ANALYTICAL ANALYSIS OF BLOOD SPATTER TRAJECTORY FOR CRIME SCENE INVESTIGATION

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ABSTRACT

This paper presents a noble theoretical method followed by an algorithm for analyzing the trajectory path of linear blood spatter drop in motion base on image of a crime scene. Linear blood spatter drop has some specific pattern with moderate droplet size (Reynolds number, Re >= 1) and elliptical or circular shaped stain. Their free flight trajectories are non-linear such as parabolic motion or motion with drag. So far, works been done using Stokes’ law only for blood-droplet free flight trajectory of motion with drag, which is suitable for very small droplet (Reynolds number, Re << 1), like mist. However, it is not suitable for moderate droplet size. Hence, we introduce two regions of the drag force working on the droplet, Stokes’ law and Newton’s law region depending on their size, velocity and Re values. It also takes into consideration the perturbed path for the droplet’s movement, given room for a small error, δ, for the angle and speed. Hence, more realistic reconstruction of trajectory path along with angle and speed of blood spatter drop compared to available ones. The proposed method could be very helpful for blood spatter image analysis for crime scene investigation in near future.

Keywords: Newton’s Law, Stokes’ Law, Reynolds’ Number, Trajectory Construction, Blood Spatter Image Analysis.

1. INTRODUCTION

When blood drops are ejected from a wound in the events of a violent crime, the blood is assumed to move through air and touch the ground/destination in a certain pattern [1, 2]. The forces acting on the blood particles are assumed to be gravity and drag. The shapes of the stains are usually projected spheres onto the recipient surface, which are ellipses on planar surfaces [2, 3, 4, 5, 6, 7]. We can only observe the impact positions and angles after the event occurred, thus leaving the original emitting position at the time of the event unknown.

Traditional techniques to determine trajectory includes using (i) physical strings and (ii) software. The string method involves identifying the angle of the impact and assuming the droplet takes a linear path to physically construct a 3D trajectory at the crime scene [2]. It is done manually, hence the measurement of impact angle vary person to person, leaving a degree of error. Also assumptions of linear path for the droplet add some additional error to estimate the final trajectory of blood droplet path through the air. This trajectory path (along with impact angle) is very useful for the investigator to estimate the original blood emitting position (weapon hitting height from ground, body position, etc.), heating direction, types of used weapons etc. As a result, a little deferment in trajectory path may result some kind of disagreement in the investigation. The 2nd available method is software method, such as BackTrackTM [8]. It has been used to reconstruct the trajectory of blood droplets in complicated way. A lot of information has to input into the system for accurate results [8], which is not possible normally. Due to insufficient available information, the output trajectory from the software system is not reliable as per expectation. Hence, for result validity up to investigators satisfaction, both string and software methods are used simultaneously. These results extra work load and hassle besides complexity and time consumption.

So far from authors’ knowledge, most of the previous conducted researches [1, 2, 7, 8, 9, 10, 11], trajectory path has been calculated using Stokes’ Law [12]. These methods using Stokes’ Law work on tiny blood droplets (like mist), usually smaller than 1mm in diameter with Reynolds number, Re<<1 [6, 12]. This scenario only occurs for gun shoot from a distance. Therefore, the available methods using Re<<1 is not suitable for the crime scenes (where other types of weapons are used than gun) for moderate or
bigger blood droplets ($Re >= 1$). From laws of physics, Newton’s law is applicable for moderate and bigger particle size. Hence, to overcome the above issues, this paper aims to investigate the use of Newton’s Law to find the trajectory for moderate size blood droplets (greater than 1mm in diameter) with $Re >= 1$ to estimate the final blood flight trajectory in the air towards a reliable investigation.

