

A NEW CLUSTERING BASED SEGMENTATION TECHNIQUE FOR BREAST LESIONS DETECTION

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ABSTRACT

Microcalcifications can be a very important sign of breast cancer. As their detection is very crucial to further investigation, automatic detection in mammograms can help practitioners to locate missed abnormalities. The aim of this work is to propose a simple method based on fuzzy clustering to efficiently segment microcalcifications. This method which is derived from two existing methods, automatically determines the number of classes in each image and then isolates potential microcalcifications. Compared to previous methods, the proposed method was tested on 7 Regions of Interest and demonstrated higher performance reaching up to 0.93 in terms of F1-Score and an overall best performance.

Keywords: *Computer Aided Detection, Machine Learning, Image Analysis, Fuzzy Clustering, Image Segmentation*

1. INTRODUCTION

Breast cancer is one of the deadliest forms of cancer among women around the world and early detection can significantly improve the chances of survival and allows effective treatment. Mammogram screening is still the most reliable mean for breast cancer detection and Computer Aided Detection (CADx) in mammography has tried to provide numerous techniques to improve breast lesions detection at an early stage.

The presence of microcalcifications (mcs) is one of the earliest signs of breast lesions and can be a good indicator of breast cancer. Anyway, their detection cannot only be considered as a standalone goal but also as prerequisite to the breast lesion diagnosis. In fact, any system that is seeking mcs detection has to include a component for mcs segmentation.

This task which consists of dividing an image into a set of homogeneous regions according to a chosen criterion [1] aims in this context to isolate mcs from the rest of breast tissue for further investigations or to highlight suspected abnormalities. Different approaches to Mcs segmentation in digitized mammograms have included various techniques such as Local Thresholding, Region Growing, Markov Random Fields, Wavelets and Fuzzy Thresholding amongst others. Cheng and al.

reported a modified local thresholding technique to segment mcs. The thresholding is based on an expected bimodal intensity distribution in a selected window size that contains the sub-image to be segmented. The level histograms of the surrounding sub-images are smoothed using a median filter and each histogram is classified as being unimodal or bimodal. Once all the sub-images have been processed, each threshold is replaced by a value that is interpolated from the thresholds of the neighboring sub-images [2]. Cheng al. used a variance occurrence to measure heterogeneity within each region of the image. The minimum error thresholding is then applied to determine a proper threshold for the regions with a high heterogeneity [3]. Bankman and al. developed a technique to segment candidate mcs using a region growing algorithm that does not require any parameter selection. The method was compared to the multi-tolerance region growing and to active contours showing similar performances [4]. Yu and al. proposed a method where the suspicious mcs are preserved by thresholding a Wavelet filtered image according to the Means Pixel Value of that image. Markov Random Field features are then extracted from the neighborhood of every suspicious mcs as the primary texture features for mcs recognition and false positive rate reduction [5]. Kabbadj et al.



proposed an approach based on fuzzy enhancement and thresholding where two new fuzzy features called respectively Local Contrast and Weighted Local Differences are used to build a fuzzy inference system that enhances the image giving a higher membership to the pixels that belong to mcs. A global thresholding is adopted to isolate the candidate mcs.

Fuzzy logic is a multi-valued probabilistic theory that generalizes the original set theory [26]. Since it is a better tool for the description and the understanding of the human nature, it was successfully used for the enhancement, segmentation and classification of microcalcification in mammograms [2].

A raw version of fuzzy c-means was used for an intensity based segmentation of mcs in mammograms [25]. Anyway, to our knowledge no other work has utilized fuzzy c-means for the task of mcs segmentation. The present work focuses on the particularities of mcs and tries to adapt fuzzy c-means to their nature and appearance on mammograms. It is based on two complementary techniques. The first one is an automatic number of classes' determination method and the second is a modified version of the Fuzzy C-Means. The first one was used to select the number of classes and hence the number of objects within each region of interest while the second segments each region of interest using that number of objects. This approach does not make any assumption on the nature of the data and does not need any parameter except for the region of interest size which does not have a too much influence since the number of classes is chosen for each region of interest.

