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A NOVEL BINOMIAL TREE APPROACH TO CALCULATE COLLATERAL AMOUNT FOR AN OPTION WITH CREDIT RISK

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Abstract: Options traded over-the-counter are associated with credit risk and are called vulnerable options. Collateral can be taken to mitigate the credit risk. However the amount of collateral should be such that it shouldn't take into account the almost not plausible states of the underlying which when considered increases the required collateral amount. This paper proposes a methodology to calculate the optimum collateral amount that is required from the seller of a vulnerable option. The calculated collateral makes the vulnerable option as risky as the exchange traded option. The algorithm to calculate the optimum collateral uses the novel binomial decision tree built without any assumption on the underlying distribution. The study reveals that the price of a vulnerable option converges to an exchange traded option as the collateral amount reaches a certain optimum value. The proposed methodology will be of interest to the option seller so that the excess collateral above the optimum collateral can be used for some other purposes.

Keywords: *Binomial Tree, Credit Risk, Collateral Amount, Option pricing, Venerable options, Margin Computation*

1 INTRODUCTION

Derivatives are financial instruments that are used to transfer the risk in an underlying to counterparty. One such derivative is an option which gives the buyer the right but not the obligation to exercise the same. However the counterparty which sells the option has to obey the option contract if it turns in-the-money. For entering into an option transaction, the option buyer has to pay certain premium for transferring the risk in the underlying to the option seller. If the option is exchange traded, the risk associated with it is considered free of credit risk, as the probability of default of an exchange is almost zero. In case of options traded over-the-counter (OTC) which are called vulnerable options a considerable amount of credit risk is associated with the option seller. Hence the option buyer can demand collateral to mitigate the credit risk associated with the vulnerable option.

Many works have been presented [4][6][7][8][10]in the literature which focus on pricing of vulnerable European options. Binomial trees [11] [14] can be used to price both European and American style vulnerable options. Although these methods, price vulnerable options for a given collateral amount they do not give an estimate on the optimum collateral amount at which the vulnerable option is as risky as an exchange-traded option. It has been found that [8] the collateral calculated is constant irrespective of strike price and covers almost implausible states that the underlying can take.

In this work, the problem of calculating the optimal collateral amount that makes the vulnerable option as risky as an exchange-traded option is presented. The collateral calculation algorithm overcomes the constant collateral amount for all strikes as in [8] and considers only the plausible states at a given percentile.

The collateral amount calculated using the pricing formula proposed in [8] is compared with the collateral amount calculated based on the algorithm presented in this paper, which prices the vulnerable based on a binomial tree proposed in [11].

The rest of the paper is organised as follows: Section 2 provides the literature survey for pricing vulnerable options. Section 3 discusses the tree building methodology as described in [11]. Section 4 describes pricing of vulnerable

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option and discusses the algorithm to calculate optimum collateral. The convergence of the collateral amount and the sensitivity of the price of vulnerable option are modelled and presented in this paper. A comparative study of the collateral amounts calculated using pricing formula proposed in [8] and with the algorithm discussed in section 4 using the binomial tree proposed in [11] is presented in section 4.3. Section 5 concludes and gives a direction to future work.

2 LITERATURE SURVEY

The breakthrough in pricing of options is the famous Black-Scholes model [1], which prices European options on an underlying following Brownian motion. While considering credit risk, derivatives are viewed as compound options on the assets underlying the financial securities as in the case of Merton [12] [13], Black and Cox [2], Ho and Singer [5], Chance [3] and Kim, Rama Swamy and Sundaresan [9]. Johnson and Stulz [8] proposed the valuation model concerning the issue of counterparty credit risk, assuming that the option itself is the only liability of the option writer, and the default occurs when the option writer's collateral assets cannot afford the promised payment in the option contract. They also introduced the term “vulnerable option” for an option that contains counterparty credit risk. Their approach is an extension of the corporate bond model presented by Merton [12]. Hull and White [7] extended the credit risk model of Johnson and Stulz [8] to bond pricing models related to the first passage time. Jarrow & Turnbull [7] also considered early default in studying the effect of default risk on fixed income and other options. Considering the correlation between the underlying asset price and the assets value of the option writer, they assumed that an option writer defaults on its obligations when an exogenously specified boundary is reached by the assets value of the option writer. Klein [7] further modified Johnson and Stulz [8] assumption by allowing the option writer to have other liabilities of equal priority payment under the option. Klein [10] developed close-form solutions for Black-Scholes options subject to credit risk of the option writer's default event. Cochrane and Saá-Requejo [4] developed a methodology to price vulnerable options with stochastic and deterministic interest rates in an incomplete market. They have proposed a methodology that can price vulnerable options in incomplete markets considering liquidity risk into account. Hui, Lo and Ku [4] developed a valuation model of European options incorporating a stochastic default barrier extending the constant default barrier proposed in the Hull-White model.