2. THEORETICAL ANALYSIS

2.1. Newtonian Derivation for a Parabolic Trajectory

One of the most basic types of physical motion where objects fall only under the effect of gravity is projectile motion, without considering the air resistance. Newton’s laws state that the net force $F_{net}$ on an object is related to the mass $m$ and acceleration $a$ by the equation [3, 13, 14]:

$$F_{net} = ma$$

(1)

If we assume the only downward force acting on a particle/droplet is gravity, the above equation can be modified as:

$$F_g = mg$$

(2)

Where $g = (0, -g)$ is the 2D acceleration vector of gravity with $g = 9.81m/s^2$. For particles only affected by gravity the spatial motion is parabolic [4, 5].

Integrating $a = g$ with respect to time $t$, since gravity is the only force, the result obtained is:

$$v(t) = gt + v_0$$

(3)

where, $v_0$ is the initial velocity. To obtain the initial position $x_0$, we integrate the above equation again to find analytical description of the trajectory:

$$x(t) = \frac{1}{2}gt^2 + vt + x_0$$

(4)

From the Equation (4), it can be seen that the trajectory of the particle/droplet would be parabolic in motion. This equation is true for all particles since any particle/droplet falling under gravity has the same acceleration.

2.2. Trajectory Derivation with Drag Force

In Section 2.1, the drag force of air resistance was not taken into account. By considering the drag force into the derivation, the resulting trajectory can be parabolic initially, but turns to linear as the particle approaches its terminal velocity [3, 5, 15]. This drag force and therefore terminal velocity vary for different particles depending on their size and shape. In this paper, we investigate two scenarios considering the drag force on a particle/droplet in terms of the Stokes law region and the Newton’s law region. The drag force in these two regions depends on the Reynolds number of the particles/droplets as it moves through air, which usually varies with their size and velocity [6, 16]. Reynolds number is defined as $Re = \frac{\rho v d}{\mu}$, with $v$: velocity of particle, $d$: diameter of the particle, $\rho$: density of fluid/air, $\mu$: viscosity of fluid/air.

2.2.1. Stokes’ law region

Stokes Law region applies on spherical objects in a fluid with low Reynolds number ($Re < 1$) and high velocity movement [6, 9, 12]. When a liquid droplet is small, it is usually spherical in nature as it falls through air. This situation is common in crime scenes where a gun has been used, and mist spatter pattern is found in the scene. The mist droplets being very small in size and had flown through the air with high velocity causing it to retain its spherical shape [10].

This law along with Equation (2) states the drag force, particle velocity, terminal velocity, and trajectory path as shown in Equations (5) to (8) respectively [9]:

$$||F_d|| = 6\pi \mu r ||v||$$

(5)

$$v(t) = \frac{mv}{k} + (v_0 - \frac{mv}{k}) e^{-\frac{kt}{m}}$$

(6)

$$v_{eq} = \frac{mv}{k}$$

(7)

$$x(t) = v_{eq}t + \frac{mv}{k}(v_0 - v_{eq})(1 - e^{-\frac{kt}{m}}) + x_0$$

(8)

where, $\mu$ is the dynamic viscosity of the fluid, which for air is $1.78 \times 10^{-5}$ kg/m.s, $v$ is the particle velocity, $k = 6\pi \mu r$, $r$ is the droplet/particle radius, $m$ is the droplet/particle mass, $v_0$ is the initial velocity, $v_{eq}$ is the terminal velocity, and $x_0$ is the initial position.
2.2.2. Newton’s law region

Newton’s law region applies objects in a fluid with Reynolds number \( 1 < R_e < \frac{2 \times 10^5}{9} \) [9]. This applies for most of the particles/droplets that are medium to large in size (approximately, 1mm \( \leq d \leq 4\text{mm} \)) with medium to low velocity [15]. These kinds of stains are most commonly found in crime scenes with liner blood spatter pattern where firearms not been used. However, so far, no analysis is done to form a theory for blood spatter trajectory reconstruction investigation for this kind, which is our focus here. Hence, detail derivation and steps for this law are presented here.