The present article is organized as follows: Section II introduces the general framework of the fuzzy c-means and the modified version. The automatic choice of the number of classes is then discussed in Section III. Then, the experimentations as well as the results are addressed in Section IV. The final Section is a general discussion and conclusion.

2. FUZZY C-MEANS, INSENSITIVE TO CLUSTER SIZE WITH LOCAL CONSTRAINTS

2.1 Fuzzy C-means

The fuzzy c-means is one of the most classical clustering algorithms and was originally introduced by Bezdek [7]. The algorithm presents itself as a generalization of k-means method with an objective function to minimize.

$$J_m(U, V) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m \|x_j - v_i\|^2 \quad (1)$$

Where u_{ij} is the membership degree of data point x_j to the cluster with the center or prototype v_i . The parameter m controls the fuzziness of the resulting classification. The original algorithm is subject to the two following constraints:

$$\sum_{i=1}^c u_{ik} = 1 \quad \text{and} \quad 0 < \sum_{k=1}^n u_{ik} < n \quad (2)$$

The previous constraints ensure that each data point belongs to all clusters with a different degree for each one and that there is no empty cluster. Minimization of the objective function must satisfy the two necessary but not sufficient conditions:

$$u_{ik} = \left(\left(\sum_{j=1}^c \frac{\|x_k - v_j\|^2}{\|x_k - v_i\|^2} \right)^{\frac{2}{m-1}} \right)^{-1} \quad (3)$$

$$v_i = \frac{\sum_{k=1}^n u_{ik}^m x_k}{\sum_{k=1}^n u_{ik}^m} \quad (4)$$

In practice, cluster centroids and membership functions are typically initialized with specified or random values, and then in each iteration both are updated according to the previous conditions. The iterative process stops when no significant improvement of the objective function is reached or when the number of iterations exceeds the predefined *Maximum Number of Iterations*.

2.2 Fuzzy c-means with spatial constraints (SC)

In digital images, pixels in a local neighborhood have similar properties especially when they belong to the same object. Furthermore, every pixel that shows different properties compared to his neighbors due to the presence of noise may be wrongly classified. For this reason, fuzzy c-means applied to digital images has to take into account not only the current pixel features but also the features of its neighbors. These reasons led a number of researchers to incorporate certain spatial constraints to the objective function [8, 9 and 10].

Each pixel is influenced by the neighbor's features in addition to its own features for effective image regularization. In order to reach this end, many extensions of the objective functions are calculated. A simple and computationally acceptable version where the pixel is influenced by the mean value of its neighborhood was used as a starting point for the current work [11]:

$$J_m(U, V) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m \|x_j - v_i\|^2 + \alpha \sum_{i=1}^c \sum_{j=1}^k u_{ij}^m \|\bar{x}_j - v_i\|^2 \quad (5)$$



\bar{x}_j is the mean of the neighboring pixels lying in a window around x_j . Unlike other versions this one is relatively faster because the mean image can be computed in advance [12].

2.3 Fuzzy c-means insensitive to cluster size (ICS)

Fuzzy c-means in its standard version tends to balance the number of points in each cluster and smaller clusters are drawn to larger clusters [13]. Indeed, in each centroid update, according to equation (4), a bigger influence is attributed to clusters with a relatively bigger size. This results in an inaccurate centroids positioning and a consequent inefficient segmentation. In the context of mcs segmentation, each region of interest includes a larger number of pixels that belong to breast tissue than pixels that belong to the background. Thus, fuzzy c-means has to be independent to cluster size. The particularity of ICS is to replace the constraint $\sum_{i=1}^c u_{ik} = 1$ with an adaptive constraint $\sum_{i=1}^c u_{ik} = f_{ki}$ which is dependent to the cluster size during each iteration of the process. The coefficient f_{ki} is updated in each iteration using the following formula:

$$f_{ki} = \frac{1}{1 - P_{imin}} (1 - P_i) \quad (6)$$

P_{imin} is the probability of the cluster with lesser number of members and P_i is the probability of the cluster to which x_k belongs to. The formula for the membership calculation in equation 3 is replaced by a more generic one:

$$u_{ik} = \frac{f_k}{\left(\sum_{j=1}^c \frac{\|x_k - v_j\|^2}{\|x_k - v_j\|} \right)^{\frac{2}{m-1}}} \quad (7)$$

