

## PORTFOLIO SELECTION USING THE CAT SWARM OPTIMIZATION

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### ABSTRACT

The portfolio selection is a discipline in finance interested in the optimization of the investment represented by a mixed quadratic programming problem. The approach of this paper to studying the portfolio selection problem is the implementation of the metaheuristic cat swarm optimization, a method inspired from the behavior of different felines and characterized by two modes: the seeking and the tracing mode; the seeking mode is when a cat is at rest observing its environment, the tracing mode is when the cat is hunting. In this article, we have adapted this method to the cardinality constrained efficient frontier (CCEF) compared to the data of mean return and risk obtained by the unconstrained efficient frontier (UEF) for five indexes markets and we have obtained efficient results.

**Keywords:** *Portfolio Selection Problem, Metaheuristic, Cat Swarm Optimization, Efficient Frontier, Cardinality Constrained Efficient Frontier.*

### 1. INTRODUCTION

The cat swarm optimization (CSO)[1] technique is a new meta-heuristic proposed in 2006 by Shu-Chuan Chu and Pei-Wei Tsai. It is characterized by two modes: the seeking and the tracing mode; the seeking mode is when a cat is at rest observing its environment, the tracing mode is when the cat is hunting. The application of this method showed that it is more efficient than an important metaheuristic techniques.

This paper presents an adaptation of this method to the portfolio optimization problem (PO).

The PO consists of a selection of assets combinations having the best mean return and risk in order to help the investor to invest his money efficiently, and at the same time by investing we can help businesses grow their activities and create jobs. To illustrate that among the models used, the fundamental modern theory of portfolio MPT introduced by Harry Markowitz[2,3] by using statistical notion, the mean and the variance for computing the expected return and the risk of a portfolio, respectively. The principal study in this paper focuses on the CCEF[4,13,14] model inspired from the standard model of

Markowitz; this model is used because, in practice, investing can confront some constraints like

the portfolio size or the limitation of the assets proportion and those constraints are not considered in the standard model of Markowitz.

Many metaheuristics have been adapted to solve this problem, namely the simulated annealing[4,5], tabu search[4], genetic algorithm[4,6,7], particle swarm optimization [8,9,10], ant colonies[11], neurons network[12]. The contribution of this article is the application of the metaheuristic cat swarm optimization to the CCEF model and the achievement of efficient results compared to the UEF[4].

This paper is organized as follows: in the second section, we give a definition of the portfolio selection problem, in the third section, a presentation of the CSO algorithm, in the fourth section, CSO algorithm adapted to the portfolio problem, in the fifth section, the results obtained, and in the last section, a conclusion.

### 2. PORTFOLIO SELECTION PROBLEM

The model of unconstrained efficient frontier UEF is an optimization problem defined by introducing a parameter  $\lambda$  in order to simplify the optimization of the risk and the return. We have used in this paper the

data of 2000 couple mean return risk obtained with the UEF in order to have a clear comparison with the results programmed with the CCEF, the UEF model is defined as follows:

$$\begin{aligned} & \text{Min } \lambda[X'MX] - (1-\lambda)[X'\mu] \\ & \sum_{i=1}^n x_i = 1 \\ & 0 \leq x_i \leq 1, i=1, \dots, n \\ & \lambda \in [0,1] \end{aligned}$$

$n$  : the number of assets

$x_i$ : the proportion of the capital invested in  $i$

$M$  : covariance matrix,

$\mu$ : mean return of the portfolio

$\lambda$  : risk parameter

For  $\lambda=0$  the return is maximal and for  $\lambda=1$  the risk is minimal, if  $0 < \lambda < 1$  the risk and return are optimal as possible. The solution of the UEF problem is represented by the couple (return/risk) with these two objectives the efficient frontier can be obtained increasing and continuous[3].

The model with constrained (CCEF) is defined by including some parameters like  $K$  representing the number of assets considered, the limitations of the assets proportions  $s_i$  and  $e_i$ ,  $z_i$  for selecting the asset, the CCEF model is defined as follows :

$$\begin{aligned} & \text{Min } \lambda[X'MX] - (1-\lambda)[X'\mu] \\ & \text{with } \sum_{i=1}^n x_i = 1 \\ & \sum_{i=1}^n z_i = K \\ & s_i z_i \leq x_i \leq e_i z_i, i=1, \dots, n \\ & z_i \in \{0,1\}, i=1, \dots, n \end{aligned}$$

$K$ : the desired number of assets.

if an asset  $i$  is held  $z_i=1$

$s_i$  : the lower limit of the interval

$e_i$  : the upper limit of the interval.

with the solutions of this problem result a cardinality constrained efficient frontier curve.