This law states that the drag force:

\[
F_D = C_D \frac{1}{2} \rho \, d^2 \, v^2
\]

(9)

where, \( \rho = 1.225 \text{ kg/m}^3 \), is the fluid (air) density, and \( C_D \) is the coefficient of drag, which is dependent on Reynolds number. For the Newton’s law region, usually, \( C_D \cong 0.44 \). Then, substituting this back to drag equations, we get:

\[
F_D = 0.44 \frac{1}{2} \rho \, d^2 \, v^2
\]

(10)

Combining all constants into one term \( l \), where:

\[
l = 0.055 \pi \mu \, d^2
\]

(11)

we can write the drag force as follows:

\[
F_D = -l v^2
\]

(12)

According to Newton’s law in Equation (1),

\[
F = F_g + F_D = ma,
\]

mg – hv^2 = ma.

Substituting \( a = \frac{dv}{dt} \) and simplifying it:

\[
mg – hv^2 = \frac{dv}{dt}
\]

(13)

Integrating this to find the velocity (by considering one component of the vector at a time), we get:

\[
v = tanh^{-1} \left( \frac{C_s \, l}{mg} \right) \frac{1}{mg}
\]

(14)

For both the \( x \) and \( y \) components, we obtain the equation:

\[
\nu = tanh^{-1} \left( \frac{C_s \, l}{mg} \right) \frac{1}{mg}
\]

(15)

where, the constant \( C_3 \) is different for each component. If initial conditions \( v(t) = v_0 \) for \( t = 0 \), we can find:

\[
v_0 = tanh^{-1} \left( \frac{C_s \, l}{mg} \right) \frac{1}{mg}
\]

Simplifying this,

\[
C_s = \frac{mg}{l} \tanh^{-1} \left( \frac{1}{mg} \, v_0 \right)
\]

(16)

Then substituting this back in Equation (15), the velocity equation becomes:

\[
v(t) = \frac{1}{l} \sqrt{\frac{1}{C_s} \log \left( \cosh \left( \sqrt{\frac{2}{C_s}} \, \left( \tanh^{-1} \left( \frac{1}{mg} \, v_0 \right) \right) \right) \right) + \frac{1}{l} \sqrt{\frac{1}{C_s} \log C_s}}
\]

(17)

Then, the terminal velocity from analyzing this, as:

\[
v_{t} = \lim_{t \to \infty} v(t) = \frac{mg}{l}
\]

(18)

Again, integrating the velocity Equation (17), we can obtain the trajectory or the distance as:

\[
x(t) = \sqrt{\frac{mg}{l} \log \left( \cosh \left( \sqrt{\frac{2}{C_s}} \, \left( \tanh^{-1} \left( \frac{1}{mg} \, v_0 \right) \right) \right) \right) + \frac{1}{l} \sqrt{\frac{1}{C_s} \log C_s}}
\]

(19)

Now we substitute the initial condition \( x(t_0) = x_0 \) at time \( t = 0 \) in Equation (31), to solve for \( C_s \) as:

\[
x_0 = \sqrt{\frac{mg}{l} \log \left( \cosh \left( \sqrt{\frac{2}{C_s}} \, (0) \right) + \frac{1}{l} \sqrt{\frac{1}{C_s} \log C_s} \right)}
\]

\[
C_s = \frac{mg}{\sqrt{\frac{1}{l} \log C_s}} \sqrt{1 - \left( \frac{1}{mg} \, v_0 \right)}
\]

(20)
Substituting $C_d$ back in Equation (19), we obtain the final drag trajectory as,

$$x(t) = \sqrt{\frac{4m}{\nu}} \log \left( \frac{\nu}{\nu_0} \left( 1 - \frac{v_0}{\nu_0} \right) \right) 
\cosh \left( \frac{E}{m} t + \text{tanh}^{-1} \left( \frac{v_0}{\nu_0} \right) \right)$$

(21)

However, for the terminal velocity, this drag trajectory yields:

$$x(t) = \nu_0 \log \left( \frac{\nu}{\nu_0} \left( 1 - \frac{v_0}{\nu_0} \right) \right) \cosh \left( \frac{E}{m} t + \text{tanh}^{-1} \frac{v_0}{\nu_0} \right)$$

(22)

From the above analysis of Stokes law and Newton law regions we can get the analytical description for the particle/droplets flight paths.

