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ADAPTIVE BACKSTEPPING SLIDING MODE CONTROL OF SPACECRAFT ATTITUDE TRACKING

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ABSTRACT

This paper investigates the attitude tracking problem for rigid spacecraft. An adaptive backstepping sliding mode control law is proposed to solve this problem in the presence of inertia uncertainties and external disturbances. The control scheme is developed by combining sliding mode control with adaptive backstepping technique to achieve fast and accurate tracking responses. The proposed control law provides complete compensation of uncertainty and disturbances. Although it assumes that the uncertainty and disturbances are bounded, this control law does not require information about the bounds on the uncertainties and disturbances. The asymptotic convergence of attitude tracking is ensured by Lyapunov approach. Numerical simulations on attitude tracking control of spacecraft are provided to demonstrate the performance of the proposed controller.

Keywords: *Attitude Tracking Control, Sliding Mode Control, Backstepping Design, Adaptive Control*

1. INTRODUCTION

In recent years the attitude control of rigid spacecraft which has already attracted a great deal of attention. It is an important and practical problem due to its many different types of space missions such as spacecrafts, satellites maneuvering, satellite surveillance, spacecraft formation flying, and space station docking and installation. Since the attitude dynamics of spacecraft is coupled and highly nonlinear, the attitude controller designs are usually difficult. In practical situations, the model parameters of the spacecraft may not be exactly known and the spacecraft is always subject to external disturbances. Thus, the attitude control problem with external disturbance has become a very challenge and interesting problem.

A large variety of nonlinear control sechemes [13] have been proposed for solving the attitude tracking control problem. These control schemes include adaptive attitude control [4,5] sliding mode control [6,7], output feedback control [8,9], LMI-based control [10], passivity-based control [11,12], and fuzzy control [13]. Among these methods, sliding mode control (SMC) has been shown to be a potentially approach when applied to a system subject to disturbances which satisfy the matched uncertainty condition [14]. SMC can offer many good properties, such as insensitivity to model uncertainty, disturbance rejection and fast dynamics responses. Robust attitude controllers based on the SMC scheme have been proposed in [15,16]. These control laws can achieve global asymptotic stability and provide good tracking resultsThe terminal sliding mode (TSM) method [17,18] can be used to design a robust controller that will guarantee a finite time convergence to the origin. This makes it a welcome method for attitude controller designs (see e.g., [19,20]).

Since the bounds on the uncertainty and disturbance of a spacecraft are often unknown, it is important to develop controllers that do not require this information. The adaptive sliding mode control (ASMC) method has been developed to solve this problem. In the ASMC method an adaptive scheme is combined with the SMC design. For example, an ASMC strategy has been proposed for the attitude stabilization of a rigid spacecraft, where the lumped uncertainty is assumed to be bounded by a linear function of the norms of angular velocity and quaternion (see e.g., [20,21]). On the other hand, backstepping design techniques have been widely applied to control nonlinear systems (see [22] and reference therein). The main concept in backstepping technique (BT) is the use of a step-by-step recursive process to stabilize the state of the system. Once the final step is completed, the

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stability of the entire systems is ensured. However, the conventional backstepping design assumes that the lumped uncertainty in the system is constant or slowly changing. When, the derivatives of the model uncertainty and disturbance can not be regarded as zero, the backstepping design with integral adaptive laws is no longer useful. Control strategies which combine BT with SMC have been developed by [23].

In this paper a new attitude control algorithm is developed. The finite-time stability of the algorithm is analyzed using Lyapunov concepts and differential inequalities with backstepping method. The proposed adaptive backstepping sliding mode control (ABSMC) strategy ensures that asymptotic convergence of the errors is achieved.

The paper is organized as follows. In Section 2, the dynamic and kinematic equations that govern the attitude model [24, 25]. Section 3 states the control objective of the proposed control design. Section 4 proposes an adaptive backstepping sliding mode control (ABSMC) algorithm for a rigid spacecraft. The sliding manifold is chosen and the sliding control law is studied and a proof of finite time convergence is given by using the Lyapunov stability theory. In section 5, numerical simulations on attitude tracking control is presented to show the usefulness of the proposed controller. In Section 6, we present conclusions.