2.4 Combining Local Information And Insensitivity To Cluster Size

Each one of the two cited modifications to fuzzy c-means is designed for a particular situation. While SC is designed to cancel the effect of noise which tends to corrupt pixel values, ICS gives similar performances in each clustering process, independently on the cluster size. Mammography presents often an important amount of noise due to the radiation source and to the choice of the filters that are used [14], which induces a lack of details and corrupted information. To compensate for this, segmentation has to make a decision for each pixel based on its characteristics as well as those of its own neighbors. Furthermore, in each region of interest which contains suspicious mcs, the tissue pixels outnumber the abnormality pixels. Thus, the membership update formula in equation (7) is a

good alternative to that in equation (3) and ICS and SC are merged together to form a stronger algorithm as described below.

The SCICS ALGORITHM :

1. **Fix c: The number of clusters, t_{max}**
The maximum number of iterations,
 $m > 1$: The exponent, $\epsilon > 0$: The Minimum improvement from iteration to iteration and α the neighborhood impact
2. **Initialize the membership function u_{ik} and cluster prototypes c_i**
3. **For $t = 1, 2, \dots, t_{max}$**
 - a) **Update the cluster prototypes with the following formula :**

$$v_i = \frac{\sum_{k=1}^N u_{ik}^m (x_k + \alpha \bar{x}_k)}{(1 + \alpha) \sum_{k=1}^N u_{ik}^m} \quad (8)$$
 - b) **Update the membership function :**

$$u_{ik} = \frac{f_k}{\sum_{j=1}^c \left(\frac{\|x_k - v_i\|^2 + \alpha \|\bar{x}_k - v_i\|^2}{\|x_k - v_j\|^2 + \alpha \|\bar{x}_k - v_j\|^2} \right)^{\frac{1}{m-1}}} \quad (9)$$
 - c) **Compute $J_m(U, V)$ in equation (5)**
 - d) **If the improvement of $J_m(U, V) < \epsilon$ break;**

End For

The formulas for the cluster prototypes and the membership function update in equations (8) and (9) were obtained using Lagrange Multipliers [15]. The initialization of the membership function and cluster prototypes can be made randomly or using initialization method such as subtractive clustering [16]. But since fuzzy c-means algorithms are sensitive to initialization [17], the latter method is more recommended to avoid an entrapment in local minimums.

3. CHOOSING A PROPER NUMBER OF CLASSES

Since Fuzzy c-means needs to determine the number of classes in advance; it is crucial to feed the algorithm with an accurate number of classes. Indeed, we are concerned with accurately segmenting each region of interest (ROI) by distinguishing mcs from the remaining breast tissue. So first, we have to find the number of classes which are present in the current ROI and, depending on it, we decide if the ROI contains eventual mcs. The second step is to use it as a



parameter for our segmentation algorithm. The segmentation will produce accurate objects provided that the number of classes given by the user is correct. So since we are building an automatic segmentation, the method must satisfy two conditions:

1. Being able to determine if mcs are present in the current ROI
2. Being able to segment the ROI with the proper number of classes if mcs are present

We have chosen Fuzzy c-means for its proven robustness and simplicity and we have intentionally modified its objective function for an efficient use in the context of mcs segmentation to fulfill the second condition as described in the previous section. To achieve the second goal (determining the number of classes), different indexes that tell if a distribution is bimodal such as *Ashman's D*, *The Bimodality Index and the Bimodality Coefficient* [18, 19, 20] could be used. However; to our knowledge, no measure exists to determine the number of classes present within an image. Among the few approaches that have been developed to determine the number of classes, which is presented by authors in [21] basically uses the *k-means* as a support. Only the minimum and the maximum number of classes: min_c and max_c respectively need to be known. The method performs then an initial partition and computes the following measures:

$$validity = \frac{intra}{inter} \quad (10)$$

$$where \quad intra = \frac{1}{N} \sum_{i=1}^k \sum_{x \in C_i} \|x - C_i\|^2 \quad (11)$$

$$And \quad inter = \min(\|Z_i - Z_j\|^2) \quad (12)$$

$$for \quad i = 1, 2, \dots, K - 1;$$

$$and \quad j = i + 1, \dots, K$$

K and z_i are respectively the number of classes and the center of class C_i . The validity index is stored and the procedure is repeated by splitting the cluster with maximum intra-class variance into clusters resulting in an additional cluster and then computing and storing again the same index. The algorithm repeats the same process (splitting the cluster with the biggest intra-class variance and

computing the validity index) until max_c is reached. The number of classes that minimizes (10) which is considered as the ideal number of classes is the output of the method. We used the previous method to detect the presence of mcs and automatically segment the ROI for its simplicity and independence of any parameter.

4. EXPERIMENTATION AND RESULTS

4.1 Experimentation Images

To evaluate the described method for mcs segmentation, we used the MIAS database [22]. The version that was used for the purpose of this work consists of 322 cases. The image resolution was originally of 50 microns/ pixel and have been re-digitized to have a resolution of 200 microns/ pixel. We tested our method using seven cases with visible mcs. To further capture the strengths and weaknesses of the present work, only cropped regions of interest that contain mcs were used. Indeed, the MIAS includes a description file that shows for each image, the center coordinates of eventual anomalies, their type and the approximate radius of the circle that encloses them. This description file was used to effectively evaluate the present work.

4.2 Evaluation And Results

Segmentation traditionally incorporates two main approaches: a dividing approach and a discriminative approach [23]. The first approach tries to find method that best separates an image into a set of regions independently on its semantic content. The second approach differs from the first for its search for the generative model i.e. the model that is responsible for the image generation. It is clear that the second approach is application dependent which is the case for the present work. And for this kind of applications, various evaluation methods exist [23]. All these evaluation methods require a manual segmentation to compare an automatic segmentation to reference segmentation (manual) quantitatively. But, since only an approximate description of the anomalies is available, such methods are not applicable for this work. So in order to choose an alternative evaluation method, we can reformulate our needs in terms of questions that would be answered by the evaluation methods.

Indeed, since the rationale behind a CAD is the detection of mcs, and that this latter is perfectly fulfilled if this system can detect all the existing mcs without making mistakes, some questions arise

naturally to know the extent to which this goal is achieved and these questions are respectively:

-Among all the existing mcs, how many have been successfully detected?

-Among all the lesions that were labeled as mcs, how many are truly mcs?

The F1-score which is a way to answer these two questions either separately and simultaneously is defined as:

$$F1 = \frac{2P.R}{P + R} \quad (13)$$

where

$$P = Precision = \frac{TP}{TP + FP} \quad (14)$$

and

$$R = Recall = \frac{TP}{TP + FN} \quad (15)$$

TP, FP and FN are respectively the number of True Positive cases, False Positive cases and False Negative cases. The P and R measures answer respectively the questions above and both can be combined to provide a unique measure (13).

The method begins by looking for the right number of classes which can vary from an image to another. After being found, it is used as an argument for the SPC, ICS and SPCICS algorithms. The three methods end while providing for each pixel the class that maximizes each membership function. We select then the class with the maximal gray level since Mcs are generally brighter than the other tissue. The Mias description file which contains the center coordinates and the radius of each abnormality is used to evaluate the three methods. For this work we have used 7 Regions of Interest which contain all Mcs. Table 1 shows the evaluation in terms of sensitivity specificity and F1-Score of the 7 ROIs using parameters $m=2$ and $\alpha=1$ for this part. In terms of precision, the SC method outperforms ICS and SCICS which is perfectly normal since SC cancels the effects of impulsive noise and thus makes less false positive cases.

