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A METHOD OF COMPUTING THE TRAFFIC FLOW DISTRIBUTION DENSITY IN THE NETWORK WITH NEW FLOW-FORMING OBJECTS BEING PUT INTO OPERATION

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ABSTRACT

The problem of efficient designing of new traffic networks and operation of existing ones is an active topic of research. This problem is being solved in most countries by using Intelligent Transportation Systems and Decision Support Systems. The core of these systems is a mathematical model of processing the data of monitoring traffic flow parameters and road conditions. The efficiency of managerial decisions depends on minimizing the time interval from the moment of obtaining data till the moment of making a decision. We developed a mathematical model based on the Erlang time distribution which allowed us to deduce in explicit form a function of transportation costs for traffic network routes. Of great importance is the practical problem of determining the changes in the network traffic flows distribution with new flow-forming objects being put into operation. This article provides and justifies a new method that helps solve the given problem. The algorithms developed are based on the theory of flow equilibrium. Their adequacy was proved in practice.

Keywords: *Traffic Flow, Statistical Distribution, Mathematical Model, Function of Transportation Costs, Origin/Destination Pair.*

1. INTRODUCTION

Efficient designing of traffic networks as well as optimal operation of those existing is a topical problem nowadays. Around the world, this problem is being solved by applying Intelligent Transportation Systems (ITS) and Decision Support Systems (DSS). Originally, the basic idea of ITS was application of modern information technology to the automated control of transportation systems and monitoring their condition. Progress in IT made it possible to carry out an automatic analysis of efficiency of various scenarios of control over transportation systems [1]. Systems which predict mean speed and travel time along the routes have been increasingly applying in recent years. Such systems obviously have a significant impact on traffic flow distribution. Unlike ITS, DSS do not make decisions automatically. The decision-making process involves experts who consider the obtained information. The initial data for DSS are mainly obtained with the help of ITS instrumental monitoring subsystems [2]. But the core of these systems is a mathematical model of processing the data of monitoring traffic flow parameters and road conditions which provide the validity of output data.

Since the middle of the last century many researchers [3]-[8] have studied modelling and distribution of traffic flows in networks and developed methods of predicting the changes in the traffic intensity in particular sections of transportation network under various managerial actions. Mathematical models applied to the analysis of transportation networks are various due to the problems solved, the data, the mathematical tools, and the degree of specification in traffic description. Difficulties in modelling are caused by a plenty of parameters to be taken into account, the complicated process of collecting initial data and the ambiguity of optimization criteria as well as by the necessity of connecting the developed model with the particular transportation network.

According to the specification of traffic flow mathematical models applied to the urban transportation network are classified as macroscopic, mesoscopic, microscopic and submicroscopic [2]. Each group has its own advantages and disadvantages. Macroscopic models

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can help rather quickly solve the problems of urban transportation network globally and they do not require complex computer equipment. Calculation accuracy is, however, not very high. Microscopic and submicroscopic models, due to a higher degree of specification, allow us to solve local problems with high precision but at a low speed of calculation, and they need high-power computers. Mesoscopic models (CONTRAM, DynaMIT, MEZZO) can help solve both global and local problems [9]-[10]. Medium-scale specification results in more accurate calculations than in macroscopic models but much faster than in microscopic models.

2. STATISTICAL DISTRIBUTIONS IN THE TRAFFIC FLOW THEORY

Microscopic and mesoscopic models are based on the hypothesis of statistical distribution of vehicle arrivals at the given section of the urban transportation network (UTN). Methods of optimal control of traffic flows must be first and foremost based on a mathematical model which adequately describes a real traffic flow. Otherwise, the results will not provide accurate predictions. Besides, the choice of distribution of intervals between vehicles in a flow is critical for further methods of computing the efficiency parameters for traffic organization in the network. That is why a number of models exploit exponential distribution and shifted exponential distribution [11]-[13] to obtain in explicit form calculation formulas with a help of queuing theory.

