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CHARACTERIZATION OF SOFT COMPACT SPACES BASED ON SOFT FILTER

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ABSTRACT

In this paper, we define soft filter and maximal soft filter and give some characterizations about them. Zorlutuna, Akdag, Min, Atmaca[2]introduced and studied the notion of soft compact space. In particular, we investigate the soft compact space based on soft filter and maximal soft filter and get some new results.It provides theoretical supports for the development of the soft topological spaces.

Keywords: *Soft Compact space, Soft Filter, Maximal Soft Filter, Soft Cluster Point , Soft Set*

1. INTRODUCTION

In our life, many complicated questions in engineering, environment, medicine, economics and sociology, can not be well dealt with by the classical methods because there are various uncertainties for these questions. In order to solve these uncertainties, L.A.Zadeh[1] introduced fuzzy set theory, Z.Pawlak[3] introduced rough set theory and M.B.Gorzalzany[4] introduced interval mathematics theory. Although they have been successfully applied in pattern recognition, data mining, machine learning, and so on. All the above theories have their own difficulties, they are still restrictive for many applications because then can only deal with complete information systems.

D.Molodtsov[5] introduced the concept of a soft set as a mathematical tool for dealing with uncertainties which is relatively easy from the above difficulties. Because it needn't to find any relationship on it. Soft set theory has rich potential for practical applications in several domains, a few of which are indicated by D.Molodstov [6]. In recently years, soft set theory has been researched in many fields. Z.Kong et al.[7]introduced reduction of the notion of normal parameter of soft sets. Zou and Xiao[8]discussed the soft data analysis method. Pei and Miao[9]proof that the soft sets can form a special information systems. There are many people who worked on soft group, soft semigroup, soft semiring [10,11,12], soft ideals[13] etc. Under the special structure of the nature of the soft set attracts a lot of people to research.

The topological structures of set theories dealing with uncertainties were first studied by Chang[14]. E.F.Lashin et al.[15] generalized rough set theory in the framework of topological spaces. Shabir and Naz[19]are the first person who introduce the concept of soft topological spaces which are defined over an initial universe with a fixed parameters. Zorlutuna, Akdag, Min, Atmaca[2] introduced some new concepts in soft topological spaces and give some new properties about soft topological spaces.

Compact spaces are one of the most important classes in general topological spaces[16]. They have many well known properties which can be used in many disciplines. Zorlutuna, Akdag, Min and Atmaca[2] introduced compact soft spaces around a soft topology. Sabir and Bashir[17] investigated the properties of soft open, soft neighborhood and soft closure, they also discussed the properties of soft interior and so on. Won Keun Min[18]introduced the notion of the soft regular. Beside these, we will discuss compact soft spaces around a soft topology. In this paper, we consider only soft sets (F, A) over a universe U in which all the parameter set A are same. We denote the family of these soft sets by $SS_U(A)$. For terms which are not defined here, please refer to[20] and relate references..

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2. BSIC CONCEPTS

Definition 1.1[2] A pair (F,A) is called a soft set over U, where F is a mapping give by $F:A\to P(U)$.

Definition 1.2[21] For two soft sets (F,A) and (G,B) over a common universe U, (F,A) is a soft subset of (G,B), denoted by $(F,A)\tilde{\subseteq}(G,B)$, if $A\subseteq B$ and $F(e)\subseteq G(e)$ for any $e\in A$.

Definition 1.3[21] For two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 1.4[22] The complement of a soft sets (F,A), denoted by $(F,A)^c$, is defined by $(F,A)^c=(F^c,A)$. Where $F^c:A\to P(U)$ is a mapping given by $F^c(\alpha)=U\setminus F(\alpha)$, for any $\alpha\in A$. F^c is called the soft complement function of F. Clearly, $(F^c)^c$ is the same as F and $[(F,A)^c]^c=(F,A)$.