### 3. CSO ALGORITHM

Cat swarm optimization is an evolutionary algorithm which is based on the observation of the behavior of these animals; for each one we associate a position, velocity and, depending on the parameter MR (mixte ratio), a proportion of cats is defined in tracing mode and the rest will be in seeking mode.

#### 3.1 Seeking Mode

It is the phase where the cats are at the rest and at the same time on observation of their environments in order to prepare their next move. The seeking mode includes four parameters:

SMP (seeking memory pool): number of copies of the current position

SPC (self position consideration): if (SPC = 1) the current position is considerate candidate

CDC (counts of dimension to change): the number of elements that we will change from the current position

SRD (seeking range of the selected dimension): parameter of mutation.

The seeking mode algorithm is as follows:

**Step 1:** take  $j$  copies of the cat  $i$  where  $j = \text{SMP}$  if (SPC = 1), then  $j = (\text{SMP} - 1)$  and the current position will be a candidate.

**Step 2:** for each copy according to CDC, (SRD\*the current position) is randomly added or removed to have a new combination

**Step 3:** calculation of the new positions values.

**Step 4:** calculation of the probabilities with (1) if the values are not equal, otherwise the probabilities are all equal to 1

$$P_i = \frac{|FS_i - FS_b|}{FS_{\max} - FS_{\min}} \text{ ou } 0 < i < j \quad (1)$$

$FS_b = FS_{\max}$  if we look to minimize the objective function otherwise  $FS_b = FS_{\min}$

**Step 5:** randomly pick the next position.

#### 3.2 Tracing Mode

It is characterized by a quick movement of cats while hunting. This phase can be described in three steps:

**First step:** calculation of the velocities for each dimension.

$$v_{i,d} = v_{i,d} + (r_1 * c_1) * (x_{\text{best},d} - x_{i,d}) \quad d=1, \dots, M$$

$x_{\text{best},d}$  : is the position of the cat having the best value in the dimension  $d$ ,  $x_{i,d}$  is the position of the cat  $i$  in the dimension  $d$ ,  $c_1$  is a constant  $r_1$  is a random value between  $[0,1]$ .

**Second step:** check if the velocity is maximal

Third step: update the position:

$$x_{i,d} = x_{i,d} + v_{i,d}$$

### 4. CSO ALGORITHM ADAPTED TO THE PROBLEM OF PORTFOLIO

For every cat are associated a proportion vector  $X$  and a decision vector  $Z$  defined in seeking mode, otherwise in tracing mode.

#### 4.1 Definition Of The Optimization Function

$$f = \lambda * ((X)' * M * X) - (1 - \lambda) * (X * (R_{\text{bar}})')$$

The objective is to minimize this function with the method of cat swarm optimization. The movement can be defined in two modes: the tracing mode or seeking mode ; the tracing mode illustrates the movement of the cats depending on the best position of the group and the seeking mode is the selection of the position between a lot of combinations inspired from the present position.



**4.2 Cats Movement**

As mentioned before, each cat is initialized in seeking mode, otherwise in tracing mode, and after the movement of all the particles these latter are reinitialized. For each phase the movement is defined as follows:

**4.2.1 Tracing mode movement**

In this phase, for each dimension *i* and cat *c*, the movement is defined as follows:

$$vZ_{ci}^{t+1} = vZ_{ci}^t + (r_1 * c_1) * (GZ_{bi} - Z_{ci}^t)$$

$$Z_{ci}^{t+1} = \text{round}\left(\frac{1}{1+e^{-\xi}} - \alpha\right)$$

$vZ_{ci}^t$  : the velocity of the decision to the dimension *i* at the iteration *t*.

$r_1$  : a random number in [0,1].

$c_1$  : constant is generally equal to 2.05.