Owing to the Kimber and Hollis algorithm for computing an estimated queue length in the mass service network with random distribution of request arrivals and general form of service time, the spectrum of statistical distribution applied to traffic flow theory has broadened [14]. But some coefficients obtained by the transformation coordinates method are approximate. If we take into account the scale of real transportation networks, we realize that even minor computational errors for particular nodes accumulate and may result in serious mistakes in the network as a whole.

We developed a mathematical model of traffic flow distribution in the transportation network on the basis of hypothesis of time intervals distribution between vehicles in lanes according to the general Erlang law. With a general Erlang distribution the time interval between two successive requests has k stages $T_0, T_1, \ldots, T_{k-1}$, the duration of these stages having exponential distributions with $\lambda_0, \lambda_1, \ldots, \lambda_{k-1}$ parameters correspondingly (Ventsel and Ovcharov, 2000.). The Laplace transform for the density function $f_k(t)$ will hold:

$$f^{(*)}(s) = \frac{\lambda_0 \lambda_1 \cdot \ldots \cdot \lambda_{k-1}}{(s+\lambda_0)(s+\lambda_1)\cdot \ldots \cdot (s+\lambda_{k-1})}. \quad (1)$$

If all λ_i parameters are different, the general Erlang distribution function becomes as follows:

$$f_k(t) = (-1)^{k-1} \cdot \prod_{i=0}^{k-1} \lambda_i \cdot \sum_{i=0}^{k-1} \frac{e^{-\lambda_i t}}{\prod_{\substack{n=0 \\ n \neq j}}^{k-1} (\lambda_j - \lambda_n)}. \quad (2)$$

We deduced and presented in our publications [16] the formulas to define the general Erlang parameters by the data of monitoring the traffic flow. Our experimental researches showed good consistency between the empirical and theoretical distribution at the flow density of up to 1,000 vehicles in a lane per hour.

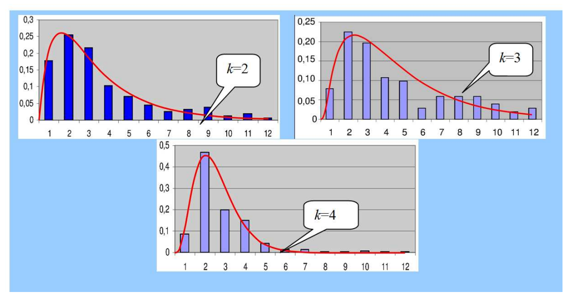

Figure 1. Plots of Density Function of Theoretical Erlang Distribution And Histograms Of Empirical Relative Frequencies For Various Parameters Values

The hypothesis about time intervals distribution between vehicles by Erlang let us, by means of differential and integral calculus by methods of function of complex variable and theory of random processes, deduce in explicit form the formulas for computing the delay value in the nods of transportation network, mean queue length at both simple intersections [17] and intersections with regulated traffic [16].

3. THE TRANSPORTATION COSTS FUNCTION IN THE NETWORK ROUTES

Under the assumption that the hypothesis about time intervals distribution by Erlang is true, we deduced in analytical form a transportation costs function as a function of parameters of flows distribution in all directions at the network nodes.

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All necessary initial data for calculations are given in the matrix presentation of network developed by us [18] in $A_{STREETS}$ and $B_{INTERSECTION}$ matrices.

The transportation cost is assumed by us to coincide with the travel time on a section of the network. The supposition about additive dependence of cost function *G(x)* on travel cost $\tau(y)$ on links of the network occurs to be the simplest and most accepted [19]. This means that the travel cost on a path is equal to the sum of the travel costs on the links which make up that path, that is,

$$G_p(x) = \sum_{l \in L} \theta_{lp} \tau_{lz}(y) = \sum_{l \in L} \theta_{lp} \left(\tau_l(y) + \tau_z(y) \right), \quad (3)$$

where G_p = travel cost function on path p,

$\tau_l(y)$ = travel cost on a link between two adjacent nodes in the network (excluding the nodes),

$\tau_z(y)$ = travel cost on passing the nodes in the network,

$$\theta_{ep} = \begin{cases} 1, & path\ p\ comprises\ link\ e; \\ 0 & otherwise \end{cases}$$

L = set of links of the network.