2. NONLINEAR MODEL OF SPACECRAFT AND PROBLEM FORMULATION

2.1 Kinematics of the Attitude Error

We now briefly explain the use of quaternions for description of the attitude error. We define the quaternion $Q=\left[q^T q_4\right]^T \in \Re^3\times\Re$ with $q=\left[q_1 \quad q_2 \quad q_3\right]^T \in \Re^3$ and

$$Q_d=\left[q_d^T \ q_{4d}\right]^T,$$

where $q_d=\left[q_{1d}\ q_{2d}\ q_{3d}\right]^T \in \Re^3$ is the desired reference attitude. The quaternion for the attitude error is $Q_e=\left[q_e^T \ q_{4e}\right]^T \in \Re^3\times\Re$ with $q_e=\left[q_{1e}q_{2e}q_{3e}\right]^T \in \Re^3$. Using the multiplication law for quaternions, we then obtain

$$Q_e=\begin{bmatrix} q_{4d}q-q_4q_d-q_r^{\times}q \\ q_4q_{4d}+q^Tq_d \end{bmatrix} \tag{1}$$

subject to the constraint

$$Q_e^TQ_e=\left(q^Tq+q_4^2\right)\left(q_d^Tq_d+q_{4d}^2\right)=1. \tag{2}$$

Remark 1 a quaternion consists of the scalar q_4 and the three - dimensional vector q, so it has four components. The scalar term is used for avoidance of singular points in the attitude representation [26]. The scalar q_4 can be calculated easily using the vector q and the condition $\|Q\|=1$. For more details of quaternion and other attitude representation please see [24, 26].

The kinematic equation for the attitude error can be written as [25]

$$\dot{Q}_e=\frac{1}{2}\begin{bmatrix} T(Q_e) \\ -q_e^T \end{bmatrix}\omega_e, \tag{3}$$

where I_3 is the 3×3 identity matrix and $T(Q_e)=q_e^{\times}+q_{4e}I_3$.

To avoid the singularity of $T(Q_e)$ that will occur at $q_{4e}=0$, we let the attitude of the spacecraft be restricted to the workspace W defined by [27]

$$W=\Big\{Q_e \mid Q_e=\left[q_e^T \ q_{4e}\right]^T, \|q_e\|\le\beta<1, \\ q_{4e}\ge\sqrt{1-\beta^2}>0\Big\}, \tag{4}$$

where β is a positive constant.

2.2 Dynamic Equations of the Error Rate

In [24], the dynamic equation for a rigid spacecraft rotating under the influence of body-fixed devices is given as

$$J\dot{\omega}=-\omega^{\times}J\omega+u+d, \tag{5}$$

where $\omega=\left[\omega_1 \quad \omega_2 \quad \omega_3\right]^T$ is the angular rate of the spacecraft, $u=\left[u_1 \quad u_2 \quad u_3\right]^T$ represents the control vector, $d=\left[d_1 \quad d_2 \quad d_3\right]^T$ are bounded disturbances, J is the inertia matrix, and the skew-symmetric matrix $\omega^{\times}$ is defined by

$$\omega^{\times}=\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{6}$$

We denote $\omega_d=\left[\omega_{1d} \quad \omega_{2d} \quad \omega_{3d}\right]^T$ as the desired reference rate and $\omega_e=\omega-C\omega_d$ with $C=\left(q_{4e}^2-2q_e^Tq_e\right)I_3+2q_eq_e^T-2q_{4e}q_e^{\times}$ as the error rate. As a result, the dynamic equation of the error rate can be obtained in form [25]

$$J\dot{\omega}_e=-\left(\omega_e+C\omega_d\right)^{\times}J\left(\omega_e+C\omega_d\right)+u+d$$

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$$+J\left(\omega_e^{\times}C\omega_d + C\dot{\omega}_d\right). \quad (7)$$

In this paper, $q_d, \dot{q}_d, \ddot{q}_d$ and ω_d are assumed to be bounded, and the objective is to design control laws which force the states of the closed-loop systems (3) and (7) to converge to a desired region contatining the origin in finite time.

2.3. Lemmas

Before giving the controller design, the following lemmas and assumptions are required.