However, in terms of Recall and F1-Score, our method is superior in most of the cases resulting in

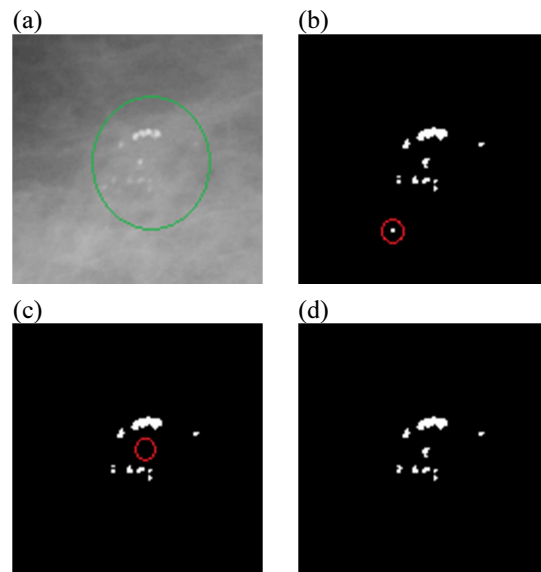


Figure 1: Results of the three methods on Roi1. (a) Original Image, (b): Results of ICS, (c): Results of SC and (d): Results of SCICS

less false negatives while keeping a good balance between precision and recall. These results demonstrate the efficiency of our method in Mcs segmentation since it uses the strong points of SC (reduction of false positives) and ICS (Insensitivity to cluster size) given that Mcs take only a little proportion of a Region Of Interest when they are present in addition to the aspect of impulsive noise is similar to their own appearance.

Figure 1 and Figure 8 show the results of the three methods on 2 images. The Mcs locations are encircled in green in (a). The red circles in the segmented images show either the regions that were wrongly considered as Mcs or the Mcs that were undetected by the segmentation method.

It is clear from Table 1 as well as from the two figures that SCICS outperforms the two other methods since it takes advantage from their respective strengths. Indeed, SCICS is at the same time insensitive to the cluster size while making less mistakes than ICS method. Its hybrid combination of SC and ICS makes it a suitable and efficient tool in the task of Mcs segmentation.

	Precision			Recall			F1		
	ICS	SC	SCICS	ICS	SC	SCICS	ICS	SC	SCICS
ROI1	0.72	0.85	0.80	0.60	0.80	0.80	0.65	0.82	0.80
ROI2	0.61	0.70	0.66	0.88	0.77	0.88	0.72	0.73	0.76
ROI3	0.86	0.95	0.91	1	0.80	0.88	0.92	0.86	0.89
ROI4	0.57	0.75	0.80	1	0.75	1	0.72	0.75	0.88
ROI5	0.65	0.86	0.87	0.76	0.76	0.82	0.70	0.81	0.84
ROI6	0.50	0.75	1	1	1	0.75	0.66	0.85	0.85
ROI7	0.80	0.90	0.90	0.95	0.90	0.95	0.86	0.90	0.93
Average	0.67	0.82	0.84	0.88	0.82	0.86	0.74	0.81	0.85

Table1: Comparative results of SC, ICS, and SCICS for $m=2$ and $\alpha=1$

Influence of parameter α

As a preliminary evaluation, the three methods were tested on the 7 ROIs using parameter $\alpha = 1$ which correspond to according to the pixel the same importance as its neighborhood in the optimization function. However, α can take an infinite range of values which can have an influence on the performances. In this paragraph, we study the influence of α on the precision, recall and F1. Figures 2, 3 and 4 show respectively precision, recall and f1 in terms of α for SC and SCICS algorithms. Precision is clearly influenced by α as we can see on figure. SC and SCICS have both medium values of precision when α is closed to 0. They reach their maximal values when α is around 1 and then drop continuously. The first figure shows that the best values of α is around 1 for both algorithms. As of the influence on recall, the curves aspect is pretty similar to the previous one except that for α between 0 and 1 recall is not as much influenced by the parameter as the precision which is probably due to the fact that small values of the parameter have a stronger impact on false positives than on false negatives while high values influence them both. finally, the f1 curve have approximately the same characteristics as that of precision and recall. It reaches its maximum for $\alpha = 0.8$. From the three figures we can note that the parameter have a very significant importance on the performances and that the best choice is to take it between 0.8 and 1.2.