Travel cost $\tau_l(y)$ on a link between two adjacent nodes in the network (excluding the nodes) is defined as:

$$\tau_l(y) = \frac{l(y)}{v(y)}, \quad (4)$$

where $l(y)$ = link length,

$v(y)$ mean velocity of travel on the link.

Depending on the objectives of optimization, as travel cost function $\tau_z(y)$ on passing the nodes we can choose:

1) $\vec{\mu}(z_n)$ – the weight of node z_n (node-point) for the flow of the given direction, that is the mean delay time of all vehicles in the given direction per hour;

2) $\mu(z_n)$ – the total weight of node z_n (node-point), that is the mean delay time of all vehicles at the given node per hour;

3) $\omega_M(z_n)$ – the mean delay time (in seconds) of requests in the chosen directions.

When computing the travel cost on the given route of the network we should take $\omega_M(z_n)$, which is the mean delay in the chosen direction, as a function. The key module (Module 1) for computing traffic flow distribution in the network by traffic equilibrium principle was adapted for our model of transportation network by employing Dijkstra's algorithm which supposes, that each node can be adjacent to no more than four other nodes.

Module1:

Each link is assigned number $l(x,y) = \vec{\mu}(x) + \mu(l_{xy})$, which denotes the link length. If the nodes are not connected by a link, then $l(x,y) = \infty$. In the case considered, travel cost function *l(x, y)* of flow of vehicles $G_p(x) \equiv G_p(x(N_p))$ from the origin $i = x$ to destination $j = y$. In performing the algorithm we calculate values *d(x)* which are equal to the shortest path from node $s = z_0$ to node x:

$$d(x) = \min\{d(x), d(y) + l(x,y)\}.$$

Data which are necessary for the solution to the problem are stored in two arrays: *MPlus* contains the data on the nodes with permanent marks; *MMinus* contains the data on the nodes with temporary marks.

Each element of the arrays has the following structure:

$$MPlus_i = \begin{pmatrix} Str1 \\ Str2 \\ TimeCr \\ Trassa \end{pmatrix} \text{ and } MMinus_i = \begin{pmatrix} Str1 \\ Str2 \\ TimeCr \\ Trassa \end{pmatrix}.$$

We use the following notations: *Str1* is the street along which the travel to the given node is performed; *Str2* is the street crossing Str1; *TimeCr* is travel time *d(x)* to the given node from the origin; *Trassa* is the list of nodes passed.

Step 1. We specify the origin and destination of travel. The node-point (ND) – the origin is entered in array *MPlus* under number *n = 0* and in array *MMinus* under number *4n*.

Step 2. We define all the nodes adjacent to ND 1 and enter the data on these under numbers (4*n* + 1), (4*n* + 2), (4*n* + 3) in array *MMinus.* If the nodes are not adjacent or the motion in this direction is not allowed, then *l(x,y)* = ∞.

Step 3. We calculate the travel cost function from NP # *n* to all adjacent NPs which are not entered in array *MPlus*.

Step 4. We choose the minimal element in field *MMinus.TimeCr* and enter the data on the corresponding NP in array *MPlus* under number

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(n+1). We remove the data on this ND from array *MMinus*.

Step 5. We repeat steps 2 – 4 until every NP receives its permanent mark in array *MPlus.*

Step 6. In array *MPlus* , we choose element $MPlus_i = \begin{pmatrix} Str1 \\ Str2 \\ TimeCr \\ Trassa \end{pmatrix}$, which denotes the end of the path. Field

$MPlus.TimeCr_i$ defines the travel cost function value on the shortest path p_O. Field $MPlus.Trassa_i$ defines the list of nodes passed on the shortest path p_O.