$GZ_{bi}$  : the best decision to the dimension *i*

$Z_{ci}^t$  : the decision to the dimension *i* at the iteration *t*

and  $\xi = Z_{ci}^t + vZ_{ci}^{t+1}$ . [9]

$\alpha$  : is a parameter defined in order to arrange the decision in some specific cases [9]

the position is calculated as follows :

$$vX_{ci}^{t+1} = \begin{cases} vX_{ci}^t + (r_1 * c_1) * (GX_{bi} - X_{ci}^t) & \text{if } Z_{ci}^{t+1} = 1 \\ vX_{ci}^t & \text{otherwise} \end{cases}$$

$$X_{ci}^{t+1} = \begin{cases} vX_{ci}^t + X_{ci}^t & \text{if } Z_{ci}^{t+1} = 1 \\ X_{ci}^t & \text{otherwise} \end{cases}$$

$vX_{ci}^{t+1} + X_{ci}^t$  must be positive

$X_{ci}^t$  : The position to the dimension *i* at the iteration *t*

$vX_{ci}^t$  : the velocity of the position to the dimension *i* at the iteration *t*

$GX_{bi}$  : the best position of all the group to the dimension *i*.

After the movement we have to arrange X and Z

If ( $Z_{ci}^{t+1} = 0$ ) the decision at the iteration *t* before

The movement was ( $Z_{ci}^t = 0$ ) or ( $Z_{ci}^t = 1$ ), in this case we must arrange the vector Z, and after arrangement we keep the best solution.

**4.2.2 Seeking mode movement**

This mode is characterized by the application of parameters defined in section 3 (SMP, SRD, CDC, SPC) to the current position in order to have a new portfolio as follows:

*Algorithm 1: Seeking Mode Movement*

**step1:** taking SMP copies of the current position if (SPC == 1) (SMP = SMP-1) and the current position is considered candidate.

**Step2:** calculating (CDC \* N) in order to get the number of assets for the mutation, the asset is randomly selected

**For** *i*=1 to SMP

copiX: copy of the current portfolio

copiZ: copy of the current decision vector associated to the portfolio.

V: indexes vector in {1,...,N} selected randomly, his size is (CDC\*N).

**Step 3:** SRD is a mutation value added or removed randomly as follows:

**For** *j*=1 to size (V)

copiZ(i,V(j))= copiZ(i,V(j)) ± (SRD\* copiZ(i,V(j)))

**If** (copiZ(i,V(j))=1)

copiX(i,V(j))=copiX(i,V(j))±(SRD\* copiX(i,V(j)))

**End**

arrangement of the vectors  
copiZ and copiX (section 4.3)  
calculation of the fitness value.  
keep the optimal portfolio.

**End**

**End**

**step 4:** calculation of the candidates fitness value, and the probabilities .section (3.1.1)

**step 5:** choose the new portfolio with the algorithm (roulette wheel selection).

**4.3 The Constrained of the Problem**

For every cat are associated a proportion X and a decision Z the first step is to verify that:

-The number of assets held must be exactly equal to K.

-The sum of the elements of X is equal to 1.

-  $s_i \leq x_i \leq e_i$  for each asset *i* selected

In order to verify those constrained we use a function of arrangement as follows [4,9]:

Z: the set of the assets indexes after a movement in tracing or seeking mode

kr : the dimension of Z.

First case: we suppose that ( $Kr < K$ ) we generate a random number between [0,1] if it is less than 0.5 randomly choose an element not belonging to Z and



we added it otherwise If the number is greater than 0.5 we choose the index of the maximum value of C (defined below) not belonging to Z and we add it  
 Second case: if  $(Kr > K)$ , if the random number is less than 0.5 we randomly select an element in Z to remove it, otherwise we take the index of the minimum value of C belonging to Z and we delete it we must to return in the initial velocity and velocity decision in this case. Finally we have  $(Kr = K)$ .

We consider:

$$\theta_i = 1 + (1 - \lambda) \mu_i, \quad i=1, \dots, N$$

$$\rho_i = 1 + \lambda \left( \frac{\sum_{j=1}^N M_{ij}}{N} \right), \quad i=1, \dots, N$$

$$\Omega = -1 * \min(0, \theta_1, \dots, \theta_N)$$

$$\Psi = -1 * \min(0, \rho_1, \dots, \rho_N)$$

$$C_i = \frac{\theta_i + \Omega}{\rho_i + \Psi}, \quad i=1, \dots, N$$

*Algorithm2: Function Of Arrangement[4]*

**If**  $(\sum_{i \in Z} s_i \leq 1)$  **And**  $(\sum_{i \in Z} e_i \geq 1)$

B: the sum of the assets i such as  $i \in Z$ .