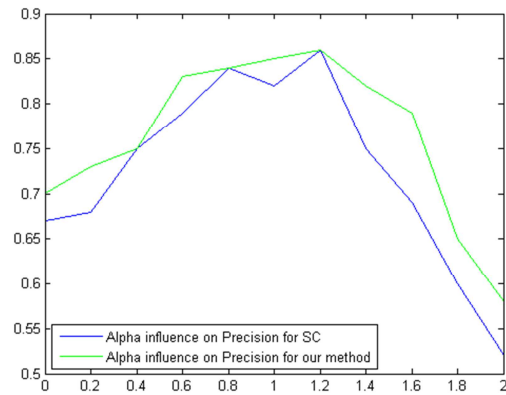


Figure 2 : The alpha influence on precision

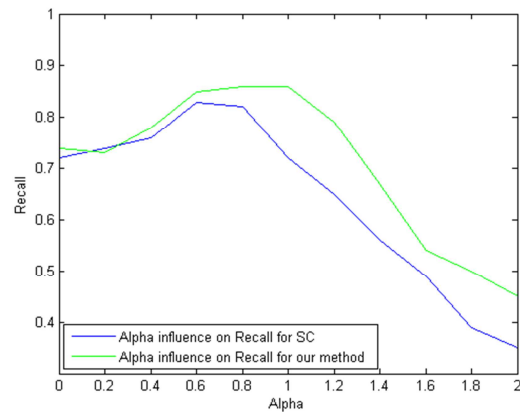


Figure 3 : The alpha influence on recall

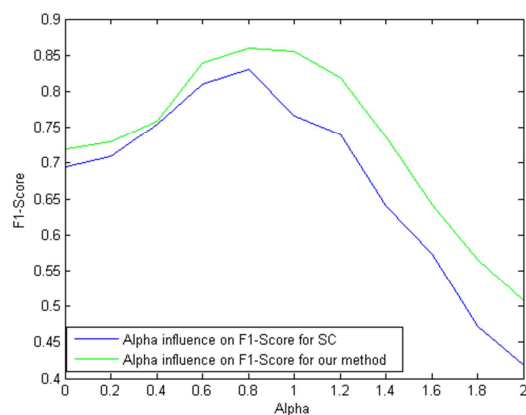


Figure 4 : The alpha influence on f1

Influence of the exponent m

The parameter m determines the degree of fuzziness of fuzzy clustering algorithms which influences the membership function i.e. that low values (close to 1) lead to crisp clusters and high values make the membership function equal for all the data-points. The choice of a certain value might also influence the performances of the fuzzy clustering algorithms. While the inventor of fuzzy c-means suggests to take the exponent between 1.5 and 2.5 [27], a more recent study has concluded to take this value between 1.5 and 4 [28]. We have chosen to take it between 1.5 and 4.5 which is conform to both studies. On the entire interval and for (SC, ICS, SCICS) especially when $m > 3.5$, precision and F1 show a constant increase which is more marked for ICS. As for recall, the three algorithms exhibit a constancy to a small decrease when $m > 3.5$. From the figures 5,6,7 we can deduce that as suggested by authors in [28], parameter m should be chosen in the interval [1.5,4] and precisely near 3.5 as it is the best trade-off between a good precision and recall.