Lemma 1 (Du et al. [28]) If $p \in (0,1)$, then the following inequality holds

$$\sum_{i=1}^{3}|x_i|^{1+p} \geq \left(\sum_{i=1}^{3}|x_i|^2\right)^{\frac{1+p}{2}} \quad (8)$$

Lemma 2 (Du et al. [28]) Suppose $V(x)$ is a smooth positive definite function (defined on $U \subset \Re^n$) and $\dot{V}(x) + cV^{\iota}(x)$ is negative semi-definite on $U \subset \Re^n$ for $\iota \in (0,1)$ and $c \in \Re^+$, then there exists an area $U_0 \subset \Re^n$ such that any $V(x)$ which starts from $U_0 \subset \Re^n$ can reach $V(x) \equiv 0$, in finite time. Moreover, if T_r is the time needed to reach $V(x) \equiv 0$ then

$$T_r \leq \frac{V^{1-\iota}(x_0)}{c(1-\iota)}, \quad (9)$$

when $V(x_0)$ is the initial value of $V(x)$.

Lemma 3 (Yu et al. [18]) For any numbers $\lambda_1 > 0,\ \lambda_2 > 0,\ 0 < \varpi < 1$, an extended Lyapunov condition of finite-time stability can be given in the form of fast terminal sliding mode as

$$\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^{\varpi}(x) \leq 0,$$

where the settling time can be estimated by

$$T_r \leq \frac{1}{\lambda_1(1-\varpi)}\ln\left(\frac{\lambda_1 V^{1-\varpi}(x_0) + \lambda_2}{\lambda_2}\right). \quad (10)$$

Lemma 4 If $P := q_{4e}I_3 + q_e^{\times}$ then $\dot{P}$ satisfies the following equation

$$\dot{P}\omega_e = -\frac{1}{2}q_e\omega_e^T\omega_e. \quad (11)$$

Proof. From P we have $\dot{P} = \dot{q}_{4e}I_3 + \dot{q}_e^{\times}$ and then

$$\dot{P}\omega_e = \left(\dot{q}_{4e}I_3 + \dot{q}_e^{\times}\right)\omega_e. \quad (12)$$

Using $\dot{q}_e = \frac{1}{2}P\omega_e$ and substituting (1) into (12), one has

$$\dot{P}\omega_e = \left(-\frac{1}{2}q_e^T\omega_e\right)\omega_e + \left(\frac{1}{2}P\omega_e\right)^{\times}\omega_e. \quad (13)$$

We know that $q_e^T\omega_e = q_e\omega_e^T$ and $\omega_e^{\times}\omega_e = 0$. Hence, $\dot{P}\omega_e = -\frac{1}{2}q_e\omega_e^T\omega_e$.

To prove the asymptotic convergence of the proposed controller, the well-known Barbalat's lemma (see e.g., [29]) is restated by Lemma 5.

Lemma 5 If a continuous differentiable dual function $V : \Re^n \times [0,\infty) \to \Re$ has lower bound $\dot{V}(x,t) \leq 0$, and $\dot{V}(x,t)$ is uniformly continuous on $[0,\infty)$, then $\lim_{t\to\infty}\dot{V}(x,t) = 0$.

Assumption 1 We assume that the inertia matrix in (7) is of the form $J = J_0 + \Delta J$ where J_0 is a known nonsingular constant matrix and ΔJ denotes the uncertainties and satisfies $\|J\| \leq \lambda_J$, where λ_J is the upper bound on the norm of the inertia matrix.

Based on Lemma 4, the time derivative of $\dot{q}_e$ along the system trajectory (3) satisfies

$$\begin{aligned}\ddot{q}_e &= \frac{1}{2}\dot{P}\omega_e + \frac{1}{2}P\dot{\omega}_e \\ &= -\frac{1}{4}q_e\omega_e^T\omega_e - \frac{1}{2}PJ_0^{-1}\omega^{\times}J_0\omega + \frac{1}{2}P\left(\omega_e^{\times}C\omega_d - C\dot{\omega}_d\right) \\ &\quad + \frac{1}{2}PJ_0^{-1}u + \tilde{d},\end{aligned} \quad (14)$$

where $\tilde{d} = \Delta g + \bar{d}$ are the total uncertainties containing inertia uncertainties and external disturbances, and where

$\Delta g = \frac{1}{2}PJ_0^{-1}\left(-\omega^{\times}\Delta J\omega - \Delta J\dot{\omega}\right)$ and $\bar{d} = \frac{1}{2}PJ_0^{-1}d$.