Most of the literature is focused on pricing of vulnerable European options under the assumption of lognormal distribution of stock prices.

The methods existing in the literature, price a vulnerable option given a collateral amount and theoretically require infinite collateral amount to make the option credit risk free. Therefore calculation of optimum collateral amount at which the credit risk is zero is a problem of interest.

While using the methodology proposed in [8] for vulnerable option pricing when the collateral is cash, it is observed that the collateral amount is constant irrespective of strike and implausible prices of the underlying are considered for collateral calculation.

KVNM Ramesh et.al [11] proposed a methodology to build a binomial tree that is independent of underlying distribution of the stock prices. Binomial tree distribution shown in [11] to found be more efficient compared to lognormal distribution of stock prices for estimating the stock prices.

In this work, we used the methodology discussed in [11] to generate the binomial tree of stock prices. The tree is used to compute the option prices of both vulnerable and non-vulnerable option. The tree can be used to price both European and American vulnerable options. We propose an algorithm that computes the optimum collateral amount at which the vulnerable option is credit risk free. The collateral calculation algorithm can also be used for both European and American vulnerable options.

Experiments were carried, keeping the risk coverage as constant and observed that the collateral varies with the strike.

The parameters that govern the algorithm are set such that the risk coverage is 99.99%. It has also been shown that the algorithm allowed the option seller to use the excess collateral covering above 99.99% of risk, for other investment purposes.

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3 BINOMIAL TREE BUILDING

The underlying stock process is assumed to follow a binomial process where the magnitudes of the upward and downward movements are governed by the risk neutral pricing principles. While building the binomial tree the stock price at time t is denoted S_t, and the next period's stock price S_{t+h} may take one of two possible values:

u S_t (an up move) w / prob p_u S_{t+h} = (1)
d S_t (a down move) w / prob p_d

where

u- Upward multiplier
d- Downward multiplier
q- Probability of up move
h- Length of time step

This can be represented in figure [1] where the magnitude of up move and down move is also represented.

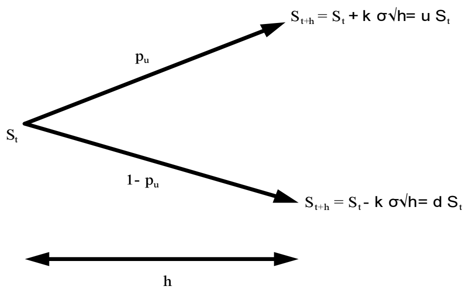

Figure 1: Binomial Tree for one step

Under risk neutral pricing, the price of the underlying should grow at a risk free rate "r". Hence the expected value at maturity should follow equation (2).

$$p_u * u\ S_t + (1- p_u)d\ S_t = S_t * e^{rh} \quad (2)$$

$$p_u = (e^{rh} - d)/(u-d) \quad (3)$$

$$u = 1 + k\ \sigma\sqrt{h}/\ S_t \quad (4)$$

$$d = 1 - k\ \sigma\sqrt{h}/\ S_t \quad (5)$$

Where

k – is the confidence factor used in building the tree
σ- Absolute volatility of the underlying daily returns

In order to make the tree recombining with constant probabilities of upward and downward movements the value of u and d are chosen as shown in equation(6) and (7).

$$u = 1 + k\ \sigma\sqrt{h}/\ S_0 \quad (6)$$

$$d = 1 - k\ \sigma\sqrt{h}/\ S_0 \quad (7)$$

The advantage with recombining trees is that the numbers of computations are decreased compared to non-recombining tree.

4 VULNERABLE OPTION PRICING

The payoff of a vulnerable option is dependent on whether the collateral amount pledged by the option writer can afford to pay the promised payment. Assuming the collateral amount is "M" the payoff of a vulnerable option is given by equation (9) which is diagrammatically represented in figure (2).

Payoff = Minimum (M, Maximum [(S_T – X), 0] for call option (9)

Minimum (M, Maximum [(X- S_T), 0]) for put option

Where

M - Risk free collateral amount at maturity
S_T – Underlying Price at Maturity
X – Strike price of the option

The payoffs at maturity are discounted by backward valuation using the tree built as discussed in section 1. The discounted value at node zero gives the premium of the vulnerable option. For an American style option the probability of early execution is also considered when the underlying price at step s satisfies the condition shown in equation (10).

$$M \leq S_s - X \quad (10)$$

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Where

S_s – Underlying price at step

Step "s" will be a part of the tree. In figure 1 only one step is shown. The same can be extended to "n" number of steps within which "s" is one of the steps. The terminal values at each step are multiplied with "u" and "d" as given in equations (4) and (5) to get the values at the next step.