Definition 1.5[12] A soft set (F,A) over U is said to be a null soft set, denoted by ϕ_A, if $F(e)=\phi$ for any $e\in A$; a soft set (F,A) over U is said to be an absolute soft set, denoted by U_A, if $F(e)=U$, for any $e\in A$.

It is obvious, $(U_A)^c=\phi_A$ and $(\phi_A)^c=U_A$.

Definition 1.6[6] The union of two soft sets (F,A) and (G,B) over a common universe U is a soft set (H,C), where $C=A\bigcup B$ and for $e\in C$

$$H(e)=\begin{cases} F(e) & e\in A\setminus B \\ G(e) & e\in B\setminus A \\ F(e)\bigcup G(e) & e\in A\bigcap B \end{cases}$$

This relationship is written as $(F,A)\tilde{\bigcup}(G,B)=(H,C)$.

Definition 1.7[6] The intersection of two soft sets (F,A) and (G,B) over a common universe U is a soft set (H,C),,

$H(e)=F(e)\bigcap G(e)$, where $C=A\bigcap B$ and for all $e\in C$. It can be denoted as $(F,A)\tilde{\bigcap}(G,B)=(H,C)$.

Definition 1.8[2] The soft set (F,A) $\in SS_U(A)$ is called a soft point in U_A, denoted

by e_F, if $F(e)\neq\phi$ and $F(e^{'})=\phi$ for the element $e\in A$ and all $e^{'}\in(A\setminus\{e\})$.

Definition 1.9[2] The soft point e_F is said to be in the soft set (G,A), denoted by $e_F\tilde{\in}(G,A)$, if for the element $e\in A$ and $F(e)\subseteq G(e)$.

3. MAIN RESULTS

Zorlutuna, Akdag, Min and Atmaca[2] introduced the soft compact spaces. They get a characterization theorem that a soft topological space is compact if and only if each family of soft closed sets with the finite intersection property has a nonnull intersection. Beside this, we give some characterizations based on soft filter.

Definition 3.0[19] Let τ be a collection of soft sets over a universe U with a fixed set A of parameters, then $\tau\subseteq SS(U)_A$ is called a soft topology on U with set A if

(T1). ϕ_A, U_A belong to τ;

(T2) the union of any number of soft sets in τ belong to τ;

(T3) the intersection of any two soft sets in τ belong to τ.

Definition 3.1[19] A soft set (G,A) in a soft topological space (U,τ,A) is called a soft neighborhood of the soft point $e_F\tilde{\in}U_A$, if there exists a soft open set (H,A) such that $e_F\tilde{\in}(H,A)\tilde{\subseteq}(G,A)$.

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Definition 3.2[6] A soft set (G,A) in a soft topological space (U,τ,A) is called a soft neighborhood of the soft set (F,A) ,if there exists a soft open set (H,A) such that $(F,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}(G,A)$.

Definition 3.3[6] A sequence of soft sets, say $\{(F_n,A): n\in N\}$, is eventually contained in a a soft set (F,A) if and only if there ia an integer m such that $(F_n,A)\tilde{\subseteq}(F,A)$ when $n\geq m$. The sequence is frequently contained in (F,A) if and only if for each integer m, there is an integer n such that $(F_n,A)\tilde{\subseteq}(F,A)$ when $n\geq m$. The sequence is in a soft topological space (U,τ,A), then we say that the sequence converges to a soft set (F,A), if it is eventually contained in each neighborhood of (F,A).

Definition 3.4[6] A family Ψ of soft sets is a cover of a soft set (F,A) if

$(F,A)\tilde{\subseteq}\bigcup\{(F_i,A)\in\Psi : i\in I\}$. If each member of Ψ is a soft open set, then Ψ is called a soft open cover.

Definition 3.5[6] A soft topological space (U,τ,A) is called soft compact space if each soft open cover of U_A has a finite subcover.

