \}$

Nearest-Neighbor classifiers provide good image classification when the query image is similar to one of the labeled images in its class.

4. RESULTS AND DISCUSSION

The database used here is ETH-80 [41]. It contains 80 objects from 8 categories. The contours version set is chosen from ETH-80 database. It consists of 3280 images. The images are resized into 60x60 pixels. Some examples of original colored images and the contours are given in Fig. 1 and Fig.2, respectively. Our work is implemented using Matlab 6.1.

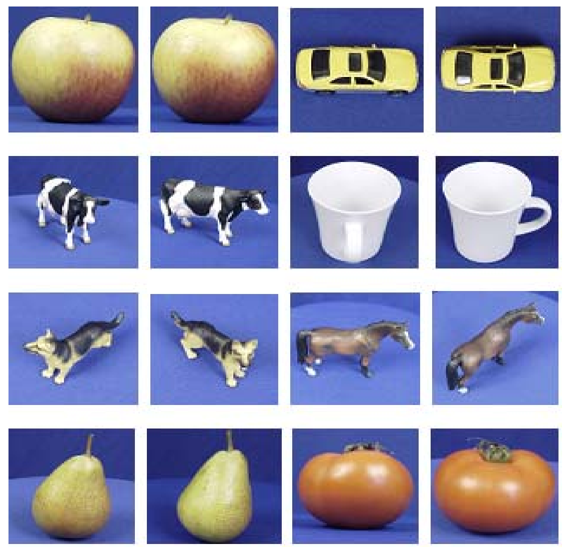

Fig. 1. Some examples of objects from ETH-80 database

Fig. 2. Samples of image contours for the objects in Fig.1.

The experiments are made based on eight classes as shown in Fig. 1. The number of prototypes per class is 5. The number of shapes in testing data set is 160.

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In our experiments, the number of input featyures(h1,h2,…,h7) extracted using Hu invariants feature extraction method is 7 while the number of inputs(Zernike features) extracted using Zernike moments is 5 and legendre is 16. These inputs are presented to the nearest neighbour classifier for testing to do matching with the feature values in reference database.

The experimental results showed that the recognition rate of the nearest neighbour classifier based on Legendre moments is higher than the recognition rate of Hu and Zernike moments. The results are given in Table 1.

Table 1. Recognition Rate of Hu, Zernike and Legendre Moments using Nearest Neighbor classifier

Objects	Hu	Zer.	Leg.	Ave.
Apple	85%	75%	95%	85%
Car	65%	65%	65%	65%
Cow	65%	65%	65%	65%
Cup	65%	65%	85%	72%
Dog	65%	75%	85%	75%
Horse	85%	75%	95%	85%
Pears	75%	65%	95%	78%
Tomato	65%	65%	65%	65%
Average	71%	69%	81%	74%

5. CONCLUSION

This paper introduced a comparative study of three most popular moments feature extraction methods (Hu, Zernike, and Legendre) to recognize the images of 3D objects using Neatest Neighbor classifier. The experimental results showed that the recognition rate of the nearest neighbour classifier based on Legendre moments is higher than the recognition rate of Hu and Zernike moments.

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