C: the sum of  $s_i$  such as  $i \in Z$ .

$D = 1 - C$

$w_i = s_i + (x_i * D) / B$  for each  $i \in Z$  // proportion satisfying the limit  $s_i$  and the sum of the elements equal to 1.

$w_i = 0$  otherwise.

$E = \emptyset$ , the set of assets i such as  $(w_i > e_i)$

**Repeat** If they exist an assets i for each  $i \in (Z - E)$  such as  $(w_i > e_i) \quad E : E \cup \{i\}$

$$G = \sum_{i \in (Z - E)} x_i$$

$$H = 1 - (\sum_{i \in (Z - E)} s_i + \sum_{i \in E} e_i)$$

$$w_i = s_i + (x_i * H) / G \text{ for each } i \in (Z - E)$$

$$w_i = e_i \text{ for each } i \in E$$

**End**

**End**

*Algorithm3: CSO Adapted To The Problem Of Portfolio*

**Begin**

$$\lambda = 0;$$

$N, K;$

Initialisation of the parameters (mode tracing)

$$r_1 = \text{rand}(1);$$

$$c_1 = 2.05;$$

Initialisation of the parameters (mode seeking)

$$SMP, SPC, SR, MR, CDC, P, N_t = MR * P, N_s = P - N_t;$$

**While**  $(\lambda \leq 1)$

initialisation of the particles mode

**For**  $p=1 \dots P$

Initialisation of the particles

proportion and decision ;

Arrange(p);

**End**

Calculation of the values  $f(p)$ ;

keeping the proportion of the particle b associated to the minimal value  $G_b$

**For**  $c=1$  **To** 279

**For**  $i=1$  **To** P //P=111

**If** i is initialised in seeking mode do

seeking mode movement

keep the best solution

**Else**

tracing mode movement

keep the best solution

**End**

**End**

reinitialisation of the particles mode

**End**

$$\lambda = \lambda + 0.02;$$

**End**

**End**

Table 1: CSO Parameter[1]

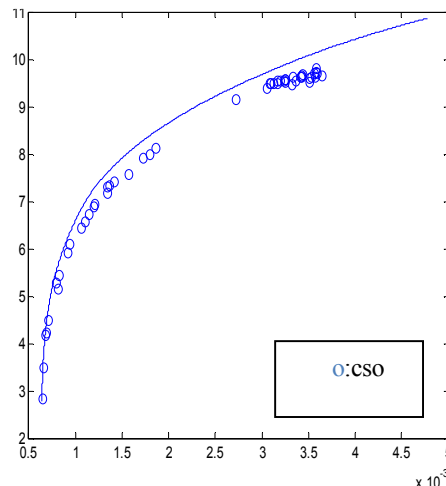
SMP	SRD	CDC	MR	c1	r	w
5	0.2	0.8	0.3	2.05	[0,1]	0.729

5. RESULTS OBTAINED

Table 2: CSO Results

Index	Assets		CSO
Hang Seng	31	Mean euclidian distance	0.000212
		Variance of return error	6.4806
		Mean return error	1.5745
		Time (s)	52
DAX 100	85	Mean Euclidian Distance	0.000397
		Variance of Return Error	31.2654
		Mean Return Error	1.7438
		Time (s)	92
FTSE 100	89	Mean Euclidian Distance	0.000082
		Variance of Return Error	7.8
		Mean Return Error	0.5486
		Time (s)	374
S&P 100	98	Mean Euclidian Distance	0.000209
		Variance of Return Error	7.8830
		Mean Return Error	2.3609
		Time (s)	210
Nikkei	225	Mean Euclidian Distance	0.0000497
		Variance of Return Error	3.1372
		Mean Return Error	1.0390
		Time (s)	539