Definition 3.6 Let F be an nonempty family of soft sets over U_A satisfies:

(F1) $\phi_A \tilde{\notin} \mathrm{F}$;

(F2) $(F,A)\tilde{\in}\mathrm{F}$ and $(F,A)\tilde{\subseteq}(G,A)$, then $(G,A)\tilde{\in}\mathrm{F}$;

(F3) If $(F,A)\tilde{\in}\mathrm{F}$ and $(G,A)\tilde{\in}\mathrm{F}$, then $(F,A)\tilde{\cap}(G,A)\tilde{\in}\mathrm{F}$.

F is called soft filter. Further, F is called maximal soft filter if for any soft filter H such that $\mathrm{F}\tilde{\subseteq}\mathrm{H}$, then $\mathrm{H}\equiv SS(U)_A$.

Theorem1. A family F' of soft sets has the finite intersection property. Then there exists a maximal soft filter F such that $\mathrm{F}'\tilde{\subseteq}\mathrm{F}$.

Proof. Let $\square = \{\mathrm{F} : \mathrm{F}'\tilde{\subseteq}\mathrm{F}$ and F' of soft sets has the finite intersection property$\}$, we define a relation on $\square$ by $\mathrm{F}_1\tilde{\subseteq}\mathrm{F}_2$ when F_1 and F_2 belong to $\square$, then $\square$ is a poset. It is obvious that every chain has an upper bound in $\square$. From Zorn's lemma, $\square$ has a maximal element F . We shall proof that F is a soft filter. Obviously F satisfies (F1). Let $(F,A)\tilde{\in}\mathrm{F}$ and $(G,A)\tilde{\in}\mathrm{F}$, since F has the finite intersection property, then $\phi_A \neq (F,A)\tilde{\cap}(G,A) \tilde{\in}\mathrm{F}$ and $\mathrm{F}'' = \{(F,A)\tilde{\cap}(G,A)\}\tilde{\cup}\mathrm{F}$ has the finite intersection property, then $\mathrm{F}''\tilde{\in}\square$, therefore $(F,A)\tilde{\cap}(G,A)\tilde{\in}\mathrm{F}$ as F is a maximal element. Then F satisfies (F3). $(F,A)\tilde{\in}\mathrm{F}$ and (F,A) $\tilde{\subseteq}(G,A)$ then $(G,A)\}\tilde{\cup}\mathrm{F}\tilde{\in}\mathrm{F}$, since F is a maximal element, then $(G,A)\tilde{\in}\mathrm{F}$ satisfies (F2).

Theorem2 The following are equivalent over U_A.

(1) A soft filter F is a maximal soft filter.

(2) for any soft set (G,A) and $(F,A)\tilde{\cap}(G,A)\neq\phi_A$ for any $(F,A)\tilde{\in}\mathrm{F}$, then $(G,A)\tilde{\in}\mathrm{F}$.

Proof. $(1\Rightarrow 2)$ Let F is a maximal soft filter , (G,A) is a soft set and $(F,A)\tilde{\cap}(G,A)\neq\phi_A$ for any $(F,A)\tilde{\in}\mathrm{F}$. Let

$$\mathrm{F}' = \{(H,A) : (F,A)\tilde{\cap}(G,A)\tilde{\subseteq}(H,A), (F,A)\tilde{\in}\mathrm{F}\}$$

. It is easy to proof that F' is a soft filter and $\mathrm{F}\tilde{\subseteq}\mathrm{F}'$. Since F is a maximal soft filter, then $\mathrm{F} = \mathrm{F}'$, therefore $(G,A)\tilde{\in}\mathrm{F}$.

$(2\Rightarrow 1)$ Let F is a soft filter and satisfies (1) and $\mathrm{F}\tilde{\subseteq}\mathrm{F}'$ for any soft filter F' . We shall prove $\mathrm{F}'\tilde{\subseteq}\mathrm{F}$. Let $(G,A)\tilde{\in}\mathrm{F}'$., from (F3) and

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(F1), $(H,A)\tilde{\cap}(G,A) \neq \phi_A$ for any $(H,A)\tilde{\in} \mathrm{F}'$, then $(F,A)\tilde{\cap}(G,A) \neq \phi_A$ for any $(F,A) \tilde{\in} \mathrm{F}$. From (2), $(G,A) \tilde{\in} \mathrm{F}$, therefore $\mathrm{F}' \tilde{\subseteq} \mathrm{F}$.