Thus the travel cost function in the developed model of traffic flow distribution in the network takes into account the delays on intersections both simple and with regulated traffic. This favourably compares with a number of existing models which neglect these costs since such costs are time-consuming to calculate. What is also important, we give a detailed description of intersections (nodes), considering all traffic flow directions, which makes it possible to predict congestion on any path.

4. THE ALGORITHM OF COMPUTING THE TRAFFIC FLOW DENSITY DISTRIBUTION IN LANES OF TRANSPORTATION NETWORK WITH PUTTING INTO OPERATION AN "ORIGIN/DESTINATION" (O/D) PAIR

The developed algorithm is based on the principle of traffic (flow) equilibrium which satisfies to the first Wardrop principle (Wardrop, 1952) where every driver chooses a path with the least travel cost. Since each individual choice has impact on the network density, it, therefore, influences the other users' choice for the same O/D pair. The proposed algorithm takes account of this fact. For defining the users' travel costs in the network we used developed by us methods of computing the travel cost function. Besides, when developing the algorithm, we considered the fact that with putting into operation a new O/D pair the travel cost function is monotonous in relation to the density on links.

Algotrithm 1:

1) By employing the Dijksra-based algorithm (Module 1) design the optimal for user path p_{O1} from the origin to the destination for a particular request; calculate mean travel time t_{01}.

2) Find the number of requests on path p_{O1} per time unit.

3) Determine the estimated increase in density ΔN_e on the links which is related to the given O/D pair:

$$\Delta N_e = \frac{y_e}{t_{01}},$$

where $y_e = \sum_{p \in P} \theta_{ep} x_p$, where

$\theta_{ep} = \begin{cases} 1, & path\ p\ comprises\ link\ e; \\ 0 & otherwise \end{cases};$

$y = (y_e : e \in E)$ = vector describing the density on the links of network Γ;

x_p = flow on path p ;

$\Theta = \{\theta_{cp} : e \in E,\ p \in P\}$ = links and paths incidence matrix.

4) Compile a new database $\mathbf{A_1}$, where increase in density on the links of path p_{O1} is fully made by value $\Delta N_e = \frac{y_e}{t_{01}}$.

5) Check whether path p_{O1} remains equilibrium (optimal from a user's perspective) for matrix $\mathbf{A_1}$. If it does, it is the end of the algorithm. Database $\mathbf{A_1}$ contains the desired changes in density distribution in the network; assume that $\mathbf{A_0} = \mathbf{A_1}$. If it does not, go to point 6.

6) Reduce the estimated change in density on the links of the path by half: $\Delta N_{e1} = \frac{\Delta N_e}{2}$ and compile database $\mathbf{A_2}$.

7) Check whether path p_{O1} remains equilibrium (optimal from a user's perspective) for matrix $\mathbf{A_2}$. If it does, increase the density on the links of the path by value $\Delta N_{e2} = \frac{\Delta N_{e1}}{2}$ and compile a new database $\mathbf{A_3}$ with new density on the links

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of path p_{O1} which equals to $N = \Delta N_{e1} + \frac{\Delta N_{e1}}{2}$. If it does not, reduce the density on the links of the path by value $\Delta N_{e2} = \frac{\Delta N_{e1}}{2}$ and compile a new database **A₃** with new density on the links of path p_{O1} which equals to $N = \Delta N_{e1} - \frac{\Delta N_{e1}}{2}$.

8) Repeat point 7 until value ΔN_{ei} is less than the given ε. Compile a new database $\mathbf{A_0}$.

9) Compute the undistributed rest of requests flow $\Delta\widetilde{N}_e$ for the given O/D pair. If $\Delta\widetilde{N}_e < \varepsilon$, then it is the end of the algorithm. Otherwise repeat points 1-8 of the algorithm for a new path p_{02}.