3. APPROXIMATION PERTURBATION OF TRAJECTORY

Now we solve for approximate solution errors in the impact variables which will be measured. This may show where the particle may have come from at some time instant, $t$.

We reconstruct the simple parabolic trajectory described in Equation (4) to show how to reconstruct accurately the area of origin with only knowing the impact positions and velocities, and not the mass and drag coefficient of the particle. We have considered the 2D Cartesian plane with $z$ representing vertical height, and $x$ representing horizontal distance. Since parabolas are symmetric, we can obtain the same flight path by firing a projectile from the same impact position with the same velocity as the impact velocity. For simplicity, we have considered the initial position as $x_0 = (0, 0)$ with impact speed $v_0$ and angle $\theta$. Therefore, the impact position and velocity parameters are $x_i$ and $v_i$ for each measured stain. This main reconstruction path can be denoted as,

$$c_m(t) = (x_m(t), z_m(t))$$

(23)

where:

$$x_m(t) = v_0 \cos(\theta) t,$$

$$z_m(t) = v_0 \sin(\theta) t - \frac{1}{2} g t^2.$$

(24)

(25)

If we consider projectile fired at a slightly different angle, $\theta + \delta \theta$, for some small positive or negative, $\delta \mu$, as well as a slightly different initial speed, $\delta v$, then this would yields a new path as a function of small perturbations in terms of angle and initial speed:

$$c_t (t, \delta \mu, \delta \nu) = (x_t (t, \delta \mu, \delta \nu), z_t (t, \delta \mu, \delta \nu))$$

(26)

where:

$$x_t (t, \delta \mu, \delta \nu) = (v_0 + \delta v) \cos(\theta + \delta \theta) t$$

$$z_t (t, \delta \mu, \delta \nu) = (v_0 + \delta v) \sin(\theta + \delta \theta) t - \frac{1}{2} g t^2$$

(27)

(28)

We can also consider this path $c_t (t, \delta \mu, \delta \nu)$ relative to the known reconstructed trajectory $c_m(t)$ with the equation

$$c_t (t, \delta \mu, \delta \nu) = p(t, \delta \mu, \delta \nu) + c_m(t),$$

(29)

where, $p(t, \delta \mu, \delta \nu)$ is a perturbation vector from the main path varying over time. We can solve for $p(t, \delta \mu, \delta \nu)$ by rearranging the above equation,

$$p = c_t - c_m$$

(30)

The perturbation vector can be solved as follows:

$$x_p = x_t - x_m = t ((v_0 + \delta v) \cos(\theta + \delta \theta) - v_0 \cos(\theta))$$

$$z_p = z_t - z_m = t ((v_0 + \delta v) \sin(\theta + \delta \theta) - v_0 \sin(\theta))$$

(31)

(32)

In conclusion the equation for the relative vector from the main path to a perturbed path is given by the equations as:

$$p(t, \delta \mu, \delta \nu) = (x_p (t, \delta \mu, \delta \nu), z_p (t, \delta \mu, \delta \nu))$$

(33)

$$x_p (t, \delta \mu, \delta \nu) = t ((v_0 + \delta v) \cos(\theta + \delta \theta) - v_0 \cos(\theta))$$

$$z_p (t, \delta \mu, \delta \nu) = t ((v_0 + \delta v) \sin(\theta + \delta \theta) - v_0 \sin(\theta))$$

(34)

(35)
4. PERTURBATION OF TRAJECTORY WITH DRAG FORCE

Now the perturbation vector is derived for trajectories undergoing drag. To define the perturbation vector, we must first state the perturbed path for a trajectory with drag because it is not symmetric unlike the case of parabolic motion. Therefore, the reconstruction path of the Newton’s law region can be derived as follows.