Definition 3.7 A soft filter F converges to a soft point $e_F \tilde{\in} U_A$ in a soft topological space (U,τ,A), if every soft neighborhood of the soft point e_F belong to the soft filter F. It can be denoted by $\mathrm{F} \to e_F$.

Remark 1 From (F2) of the definition 2.6, $\mathrm{F} \to e_F$ if and only if every soft neighborhood U_{e_F} of the soft point e_F, there exists a soft set $(F,A) \tilde{\in} \mathrm{F}$ such that $(F,A)\tilde{\subseteq} U_{e_F}$.

Definition 3.8 F is a soft filter in a soft topological spaces (U,τ,A), a soft point e_F is called soft cluster point of F, if $e_F \tilde{\in} \overline{(G,A)}$ for any $(G,A) \tilde{\in} \mathrm{F}$.

Theorem3. A soft filter F converges to a soft point e_F, then e_F is the soft cluster point of F; if F is a maximal soft filter and e_F is a soft cluster point of F, then the soft filter F converges to the soft point e_F.

Proof. Let $\mathrm{F} \to e_F$ and $(G,A) \tilde{\in} \mathrm{F}$, from the Definition2.7, every soft neighborhood U_{e_F} of the soft point e_F belong to soft filter F, from (F3) and (F1), $U_{e_F}\tilde{\cap}(G,A) \neq \phi_A$, it is easy to check $e_F \tilde{\in} \overline{(G,A)}$; conversely, let soft point e_F be a soft cluster point of F, from the Definition 2.8, $U_{e_F}\tilde{\cap}(G,A) \neq \phi_A$ for every soft neighborhood U_{e_F} of e_F and $(G,A) \tilde{\in} \mathrm{F}$, from the Theorem 2, $U_{e_F} \tilde{\in} \mathrm{F}$, from the Definition 2.7, $\mathrm{F} \to e_F$.

Inspired by Zorlutuna, Akdag, Min and Atmaca[2]proposed some definitions and properties about soft compact spaces, We will investigate some others properties and it's applications.

Theorem 4 (U,τ,A) is a soft compact space and (F,A) is a soft closed set over U, then (F,A) is a soft compact space.

Proof. It is clear from the Definitions.

Lemma 1[2] A soft topological space is compact if and only if each family of soft closed sets with the finite intersection property has a nonull intersection.

Theorem 5 The following are equivalent for a soft topological space (U,τ,A).

(1) A soft topological space (U,τ,A) is compact;

(2) Every soft filter over U_A has a soft cluster point;

(3) Every maximal soft filter over U_A converges to a soft point.

Proof. $(1 \Rightarrow 2)$ Let F be a soft filter of the soft compact space (U,τ,A), then $\overline{\mathrm{F}} = \{\overline{(G,A)} : (G,A)\tilde{\in}\mathrm{F}\}$ is a closed family and has the finite intersection property. From the Lemma 1, $\tilde{\cap}\{\overline{(G,A)} : (G,A)\tilde{\in}\mathrm{F}\} \neq \phi_A$, there exists a soft point $e_F \tilde{\in} \overline{(G,A)}$ for every $(G,A)\tilde{\in}\mathrm{F}$. From the Definition 2.8, e_F is a soft cluster point of F.

$(2 \Rightarrow 3)$ Every maximal soft filter F over U_A has a soft cluster point e_F, from the Theorem 3, then $\mathrm{F} \to e_F$.