5. THE ALGORITHM OF COMPUTING THE TRAFFIC FLOW DENSITY DISTRIBUTION IN LANES OF TRANSPORTATION NETWORK WITH PUTTING INTO OPERATION SEVERAL "ORIGIN/DESTINATION" (O/D) PAIRS

Let $\omega_1 = (i_1, j_1),\ i \in \{1,2,...,k\}$ be the O/D pairs put into operation. We use the given database $\boldsymbol{A_0}$, compiled in accordance with the requirements of our mathematical model of transportation network. We can predict certain changes in density due to introduction of a number of new origins or destinations. Therefore, we formulated a matrix of correspondences to reflect the probable changes.

The idea of the algorithm is as follows: we find the optimal paths between O/d pairs under the existing density distribution. Then density increases by a small value in all optimal paths, and we again choose the optimal from a user's perspective path. Thus, iteration by iteration we distribute correspondences in the network taking account of the user's optimum.

In the case under study the travel cost function is not monotonous in relation to the density on the links of the path. In calculations we used the algorithm of determining the optimal path for the given pair $\omega_1 = (i_1, j_1)$ as a separate module (Module 2) and the estimated increase in density on the links of the path.

Module 2:

Step 1. Design the optimal for user path p_O from the origin to the destination for a particular request; calculate mean travel time t_0.

Step 2. Find the number of requests on path p_{O1} per time unit (the data are taken from the correspondence matrix).

Step 3. Determine the estimated increase in density ΔN_e on the links which is related to the given O/D pair.

Algorithm 2 of solution to the problem with putting into operation several O/D pairs:

1) Design the optimal path for each O/D pair where $\omega_1 = (i_1, j_1),\ i \in \{1,2,...,k\}$ (Module 2).

2) Specify precision ε, determine value $\Delta N_{\max} = \max\{\Delta N_{li}\}$ and compile a new database $\boldsymbol{A_0}$ where increase in density on the links of path p_{li} is made by value $\varepsilon \cdot \frac{\Delta N_{li}}{\Delta N_{\max}}$.

3) Adjust the correspondence matrix for the distributed requests.

4) Repeat points 1-3 until all the requests are distributed on the paths (with precision ε).

In the proposed algorithm, by analogy with point 2.5, we take account of the fact that every driver chooses a path with the least travel cost.

6. PRACTICAL APPLICATION OF THE ALGORITHMS

The adequacy of the algorithms was proved experimentally on a particular section of the urban transportation network of the town of Krasnodar (Russia).

Problem 1. There was a new residential community built in Krasnodar in the area on the corner of **Sorok Let Pobedy Street** and **Rossiyskaya Street**. We assume that from 7 am till 8 am extra 500 vehicles will travel from the intersection of these streets to the intersection of **Moskovskaya Street** and **Ostrovskogo Street** where there is a campus of the Technological University. We are to determine how the density distribution on the lanes in this direction will change in this case. The solution of the problem

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was divided into the following stages: 1) collecting the necessary for solving the problem information, compiling the database; 2) determining the optimal paths between the intersections by Algorithm 1; 3) compiling the database reflecting the probable changes.

We solved the problem according to Algorithm 1. The correspondence matrix of traditional form was used to obtain the data on the probable changes in flow density in the transportation network for the given O/D pair. The result of algorithm was not only the path itself as a list of street which it includes and the travel time but the changes in density on each section of the network.

Figure 2 shows the paths on which the flow density distribution will change as a result of the problem solution. The path in red will have an increase in density by 313 vehicles per hour. The path in orange will have an increase in density by 187 vehicles per hour.

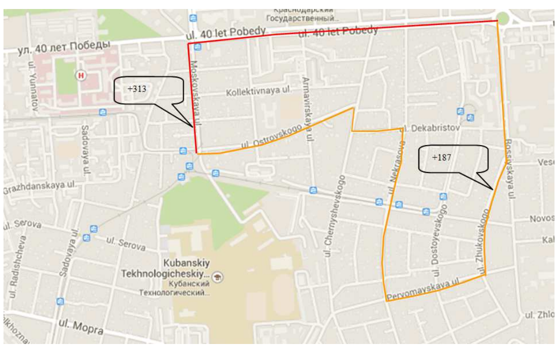

Figure 2. Paths On Which The Density Will change Subject To The Conditions Of Problem 1

Task 2. There was a new residential community built in Krasnodar in the area on the corner of **Sorok Let Pobedy Street** and **Rossiyskaya Street**. We assume that from 5 pm till 6 pm:

1) there will be 300 correspondences travelling from the intersection of **Ippodromnaya Street** and **Peredovaya Street** to the intersection of **Sorok Let Pobedy Street** and **Rossiyskaya Street**;

2) there will be 500 correspondences travelling from the intersection of **Moskovskaya Street** and **Ostrovskogo Street** to the intersection of **Sorok Let Pobedy Street** and **Rossiyskaya Street**.

We are to determine how the density distribution on the lanes will change in this case.

We solved the problem according to Algorithm 2. The solution is shown in Figure 3.The streets on which an increase in the number of correspondences will lead to the density change from the intersection of **Moskovskaya Street** and **Ostrovskogo Street** to the intersection of **Sorok Let Pobedy Street** and **Rossiyskaya Street** are marked in red.

The streets on which an increase in the number of correspondences will lead to the density change from the intersection of **Ippodromnaya Street** and **Peredovaya Street** to the intersection of **Sorok Let Pobedy Street** and **Rossiyskaya Street** are marked in blue.

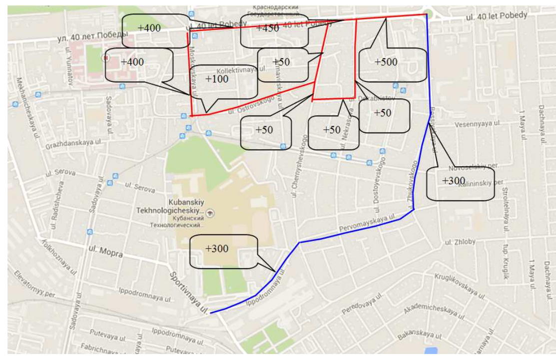

Figure 3. Paths On Which The Density Will Change Subject To The Conditions Of Problem 2

7. CONCLUSIONS

Modelling and research of traffic flows often employ the competitive noncomparative equilibrium theory which provides a fairly adequate description of transportation network operation mechanism [19].

The complexity of numerical method of solving the transportation networks-related problems mainly depend on analytical definition of travel cost function. In the developed mesoscopic model we deduced in explicit form travel cost functions for the network paths which take account of travel costs in the nodes. We solved all the problems considering the traffic flow distribution on all lanes and assuming that the general Erlang distribution of time intervals between vehicles is true. The general Erlang distribution allows us to approximate the traffic flows of high density (up to 1,000 vehicles per hour), and therefore, makes it possible to extend the applicability of the model. What is more, the minimal set of initial parameters used in our model will reduce the cost of compiling the databases for estimation of quality of reorganization in the network. The development of explicit analytical functions for determining the initial parameters without approximations and

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rounding-off will increase the accuracy of calculations. Due to the scale of real transportation networks, even minor computational errors for a single section of the network may lead to serious mistakes in determining the total travel cost function for the path. The developed analytical apparatus was proved adequate by experiment for different levels of service (LOS) and types of intersections.

The operational efficiency of managerial decisions depends on minimizing the time interval from the moment of obtaining data with the help of IT equipment till the moment of making a managerial decision. We used analytical methods of solving transportation problems which allowed us to obtain practically instant results. This contributes to the topicality of the conducted research.

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