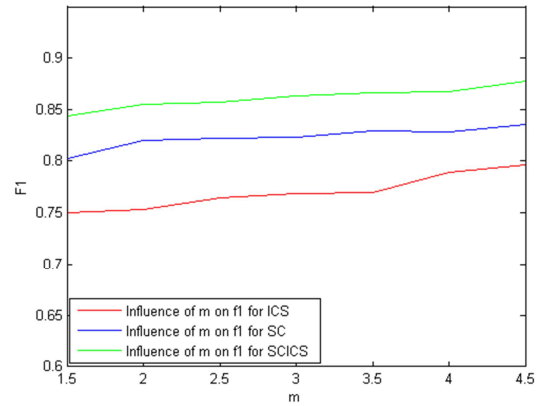


Figure 7 : The exponent influence on f1

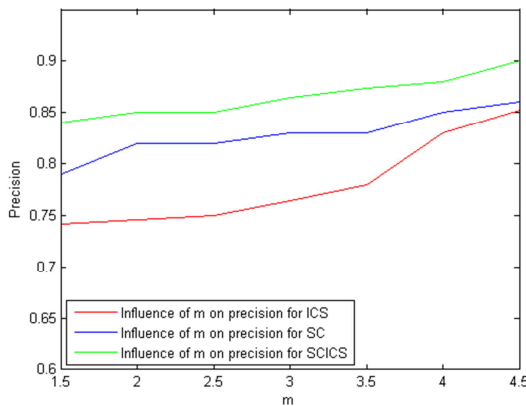


Figure 5 : The exponent influence on precision

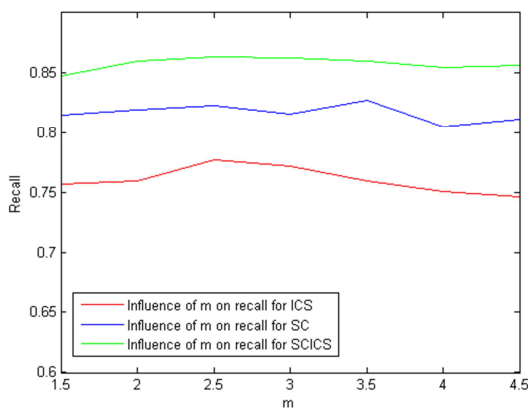


Figure 6 : The exponent influence on recall

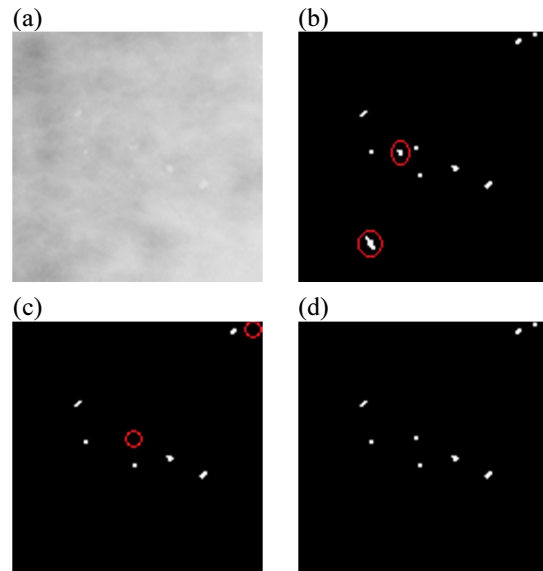


Figure 8: Results of the three methods on Roi7. (a) Original Image, (b): Results of ICS, (c): Results of SC and (d): Results of SCICS

6. CONCLUSION

Microcalcifications can be an indicator of breast cancer and their early detection can be a very crucial to efficient treatment. We proposed in this article a simple and efficient approach which combines two methods: Fuzzy C Means with spatial constraints and Fuzzy C-Means with insensitivity to cluster size. While the first one is effective when dealing with noise, the second is robust for clusters with different sizes. Our method is based on Lagrange Multipliers and a new formulation of the optimization function

for best microcalcifications segmentation especially those of very small sizes. The number of classes is predetermined using an automatic method and the class with the highest mean gray level from each image is considered as potential mcs.

Performance of the method was assessed on the basis of three measures : Precision, Recall and F1-Score. Compared to the ICS and SC methods, the proposed method improves efficiency particularly in terms of Recall and f1-score which attains in average the values 0.84 and 0.85 respectively. We showed in this work that the performance of the method depends on both the objective function exponent and the contribution of the neighboring pixels. In this work, we used for the exponent m the value 2 commonly used by authors, nevertheless we found that the value that provides the best trade-off between precision and recall is 3.5. As for the contribution factor of the neighboring pixels, best performance were obtained for a factor between 0.8 to 1.

Among the methods that we presented on this paper, the proposed method gives the best results in terms of precision, recall and f1-score.

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