Letting time be negative in Equation (21), we get:

\[ c_a(t) = \]

\[ \frac{v_{u\alpha}}{v_{u\alpha}} \log \left( \left( \frac{v_{u\alpha}}{v_{u\alpha}} \right)^2 \cosh \left( -\frac{\sqrt{v_{u\alpha}}}{v_{u\alpha}} \right) + \tanh^{-1} \left( \frac{v_{u\alpha}}{v_{u\alpha}} \right) \right) \times \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \]  

Substituting \( v_{ug} = (v_0 + \delta_0 \cos(\theta) + \delta_0 \sin(\theta + \delta_0)) \):

\[ c_a(t, \delta_0, \delta) = \]

\[ \frac{v_{u\alpha}}{v_{u\alpha}} \log \left( \left( \frac{v_{u\alpha}}{v_{u\alpha}} \right)^2 \cosh \left( -\frac{\sqrt{v_{u\alpha}}}{v_{u\alpha}} \right) + \tanh^{-1} \left( \frac{v_{u\alpha}}{v_{u\alpha}} \right) \right) \times \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \]  

To obtain the perturbation vector:

\[ \mathbf{p}(t, \delta_0, \delta) = c_x - c_{m} = \]

\[ \frac{v_{u\alpha}}{v_{u\alpha}} \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \]  

We expand this to obtain the perturbation vector equations for trajectories with drag as:

\[ \mathbf{p}(t, \delta_0, \delta) = (x(t, \delta_0, \delta), z(t, \delta_0, \delta)) \]  

\[ x(t, \delta_0, \delta) = \]

\[ \frac{v_{u\alpha}}{v_{u\alpha}} \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \times \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \]  

This is the final reconstructed path given the perturbation occurring due to small error in the angle and speed of the droplet for Newton’s law region.

Similarly, the reconstruction path for Stokes’ law region can be summarized as follows.

The perturbation vector:

\[ \mathbf{p}(t, \delta_0, \delta) = c_x - c_{m}, \]

\[ = \frac{v_{u\alpha}}{v_{u\alpha}} \left( 1 - v_{0} \right) \left( v_{u\alpha} - v_{m} \right) \]  

Therefore, the perturbation vector equations for trajectories with drag:

\[ \mathbf{p}(t, \delta_0, \delta) = (x(t, \delta_0, \delta), z(t, \delta_0, \delta)), \]  

\[ x(t, \delta_0, \delta) = \frac{v_{u\alpha}}{v_{u\alpha}} \left( 1 - v_{0} \right) \left( v_{u\alpha} - v_{m} \right) \times \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \]  

\[ z(t, \delta_0, \delta) = \frac{v_{u\alpha}}{v_{u\alpha}} \left( 1 - v_{0} \right) \left( v_{u\alpha} - v_{m} \right) \times \left( \begin{array}{c} \sin(\theta + \delta) - v_{0} \cos(\theta) \\ \cos(\theta + \delta) - v_{0} \sin(\theta) \end{array} \right) \]

5. LINEAR BLOOD SPATTER TRAJECTORY RECONSTRUCTION ALGORITHM

Based on the theoretical analysis, an algorithm is proposed to reconstruct the blood droplet free flight trajectories and point of source based on the impact image. The algorithm is useable for both very small (Re < 1, d < 1mm) and moderate size (Re > 1, approximately, 1mm ≤ d ≤ 4mm) blood droplets. But here, our aim is to reconstruct the free fly trajectory for Re > 1 using Newton’s law drag force region only. The algorithm works as follows (shown in Figure 1):

Step 1: Analyze crime scene image for blood spatter pattern and droplet shape. Estimate droplet size and mass then travelled distance x, and environmental parameters, e.g., \( \mu, \rho, \gamma \), etc.
Step 2: Estimate droplet impact angle. Then use Newton’s Law, Equation (4) to estimate initial trajectory and \( x_0 \) without drag force.