Hence, we can obtain

$$\ddot{q}_e = f + B_0 u + \tilde{d}, \quad (15)$$

where

$$f = -\frac{1}{4}q_e\omega_e^T\omega_e - \frac{1}{2}PJ_0^{-1}\omega^{\times}J_0\omega + \frac{1}{2}P\left(\omega_e^{\times}C\omega_d - C\dot{\omega}_d\right)$$

and $B_0 = \frac{1}{2}PJ_0^{-1}$. The dynamic model (15) will be used in the subsequent control design and stability analysis.

Assumption 2 The desired system states q_d, $\dot{q}_d$, $\ddot{q}_d$ and ω_d are assumed to be bounded.

Assumption 3 The time derivative of the ith

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component of the total disturbance vector is assumed to be bounded, i.e., $\left|\dot{\tilde{d}}_i\right| \le K_i^*$, $i = 1,2,3$, where K_i^*, $i = 1,2,3$ are positive constants. The term $\hat{K}_i$, $i = 1,2,3$ can then be adapted to estimate $\left|\dot{\tilde{d}}_i\right|$ and has an upper bound $K_i^*\left(i.e.\left|\hat{K}_i\right| \le K_i^*\right)$.

3. THE CONTROL OBJECTIVE

In this paper the objective is to design a control law which forces the states of closed-loop systems (1) and (2) to asymptotically converge the desired attitude motion q_d and desired angular velocity ω_d . This can be expressed as

$$\lim_{t\to\infty} e(t) = 0, \quad \text{and} \quad \lim_{t\to\infty} \omega_e(t) = 0, \tag{16}$$

where $\omega_e = \omega - \dot{\omega}_d$ is the angular velocity tracking error vector.

4. ABSMC

The adaptive backstepping method (ABT) used is a recursive Lyapunov-based scheme in which any useful nonlinearities can be introduced to improve the transient performance. SMC is a potential robust control approach for dealing with nonlinear systems with uncertainties. In this section, the SMC and ABT methods are combined to develop a control law that can achieve accurate tracking.

As an example of the application of the new method, we consider set-point control of robot systems. Let $x_1 = q_e$ and $x_2 = \dot{q}_e$, respectively. The expression (15) can then be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + B_0 u + \tilde{d}. \end{aligned} \tag{17}$$

Let x_2 be a virtual input defined by

$$x_2 = -K_1 sign^{\alpha}(x_1) - \rho_1 x_1 = \phi(x_1), \tag{18}$$

where $K_1 = diag(k_{11}, k_{12}, k_{13})$ and $\rho_1 = diag(\rho_{11}, \rho_{12}, \rho_{13})$ are positive diagonal matrices, $0 < \alpha < 1$ and $\phi(x_1) \in R^n$ with $\phi(0) = 0$. The function $sign^{\gamma}(y)$ is defined as

$$sign^{\gamma}(y) = \left[|y_1|^{\gamma} sign(y_1) \quad \dots \quad ,|y_m|^{\gamma} sign(y_m)\right]^T.$$

With $0 < \gamma < 1$ and $y \in R^m$.

Next, let $z = x_2 - \phi(x_1)$, Then the model (17) can be written as

$$\begin{aligned} \dot{x}_1 &= \phi(x_1) + z \\ \dot{z} &= f + B_0 u + \tilde{d} - \dot{\phi}(x_1) \end{aligned} \tag{19}$$

It is obvious that the derivative of $\phi(x_1)$ will be infinite at $x_{1i} = 0$ and $\dot{x}_{1i} = 0$. To avoid this problem, the term $\dot{\phi}(x_1)$ is approximated as

$$\dot{\phi}(x_{1i}) = -k_{1i}\psi(x_{1i}) - \rho_{1i}x_{2i}, \ i = 1,2,3 \tag{20}$$

with

$$\psi(x_{1i}) = \begin{cases} \alpha|x_{1i}|^{\alpha-1}\dot{x}_{1i}, \ x_{1i} \ne 0 \ and \ \dot{x}_{1i} \ne 0 \\ \alpha|\Delta_i|^{\alpha-1}\dot{x}_{1i}, \ x_{1i} = 0 \ and \ \dot{x}_{1i} \ne 0 \\ 0, \ \dot{x}_{1i} = 0, \end{cases} \tag{21}$$

where x_{1i} is the ith element of vector $x_{1i}, \dot{\phi}(x_{1i})$ is the ith element of vector $\dot{\phi}(x_1)$ and Δ_i is a small positive constant, $i = 1,2,3$. The proposed controller is designed as

$$\begin{aligned} u = B_0^{-1}\big(&-f - x_1 - K_2 sign^{\alpha}(z) - \rho_2 z \\ &-\hat{K}sign(z) + \dot{\phi}(x_1)\big) \end{aligned} \tag{22}$$

where

$K_2 = diag(k_{21}, k_{22}, k_{23})$, $\rho_2 = diag(\rho_{21}, \rho_{22}, \rho_{23})$.

Here, the adaptive law $\hat{K}_i$ is updated by

$$\dot{\hat{K}}_i = \eta|z_i|, \ i = 1,2,3, \tag{23}$$

where η is a positive scalar.

Let a sliding surface be defined as

$$s = z = x_2 - \phi(x_1). \tag{24}$$

Substituting (22) into (19), one obtains the scalar form as

$$\begin{aligned} \dot{x}_{1i} &= -K_{1i}|x_{1i}|^{\alpha} sign(x_{1i}) - \rho_{1i}x_{1i} + s_i \\ \dot{s}_i &= -K_{2i}|s_i|^{\alpha} sign(s_i) - x_{1i} - \rho_{2i}s_i \\ &\quad + \tilde{d}_i - \hat{K}sign(z)_i, \ i = 1,2,3. \end{aligned} \tag{25}$$

Theorem 1 Under Assumptions 1- 3, the action of the control law u (22), the state trajectories x_1 and the sliding variable s of the system (25) globally asymptotically converge to the origin.

Proof. The proof proceeds using the backstepping method [29] and terminal sliding mode control [18].

Consider the differential equation

$$\dot{x}_{1i} = -K_{1i}|x_{1i}|^{\alpha} sign(x_{1i}) - \rho_{1i}x_{1i}. \tag{26}$$

Consider the Lyapunov function

$$V_1 = \frac{1}{2}x_{1i}^2 \tag{27}$$

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Differentiating V_1 with respect to time yields

$$\dot{V}_1 = -x_{1i}\left(K_{1i}\left|x_{1i}\right|^{\alpha} sign\left(x_{1i}\right)\right) - x_{1i}\rho_{1i}x_{1i}$$
$$= -K_{1i}\left|x_{1i}\right|^{\alpha+1} - \rho_{1i}x_{1i}^2. \quad (14)$$

Define $k_{1m} = \min\left(k_{1i}\right)$ and $\rho_{1m} = \min\left(\rho_{1i}\right)$. the expression (14)satisfies the following inequality

$$\dot{V}_1 = -k_{1m}\left|x_{1i}\right|^{\alpha+1} - \rho_{1m}x_{1i}^2 \quad (15)$$

which can be further written as

$$\dot{V}_1 \le -\tilde{\rho}_1 V_1 - \tilde{k}_1 V_1^{\frac{\alpha+1}{2}}, \quad (16)$$

where $\tilde{k}_1 = 2^{(\alpha+1)/2}k_{1m}$ and $\tilde{\rho}_1 = 2\rho_{1m}$.

It is obvious that $\dot{V}_1$ is negative semi-definite. According to the Lemma 3, the equilibrium state of the differential equation $\dot{x}_1 = -K_1 sign^{\alpha}\left(x_1\right) - \rho_1 x_1$ is globally finite-time stable with the estimated settling time

$$T_r \le \frac{2}{\tilde{\rho}_1\left(1-\alpha\right)}\ln\left(\frac{\tilde{\rho}_1\left(V_1\left(x_0\right)\right)^{\left(\frac{1-\alpha}{2}\right)} + \tilde{k}_1}{\tilde{\rho}_1}\right).$$

We next choose the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_i^2 + \frac{1}{2\eta}(\hat{K}_i - \Gamma_i)^2, \quad (31)$$

which can be written as

$$V_2 = \frac{1}{2}\xi_i^2 + \frac{1}{2\eta}(\hat{K}_i - \Gamma_i)^2 \quad (32)$$

where $\xi = \left[x_{1i} \quad z_i\right]^T$. For analyzing the closed-loop system dynamic of (25). The time derivative of V_2 is obtained as

$$\dot{V}_2 = -x_{1i}\left(-k_{1i}\left|x_{1i}\right|^{\alpha} sign\left(x_{1i}\right) + s_i - \rho_{1i}x_{1i}\right)$$
$$+ s_i\left(-k_{2i}\left|s_i\right|^{\alpha} sign\left(s_i\right) - x_{1i} - \rho_{2i}s_i\right.$$
$$\left. - \tilde{d}_i s_i - \hat{K}_i sign\left(s_i\right)\right) + \frac{1}{\eta}(\hat{K}_i - K_i^*)\dot{\hat{K}}_i$$
$$= k_{1i}\left|x_{1i}\right|^{\alpha+1} - \rho_{1i}x_{1i}^2 - k_{2i}\left|s_i\right|^{\alpha+1} - \rho_{2i}s_i^2$$
$$+ \tilde{d}_i s_i - \hat{K}_i sign\left(s_i\right) + (\hat{K}_i - K_i^*)\left|z_i\right| \quad (33)$$

With $s_i = z_i$, it follows that

$$\dot{V}_2 \le -k_{3i}\left(\left|x_{1i}\right|^{\alpha+1} + \left|s_i\right|^{\alpha+1}\right) - \rho_{3i}\left(x_{1i}^2 + s_i^2\right)$$
$$+ \dot{\tilde{d}}_i\left|s_i\right| - K_i^*\left|s_i\right|, \quad (34)$$

where $k_{3i} = \min\left(k_{1i}, k_{2i}\right)$ and $\rho_{3i} = \min\left(\rho_{1i}, \rho_{2i}\right)$.

Letting $\varepsilon_i = K_i^* - \tilde{d}_i$ be a positive scalar, one obtains

$$\dot{V}_2 \le -k_{3i}\left(\left|x_{1i}\right|^{\alpha+1} + \left|s_i\right|^{\alpha+1}\right) - \rho_{3i}\left(x_{1i}^2 + s_i^2\right)$$
$$- \varepsilon_i\left|s_i\right|. \quad (35)$$

By Lemma 1 and $\left|\xi_i\right| \ge \left|s_i\right|$, we have

$$\dot{V}_2 \le -k_{3i}\left|\xi_i\right|^{\alpha+1} - \rho_{3i}\xi_i^2 - \varepsilon_i\left|s_i\right|$$
$$\le -\rho_{3i}\xi_i^2 \quad (36)$$

In view of (36), it is shown that ξ_i, $\hat{K}_i$, $i = 1,2,3$ are bounded and $\ddot{V}_2 \le -\rho_{3i}\xi_i\dot{\xi}_i$. Since ξ_i is bounded, then we get that x_{1i}, s_i, $i = 1,2,3$ are all bounded. Using Assumption 1 and the above analysis, one concludes that $\dot{\xi}_i$ is bounded. It follows that $\ddot{V}_2$ is also bounded. Then $\dot{V}_2$ is uniformly continuous on $[0,\infty)$. By Lemma 5, we conclude that $\lim_{t\to\infty}\dot{V}_2(t) = 0$, thus $\lim_{t\to\infty}\xi(t) = 0$. Thus, the state trajectories x_1 and the sliding variable s of the system (25) globally asymptotically converge to the origin. This competes the proof.

Remark 2 According to Theorem 1, the Lyapunov function V_2 defined in (31) is used to ensure the asymptotic stability of the whole system. Although, finite-time stability of the first equation in (25) is ensured, for the whole system, the asymptotic stability is achieved.

5. SIMULATIONS

An example of a rigid-body satellite [21] is presented with numerical simulations to verify the performance of the developed controller (22). The spacecraft is assumed to have the nominal inertia matrix

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} kg\cdot m^2$$

and the parameter uncertainties

$\Delta J = diag\left[\sin\left(0.1t\right) \quad 2\sin\left(0.2t\right) \quad 3\sin\left(0.2t\right)\right]$

$kg\cdot m^2$ The attitude control problem is considered in the presence of external disturbance $d\left(t\right)$. The external disturbances are described as

$$d\left(t\right) = \begin{bmatrix} 0.1\sin\left(0.1t\right) \\ 0.2\sin\left(0.2t\right) \\ 0.3\sin\left(0.3t\right) \end{bmatrix} N-m. \quad (17)$$

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In this numerical simulation, we assume that the desired angular velocity is given by

$$\omega_d(t) = 0.05 \begin{bmatrix} \sin\left(\frac{\pi t}{100}\right) \\ \sin\left(\frac{2\pi t}{100}\right) \\ \sin\left(\frac{3\pi t}{100}\right) \end{bmatrix} rad/s. \quad (18)$$

For the initial condition of the unit quaternion and the target unit quaternion, we set $Q(0) = [-0.3 \quad 0.1 \quad 0.2 \quad 0.9277]^T$ and $Q_d(0) = [0 \quad 0 \quad 0 \quad 1]^T$, Finally, the initial value of the angular velocity is assumed to be $\omega(0) = [-0.02 \quad 0.01 \quad 0.02]^T$ rad/s.

For the ABSMC scheme (22), the chosen parameters are given as $\rho_1 = diag(2.5, 2.5, 2.5)$ $\rho_2 = diag(2.0, 2.0, 2.0)$, $K_1 = diag(2.0, 2.0, 2.0)$, $K_2 = diag(1.5, 1.5, 1.5)$, $\eta = 0.5$, $\alpha = \frac{3}{5}$ and $\varepsilon = 0.001$.

The attitude quaternion tracking errors and angular velocity tracking errors are shown in Figs.1 and 2. It can be seen that the ABSMC scheme (22) gives smoother quaternion and angular velocity tracking errors and fast convergence to zero. From Fig. 3 it can be seen that the sliding surface $s = 0$ is reached after 10 seconds. As shown in Fig. 4, the ABSMC scheme (22) provides smoother response of control torques.

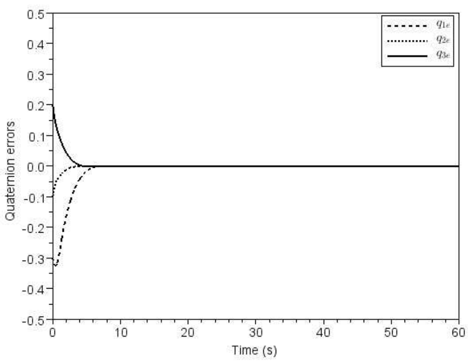

Figure 1: Quaternion tracking errors under (22).

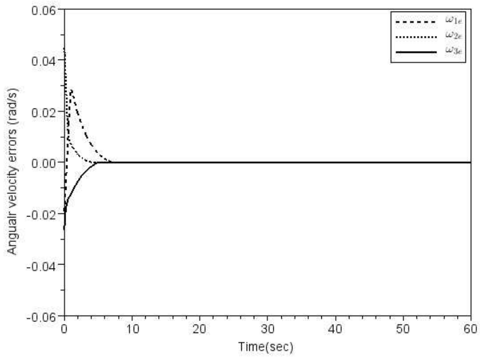

Figure 2: Angular velocity errors under (22).

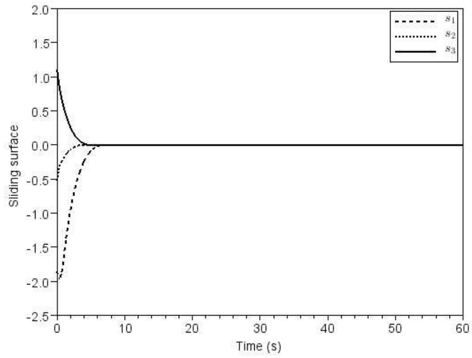

Figure 3: Switching function under (22).

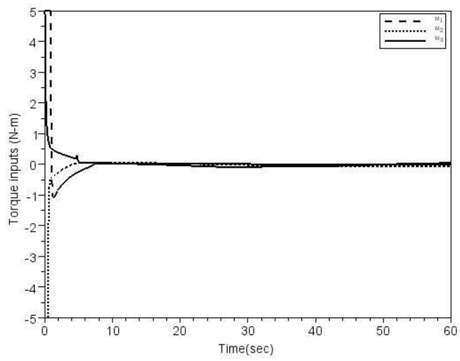

Figure 4: Control torques under (22).

6. CONCLUSIONS

The proposed robust attitude tracking controller has been successfully applied to spacecraft tracking maneuvers. For the controller design, the BASMC law is applied to deal with

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quaternion-based spacecraft-attitude-tracking maneuvers. Using the Lyapunov stability theory and adaptive backstepping design we prove that the error dynamics asymptotically converge to the origin. Numerical simulations are also given to demonstrate the performance of the proposed control law.

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