Algorithm to calculate minimum collateral

Though the vulnerable option is traded over-the-counter (OTC), the credit risk associated with the option can be mitigated by pledging a minimum amount of collateral so that the vulnerable option is as good as the exchange traded option. In this section we describe the algorithm to calculate the minimum collateral amount using the tree built in section 1.

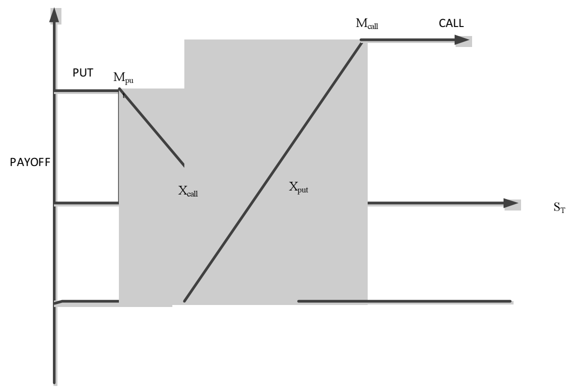

Figure 2 Payoff for a Vulnerable Put and Call options

Algorithm

1. Start
2. The range of M lies between (0, $100*S_0$) where S_0 is the spot price of the underlying stock.
3. Let M_{low} = 0 and M_{high} = $100*S_0$, Δ_{old} = 100000000
4. Let P_e be the value of exchange traded option using the binomial tree of section 1.
5. Let P_v be the value of the vulnerable option with the value of M = $(M_{low} + M_{high})/2$
6. Compute $\Delta = P_v - P_e$
7. If $\left| |\Delta_{old}| - |\Delta| \right| < \mu$ then go to step 16
8. Else
9. $\Delta_{old} = \Delta$; $M_{prev} = M$
10. End if
11. If $\Delta <$ -1 $M_{low} = M$
12. Else
13. $M_{high} = M$
14. End if
15. Go to step 5
16. The value in M_{prev} gives the Minimum collateral required.
17. End

4.1 Sensitivity of the Collateral and Vulnerable Option Price to Model Parameters

The collateral requirement increases with the strike price of the put option and decreases with the strike price of the call option. However for a given option, it can be observed from figure [3] that the price of a vulnerable option converges to the price of the riskless option as the collateral amount increases also the change in Δ with the change in value of collateral (M) after certain value of M is almost constant. From this it can be inferred that if the value of Δ is very small then the amount of collateral required is much higher to cover even the least possible payoffs which is undesirable from the view of option writer.

The inflection point at which the price of the vulnerable option converges almost to the price of the riskless option represent the minimum collateral required to cover the option position for credit risk.

The price of a vulnerable option is compared to a vanilla option which is a normal call or put option that has standardized terms and no special or unusual features and generally traded on an exchange. Figure [4] explains that the price of a vulnerable option need not always increase with maturity as opposed to the price of a plain vanilla option whose price increases with maturity. From figure [5] it can be observed that the price of the vulnerable option may decrease with increase in volatility as opposed to the price of a vanilla option whose price increases with volatility.

4.2 Comparative Analysis

When the vulnerable option is, 100% credit risk free, its price should be same as that of the exchange-traded option.

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Considering a call option the formula to price a vulnerable option as given in [8] is given in equation (11) with the option price calculated using the Black-Scholes model [1].

c(S(t), M, X, T) = c(S(t), X, T) - c(S(t), M, T) (11)

where

c(S(t), M, X, T) is price of vulnerable option
c(S(t), X, T) is price of uncapped option
c(S(t), M, T) is price of option with payoff capped to M
X is the strike price of option
T is the maturity of option
S(t) is the value of the underlying

From the equation (11) the value of collateral M that is required to cover 100% of the risk is equal to the strike price (M_{max}) at which the price of the call option is zero so that the value of the vulnerable option is same as that of exchange traded option.

The disadvantage with this approach is the value of the collateral does not change with the strike price. However as the strike price increases and the probability that the option turns in-the-money decreases.

Therefore, the collateral amount required covering that payoff should be less than M_{max}. To consider this effect the collateral calculation algorithm presented above considers vulnerable call option as a capped call option whose cap is considered as the required collateral. The cap is varied such that the price of the equivalent uncapped option is same as that of the capped option.

Table I shows the value of collateral amount computed using the algorithm given in section 4.

Table II shows the collateral calculations following equation (11). Both the computations are done with underlying price at 5000, one month maturity, 10% risk free rate and 30% volatility.