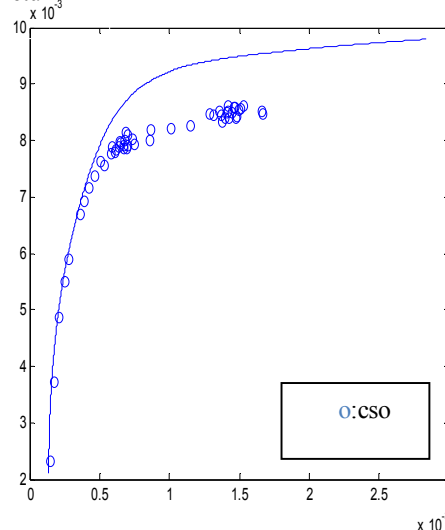
Return



Risk

Figure 1: Comparison of CSO And the Unconstrained curves for The Hang Seng Index

Return



Risk

Figure 2: Comparison of CSO And the Unconstrained curves for The Dax 100 Index

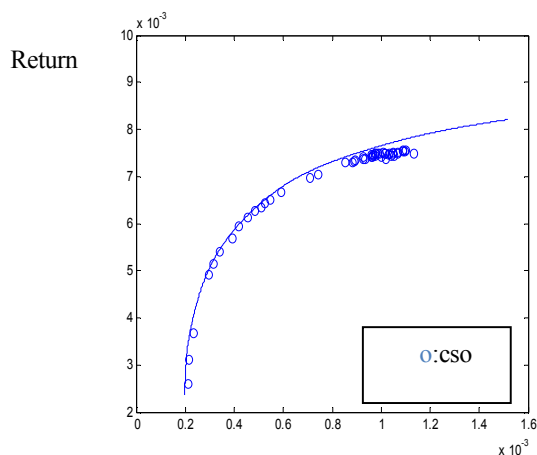


Figure 3: Comparison of CSO And The Unconstrained curves For The FTSE Index

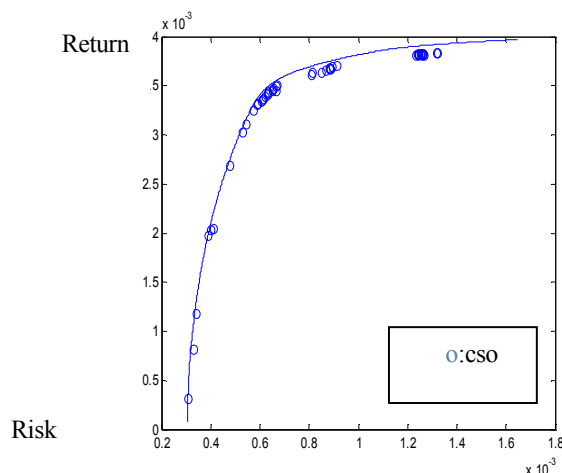


Figure 5: Comparison of CSO And The Unconstrained curves For the Nikkei Index

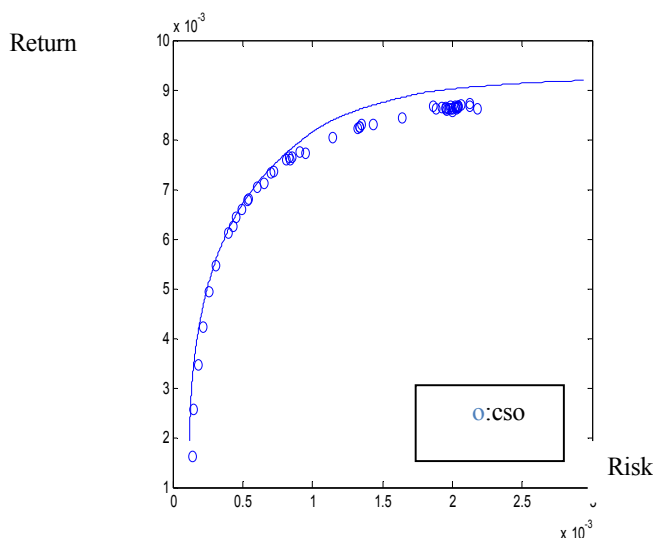


Figure 4: Comparison of CSO And The Unconstrained curves For The S&P 100 Index

The graphs represent the curves of the optimal portfolios; the points below the curves are less efficient. The results of this study are obtained by the application of the CSO method on the CCEF and they are compared to the UEF; generally we obtain very approximate curve to the unconstrained model. For low risks, we have obtained returns very approximate to those with the unconstrained model but when risks reach a certain value, the return increase systematically though a bit lower than that obtained with the unconstrained model.

## 6. CONCLUSION

In this paper we have studied the problem of portfolio selection by tracing out the efficient frontier of CCEF problem applying the method CSO. The obtained results were compared with those of the UEF problem; these results illustrate an optimal strategy for investing, carefully using five indexes, of the stock exchange. In order to develop this work, we can apply a new metaheuristic to this problem to have better results.

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