$(3 \Rightarrow 1)$ Let Ψ is a family of soft sets in a soft topological space (U,τ,A), Ψ is a open cover of a soft set (F,A), if there is no finite subcover, then

$\mathrm{F}' = \{U_A \setminus (G,A) : (G,A) \tilde{\in} \Psi\}$ has the finite intersection property. From the Theorem 1, there exists a maximal soft filter F such that $\mathrm{F}' \tilde{\subseteq} \mathrm{F}$. From (2), F converges to a soft point, therefore F has a cluster point e_F. In other words, $e_F \tilde{\in} \overline{(F,A)}$ for any $(F,A)\tilde{\in} \mathrm{F}$, it is true for any

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element in F', so $e_F \tilde{\in} \overline{U_A \setminus (G,A)} = U_A \setminus (G,A)$ for any $(G,A) \tilde{\in} \Psi$. It is a contradiction.

Definition 3.9. A soft topological space (U,τ,A) is called soft countable compact space if each countable open cover of U_A has a finite subcover.

From the Definition, it is obvious that a soft compact space is a soft countable compact space.

The following theorem is a special case as Lemma 1.

Theorem 6 A soft topological space is countable compact if and only if each family of soft closed sets with the finite intersection property has a nonnull intersection.

Definition 3.10 A soft point e_F is called ω-cluster point of soft set (G,A), if every soft neighborhood of e_F contains infinite elements of (G,A).

Definition 3.11[6] A soft point e_F in a soft topological space (U,τ,A) is a soft cluster point of a sequence if the sequence is frequently contained in every neighborhood of e_F.

Theorem 7 The following are equivalent for a soft topological space (U,τ,A).

(1)A soft topological space (U,τ,A) is a soft countable compact.

(2) Every soft sequence over U_A has a soft cluster point.

(3) Every soft set with infinite soft point has a ω-cluster point.

Proof. $(1 \Rightarrow 2)$ Let $\{(G_n,A): n \in N\}$ be a sequence of soft sets in a soft countable compact space (U,τ,A), let

$F_n = \{(G_{n+i},A): i = 0,1,2,\ldots\}$ $(n = 0,1,2,\ldots)$,

then $\mathrm{F} = \{\overline{F_n}\}$ is a closed family of soft sets with the finite intersection property. From Theorem 6, $\tilde{\bigcap}\{\overline{F_n} : F_n \tilde{\in} \mathrm{F}\} \neq \phi_A$, there exists a soft point $e_F \tilde{\in} \overline{F_n}$ for every $F_n \tilde{\in} \mathrm{F}$. From the Definition 2.8, e_F is the soft cluster point of F.

$(2 \Rightarrow 3)$ Let (G,A) be a soft set with infinite soft point, we select different soft point in (G,A) forming a sequence $\{e_{F_n}\}$, then the soft sequence $\{e_{F_n}\}$'s cluster point is also the ω-cluster point.

$(3 \Rightarrow 1)$ Let $\{F_n\}$ is a closed soft sets with family with the finite intersection property, let

$\mathrm{F} = \{ \tilde{\bigcap}_{i=1,2,\ldots,n} F_i : n = 1,2,\ldots\}$. For any

$e_{F_n} \tilde{\in} \tilde{\bigcap}_{i=1,2,\ldots,n} F_i$,$(n = 1,2,\ldots)$. If the sequence $\{e_{F_n}\}$ is a finite set, then there exists a soft point $e_F \tilde{\in} \{e_{F_n}\}$ such that e_F is the soft cluster point of the sequence $\{e_{F_n}\}$; if the sequence $\{e_{F_n}\}$ is infinite, then the sequence $\{e_{F_n}\}$ has a ω-cluster point. From the above we know that the ω-cluster point is also a soft cluster point, then the sequence $\{e_{F_n}\}$ has the soft cluster point e_F, therefore $e_F \tilde{\in} F_n$ $(n = 1,2,\ldots)$, from the Theorem 6, then (U,τ,A) is a soft countable compact space.

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