Table I: Collateral calculated at various strike prices

Strike Price	Collateral By proposed method	Covered Underlying price at maturity By proposed method	Risk Covered by collateral calculated by proposed method
1000	6000	7000	99.9900%
2000	5000	7000	99.9900%
3000	4000	7000	99.9900%
4000	3000	7000	99.9900%
5000	2000	7000	99.9900%
6000	1000	7000	99.9900%
7000	500	7500	99.9995%
8000	100	8100	100.0000%

The strike price at which the value of the call option becomes zero is 7700. From Tables I and II it can be seen that the methodology proposed in II for collateral calculation considers the most implausible events for the underlying prices at maturity and hence reports excess collateral. Also the collaterals for deep-in-the money options calculated using the proposed methodology approaches the collateral calculated following equation (11). Hence the proposed methodology will be of interest to the option sellers as the collateral requirement is not flat and considers plausible events at 99.99% confidence.

Table II: Collateral calculated at various strike prices

Strike Price	Collateral Following equation (11)	Covered Underlying price at maturity Following equation (11)	Risk Covered by collateral calculated Following equation (11)
1000	7700	08700	100%
2000	7700	97000	100%
3000	7700	10700	100%
4000	7700	11700	100%
5000	7700	12700	100%
6000	7700	13700	100%
7000	7700	14750	100%
8000	7700	15810	100%

4.2 Assumptions and Limitations

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The proposed work has the following assumptions and limitations:

1. Markets for traded assets are perfect with no taxes and transaction costs, no limit on short sales with all investors as price takers
2. There exists unlimited borrowing at constant interest rate per unit time
3. The underlying can't take negative values.
4. Markets are complete so that it is possible to construct a portfolio that pays nothing until maturity and maturity value at maturity and a portfolio that pays nothing until maturity and collateral amount at maturity.
5. The collateral is in the form of cash in the same currency as the underlying.
6. It is assumed that there exists an equivalent exchange traded option for the corresponding vulnerable option.

5 CONCLUSIONS AND FUTURE WORK

In this work we observed that the collateral requirement for the vulnerable call option, increase with decrease in strike price. The collateral calculated using the proposed algorithm approaches the collateral calculated using [8] for deep-in-the money call options. As the strike price of the call option increases, the collateral amount decreases for the same risk coverage. The proposed algorithm will be of interest to the option seller as the collateral decreases for strikes around the current underlying price. The work can be extended to vulnerable options with risky collaterals.

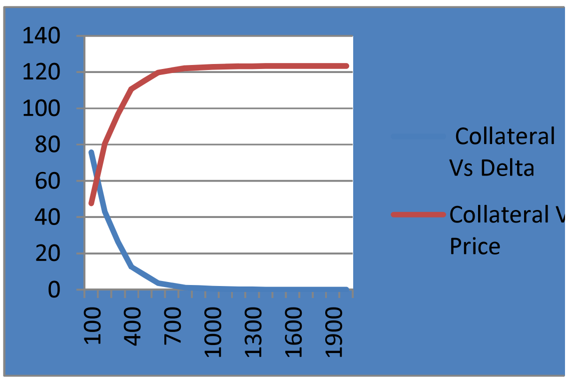

Figure 3 Collateral Vs Difference between Riskless and Vulnerable Option

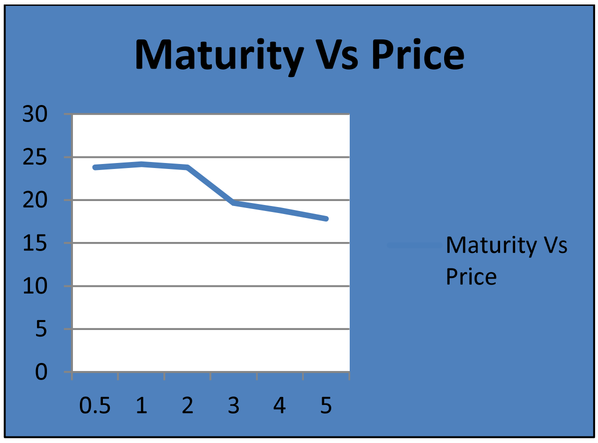

Figure 4 Maturity Vs Price of the vulnerable option

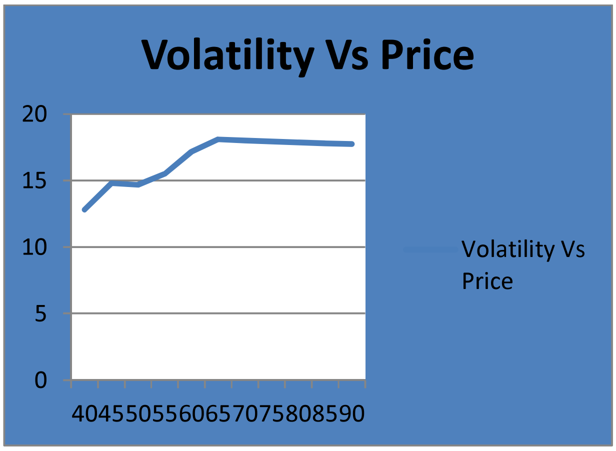

Figure 5 Volatility Vs Price of the vulnerable option

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