Step 3: Estimate droplet diameter from image, then calculate Reynolds number \( Re \) value.

Step 4: Decision to determine drag force region based on \( Re \).

(i) If \( Re < 1 \): use Stokes’ law drag force region to reconstruct the trajectory and estimate source of point using Equations (6) to (8).

(ii) Else: use Newton’s law drag force region to reconstruct the trajectory and estimate source of point using Equations (11), (17), (18) and (22).

Step 5: Based on Step 4:

(i) If \( Re < 1 \): determine the error of estimation using Equations (42) to (44) and then refine the trajectory and source of point.

If \( Re > 1 \): determine the error of estimation using Equations (38) to (40), then refine the trajectory and source of point.

6. RESULTS AND DISCUSSIONS

The algorithm is executed using MATLAB software to verify its workability and performance. The theoretical analysis through algorithm shows that it is more appropriate to measure blood droplets trajectory using Newton’ Law when diameter is greater than 1mm rather than using Stokes’ Law as shown in Figure 2.

It also shows that, in general, Newton’s Law region of drag force shows more accurate result in measuring trajectory of blood droplets compared to Stokes’ Law. Even Newton’s Law is usable for \( Re < 1 \) too as apparent in Figure 3.

From Figure 4, we also find that the perturbed errors that may occur in angle and speed, can be measured more accurately for droplets greater than 1mm in diameter, compared to the less than 1mm. Hence, Newton’s Law perturbation is more suitable for linear blood splatter trajectory analysis.

The radii used for this investigation (MATLAB Simulation) are \( r = 4 \text{mm} \) and 1\text{mm} with corresponding velocity \( v = 4 \text{ms}^{-1} \) and 2\text{ms}^{-1} respectively. For droplet radius, \( r = 4 \text{mm} \) with velocity \( v = 4 \text{ms}^{-1} \) gives the Reynolds number, \( Re = 1101.12 \) (from Figure 5). For this droplet, it
would be more appropriate to use the Newton's Law as shown in Figure 4. Whereas, droplet radius, \( r = 1 \text{mm} \), gives the Reynolds number, \( Re = 275.28 \) (from Figure 5).

The theoretical analysis through algorithm using MATLAB software shows that it is more appropriate to measure blood droplets trajectory using Newton’s Law if radius is greater than 1mm (\( r = 4 \text{mm}, Re>1000 \)) compared to Stokes’ Law, and it is more appropriate to use Stokes’ Law if radius is less than 1mm than Newton’s Law.

![Figure 2: Free Flight Trajectory Comparison Of Blood Droplet For Re > 1](image)

![Figure 3: Free Flight Trajectory Comparison Of Blood Droplet For Re < 1](image)

![Figure 4: Refined Free-Flight Trajectory (Re>1) With Perturbation](image)

![Figure 5: Reynolds Numbers Calculated Using Four Different Radii And Over 1ms^{-1}≤V≤6 Ms^{-1}.](image)

7. CONCLUSION

This paper demonstrated the analytical model for analysis of blood spatter at crime scenes using Newton’s Law. Through the model and proposed algorithm, the trajectory path of the blood droplet followed by the original emitting position can be reconstructed reliably. It is more appropriate to measure blood droplets trajectory using Newton’s Law if radius is greater than 1mm compared to Stokes’ Law and vice-versa. Finally, Newton’s Law region of drag force shows more accurate result in measuring trajectory of blood droplets contributing more reliable crime scene investigation results in near future. Even Newton’s Law is usable for smaller blood droplet size (\( Re < 1 \)) too showing its efficiency. Further work can involve more forces working on the droplet in air, such as lift force and buoyancy force.
REFERENCES:


