

A NEW CREATION OF MASK FROM DIFFERENCE OPERATOR TO IMAGE ANALYSIS

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ABSTRACT

A general theorem on a m^{th} order difference equation is presented. Specific illustration is given to support our claim. This leads to a creation of noise removal operator which can remove additive and multiplicative noises presenting in any digital image. Samples are shown to explain this new creation of mask in the field of image analysis and machine vision.

Keywords: *Mask, Entropy, Sobel, Difference Equation, Functional, Nonlinear, Oscillation, Image Analysis.*

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1. INTRODUCTION

While giving illustration for our main research, we find the coefficients of the consecutive terms of the sequence satisfying a relation form elements of the matrix. These elements omitting n and higher powers. That is treating only the constants form a new matrix of order 3×3 , whose elements added up to zero. This can be identified as an operator representing a mask to filter noise in a corrupted image by adopting like sobel operator. This is one of the techniques analyzed by Gonzalez and Woods [9]. Also this technique is adopted in the machine vision by Milan Sonka, Vaclav Hilavac and Roger Boyle [7]. The general theory of difference equations was discussed thoroughly by R.P. Agarwal [1]. Oscillatory properties were studied by R.P. Agarwal et.al. [2]. In the natural sciences, technology and population dynamics, difference equations find a lot of applications. In [5], [6], we can find the applications of the difference equations. This paper deals with the role of difference equations in image analysis.

In the following section, we present some basic concepts and results which are concerned with our research.

2. BASIC CONCEPTS AND RESULTS

The difference equation can be taken as a maximum entropy in the image content in the process of edge detection. We are concerned with the oscillatory behavior of the high order nonlinear functional difference equation of the form

$$\Delta^{m-2} \left(r(n) [\Delta^2 y(n)]^\alpha \right) - q(n) f(y(g(n))) = 0, \quad (1.1)$$

$$\text{where } \sum_{s=n_0}^{\infty} r^{-\frac{1}{\alpha}}(s) < \infty. \quad (1.2)$$

The following conditions are assumed to hold:

(i) α is the ratio of any two positive odd integers.

(ii) $\{r(n)\}$, $\{q(n)\}$ are real valued positive sequences.

(iii) $\{g(n)\}$ is a real valued increasing sequence

with $g(n) < n$, for $n \geq n_0$ and $\lim_{n \rightarrow \infty} g(n) = \infty$.

$$g = (G_x^2 + G_y^2)^{\frac{1}{2}}$$

$$= \left\{ \begin{aligned} & [(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)]^2 \\ & + [(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)]^2 \end{aligned} \right\}^{\frac{1}{2}}$$

(iv) $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function such that $xf(x) > 0$ and $f'(x) \geq 0$, for $x \neq 0$ and $-f(-xy) \geq f(xy) \geq f(x)f(y)$, for $xy > 0$. (1.3)

Here Δ is the forward difference operator defined by $\Delta y(n) = y(n+1) - y(n)$.

2.1. Oscillatory Solution

By a solution of equation (1.1), we mean a real sequence $\{y(n)\}$ which is defined for $n \geq \min_{i \geq 0} \{g(i)\}$ and satisfies equation (1.1) for large n . A solution $\{y(n)\}$ is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise it is called non-oscillatory. A difference equation is said to be oscillatory if all of its solutions are oscillatory. Otherwise, it is non-oscillatory.

2.2. Entropy

An entropy provides an excellent tool to quantify the amount of information (or uncertainty) contained in a random observation regarding its parent distribution (population). A large value of entropy implies the greater uncertainty in the data. As proposed by Shannon [10], entropy of an absolutely continuous random variable X having pdf $\phi_x(x)$ is defined as

$$H(X) = E(-\log(\phi_x(x)))$$

$$= -\int_S \phi_x(x) \log(\phi_x(x)) dx$$

where $S = \{x : \phi_x(x) > 0\}$.

2.3. Sobel Edge Detector

The sobel edge detector uses the masks in figures to approximate digitally the first derivatives G_x and G_y . In other words, the gradient at the center point in a neighborhood is computed as follows by the sobel detector:

2.4. Spatial Filtering and Mask

Neighborhood processing consists of 1. defining a center point, (x, y) ; 2. performing an operation that involves only the pixels in a predefined neighborhood about that center point; 3. letting the result of that operation be the "response" of the process at that point; and 4. repeating the process for every point in the image. The process of moving the center point creates new neighborhoods, one for each pixel in the input image. The two principal terms used to identify this operation are neighborhood processing and spatial filtering, with the second term being more prevalent.

The linear operations consist of multiplying each pixel in the neighborhood by a corresponding coefficient and summing the results to obtain the response at each point (x, y) . If the neighborhood is of size $m \times n$, mn coefficients are required. The coefficients are arranged as a matrix, called a filter, mask, filter mask, kernel, template, or window, with the first three terms being the most prevalent. For reasons that will become obvious shortly, the terms convolution filter, mask, or kernel, also are used. The process consists simply of moving the center of the filter mask w from point to point in an image, f . At each point (x, y) , the response of the filter at that point is the sum of products of the filter coefficients and the corresponding neighborhood pixels in the area spanned by the filter mask.

2.5. Image Filtering and Restoration

Any image acquired by optical, electro-optical is likely to be degraded by the sensing environment. The degradations may be in the form of sensor noise, blur due to camera misfocus, relative object-camera motion, random atmospheric turbulence, and so on. Image restoration is concerned with filtering the observed image to minimize the effect of degradations. The effectiveness of image restoration filters depends on the extent and the accuracy of the knowledge of the degradation process as well as on the filter design criterion. A frequently used criterion is the mean square error. It is reasonable local measure

and is mathematically tractable. Other criteria such as weighted mean square and maximum entropy are also used.

Image restoration differs from image enhancement in that the latter is concerned more with accentuation or extraction of image features rather than restoration of degradations. Image restoration problems can be quantified precisely, whereas enhancement criteria are difficult to represent mathematically. Consequently, restoration techniques often depend only on the class or ensemble properties of a data set, whereas image enhancement techniques are much more image dependent.

2.6. Histogram Processing and Function Plotting

Intensity transformation functions based on information extracted from image intensity histograms play a basic role in image processing, in areas such as enhancement, compression, segmentation, and description.

The histogram of a digital image with intensity levels in the range $[0, L-1]$ is a discrete function $h(r_k) = n_k$, where r_k is the k^{th} intensity value and n_k is the number of pixels in the image with intensity r_k . It is common practice to normalize a histogram by dividing each of its components by the total number of pixels in the image, denoted by the product MN , where, as usual, M and N are the row and column dimensions of the image. Thus, a normalized histogram is given by $p(r_k) = \frac{n_k}{MN}$, $k = 0, 1, 2, \dots, L-1$. $p(r_k)$ is an estimate of the probability of occurrence of intensity level r_k in an image. The sum of all components of a normalized histogram is equal to 1.

Histograms are simple to calculate in software and also lend themselves to economic hardware implementations, thus making them a popular tool for real-time image processing. The core function in the toolbox for dealing with image histograms is *imhist*, which has the following basic syntax:

$h = \text{imhist}(f, b)$ where f is the input image, h is its histogram, $h(r_k)$ and b is the number of bins used in forming the histogram. We obtain the

normalized histogram simply by using the expression $p = \text{imhist}(f, b) / \text{numel}(f)$.

3. OSCILLATORY BEHAVIOR OF DIFFERENCE OPERATOR IN IMAGE PROCESSING

We have the criterion for oscillatory sequence in the form of difference equation satisfied by the equation (1.1).

Theorem 2.1 Suppose that conditions (i)-(iv), (1.2) and (1.3) hold. Assume that there exists a real valued increasing sequence $\{\xi(n)\}$ such that $g(n) < \xi(n)$, for $n \geq n_0$. If the first order delay difference equation

$$\Delta z(n) - c(n - n_2)^{m-3} q(n) f\left(\sum_{s=n_2}^{g(n)-1} sr(s)^{\frac{-1}{\alpha}}\right) f\left(z^{\frac{1}{\alpha}}(g(n))\right) = 0 \quad (2.1)$$

is oscillatory, then the equation (1.1) is oscillatory.

Proof: Suppose to the contrary, assume that $\{y(n)\}$ be a non-oscillatory solution of equation (1.1). Also assume that $y(n) > 0$ and $y(g(n)) > 0$, for $n \geq n_0 \geq 0$.

Then from (1.1), we have

$$\Delta^{m-2} \left(r(n) [\Delta^2 y(n)]^\alpha \right) \geq 0, \quad n \geq n_0 \quad (2.2)$$

There exists a $n_1 \geq n_0$ such that

$$\left\{ \Delta^{m-2} \left(r(n) [\Delta^2 y(n)]^\alpha \right) \right\}, \{ \Delta^2 y(n) \}, \{ \Delta y(n) \}$$

are eventually monotone and one-signed for $n \geq n_1$.

There are four possible cases to consider.

(i) $\Delta y(n) > 0$, $\Delta^2 y(n) > 0$ and

$$\Delta^{m-3} \left(r(n) [\Delta^2 y(n)]^\alpha \right) > 0, \quad n \geq n_1.$$

(ii) $\Delta y(n) > 0$, $\Delta^2 y(n) < 0$ and

$$\Delta^{m-3} \left(r(n) [\Delta^2 y(n)]^\alpha \right) > 0, \quad n \geq n_1.$$

(iii) $\Delta y(n) < 0$, $\Delta^2 y(n) < 0$ and

$$\Delta^{m-3} \left(r(n) [\Delta^2 y(n)]^\alpha \right) < 0, \quad n \geq n_1.$$

(iv) $\Delta y(n) < 0, \Delta^2 y(n) > 0$ and
 $\Delta^{m-3} \left(r(n) [\Delta^2 y(n)]^\alpha \right) > 0, n \geq n_1.$

Case (i):

Since $\Delta y(n) > 0$ and $\Delta^2 y(n) > 0$, there exists a $n_2 \geq n_1$ and a positive constant $k, 0 < k < 1$ such that

$$\Delta y(n) \geq knr(n)^{\frac{-1}{\alpha}} \tag{2.3}$$

$$\left(r(n) [\Delta^2 y(n)]^\alpha \right)^{\frac{1}{\alpha}}, n \geq n_2.$$

Summing up (2.3) from n_2 to $n-1$, we can find a $n_3 \geq n_2$ with $g(n) \geq n_2$ for all $n \geq n_3$ such that

$$y(g(n)) \geq k \left(\sum_{s=n_2}^{g(n)-1} sr(s)^{\frac{-1}{\alpha}} \right) z^{\frac{1}{\alpha}}(g(n)), n \geq n_3 \tag{2.4}$$

Where $z(n) = r(n) [\Delta^2 y(n)]^\alpha$.

From (1.1), we can get

$$\Delta \left(r(n) [\Delta^2 y(n)]^\alpha \right) \geq (n - n_3)^{m-3} q(n) f(y(g(n))). \tag{2.5}$$

Using (2.4) and (1.3) in (2.5), we get,

$$\Delta z(n) - c(n - n_2)^{m-3} q(n) f \left(\sum_{s=n_2}^{g(n)-1} sr(s)^{\frac{-1}{\alpha}} \right) f \left(z^{\frac{1}{\alpha}}(g(n)) \right) \geq 0 \tag{2.6}$$

where $c = f(k), n \geq n_3.$

Now the inequality (2.6) has eventually positive solution $\{z(n)\}$.

By a well-known result (see [6]), the difference equation (2.1) also has an eventually positive solution which contradicts our assumption that (2.1) is oscillatory.

Case (ii):

Clearly $v(n) > 0, \Delta v(n) < 0$ for $n \geq n_3$ where

$$v(n) = \Delta y(n).$$

Now for $s \geq n \geq n_2$, we can see that

$$r(s) (-\Delta v(s))^\alpha \geq r(n) (-\Delta v(n))^\alpha, \text{ from which,}$$

we can obtain

$$v(g(n)) \geq b \sum_{r=g(n)}^{\infty} r(\tau)^{\frac{-1}{\alpha}}, n \geq n_3 \tag{2.7}$$

where b is a positive constant.

Using (1.3) and (2.7) in the inequality

$$\Delta^{m-2} \left(r(n) [\Delta^2 y(n)]^\alpha \right) \geq q(n) f(k) f(g(n)) f(v(g(n))),$$

we can get

$$\Delta^{m-2} \left(r(n) [\Delta^2 y(n)]^\alpha \right) \geq cq(n) f(g(n)) f \left(\sum_{r=g(n)}^{\infty} r(\tau)^{\frac{-1}{\alpha}} \right),$$

where $c = f(k)f(b).$

From the above inequality, we can easily get

$$v(n) \leq - \sum_{s=n}^{\infty} \left(\frac{1}{r(n)} \right)^{\frac{1}{\alpha}} \left[c(n - n_4)^{m-2} \right]^{\frac{1}{\alpha}} \left[q(n) f(g(n)) f \left(\sum_{r=g(n)}^{\infty} r(\tau)^{\frac{-1}{\alpha}} \right) \right]^{\frac{1}{\alpha}} n \geq n_3.$$

This implies $v(n) \rightarrow -\infty$ as $n \rightarrow \infty$, which is a contradiction.

Case (iii):

This case cannot hold.

In fact, if $\Delta y(n) < 0$ and $\Delta^2 y(n) < 0$, then we have $\lim_{n \rightarrow \infty} y(n) = -\infty$, which is a contradiction to the fact that $y(n) > 0$.

Case (iv):

For $n \geq s \geq n_0$, we have $y(n) - y(s) = \sum_{u=s}^{n-1} \Delta y(u).$

This implies, $y(s) \geq (n - s) (-\Delta y(n)).$

Replacing s and n by $g(n)$ and $\xi(n)$

respectively, we obtain

$$y(g(n)) \geq [\xi(n) - g(n)] [-\Delta y(\xi(n))], n \geq n_2 \geq n_1.$$

Let $-\Delta y(n) = z(n) > 0.$ and proceeding as in the proof of case (ii), we can easily get

$$\Delta z(n) \leq - \left(\frac{1}{r(n)} \right)^{\frac{1}{\alpha}} \left((n - n_2)^{m-3} \left(q(n) f[\xi(n) - g(n)] + b \right) \right)^{\frac{1}{\alpha}}$$

This implies, $\lim_{n \rightarrow \infty} z(n) = -\infty$, which is a contradiction to the fact that $z(n) > 0$ and this completes the proof of the theorem.

In the next section, the application of our result for $m = 4$ is illustrated.

4. APPLICATION

Consider the fourth order difference equation

$$\Delta^2 \left(n^6 (\Delta^2 y(n)) \right) - \frac{8(2n^2 + 5n + 2)}{\sqrt{e}} y(n-2) e^{\frac{y^2(n-2)}{2}} = 0. \quad (4.1)$$

Here $r(n) = n^6$.

Also,

$$\sum_{s=n_0}^{n-1} r^{-\frac{1}{\alpha}}(s) = \sum_{s=n_0}^{n-1} (s^6)^{-\frac{1}{3}} = \sum_{s=n_0}^{n-1} (s^{-2}) < \infty$$

and $g(n) = n - 2 < n$.

We can easily see that all conditions of Theorem 2.1 are satisfied and hence all the solutions of equation (4.1) are oscillatory. One of such solution is $y(n) = (-1)^n$.

Equation (4.1) can be written as

$$\begin{aligned} & (n^2 + 4n + 4)y_{n+4} - (4n^2 + 12n + 10)y_{n+3} \\ & + (6n^2 + 12n + 8)y_{n+2} - (4n^2 + 4n + 2)y_{n+1} \\ & + n^2 y_n - \frac{8(2n^2 + 5n + 2)}{\sqrt{e}} y(n-2) e^{\frac{y^2(n-2)}{2}} = 0. \end{aligned} \quad (4.2)$$

From the equation (4.2), we can see that constants form a matrix of order 3×3 , given by

$$\begin{pmatrix} 0 & -10 & 0 \\ -2 & 0 & 8 \\ 0 & 4 & 0 \end{pmatrix}$$

which is used as an operator in the removal of noise whose coefficients are added up to zero. This is explained as follows:

4.1. Data Analysis and Results:

Figure 1 shows the removal of additive and multiplicative noises in cameraman digital image using our operator and Figure 2 shows removal using sobel operator.



Figure 1: Original Image



Figure 2 : SK Operator Image



Figure 3 : Sobel Operator Image

We get a pixel value for the images with noise and after the removal of noise through sobel operator and our operator.

Figure 4 and Figure 5 show the histograms of sobel operator and our operator

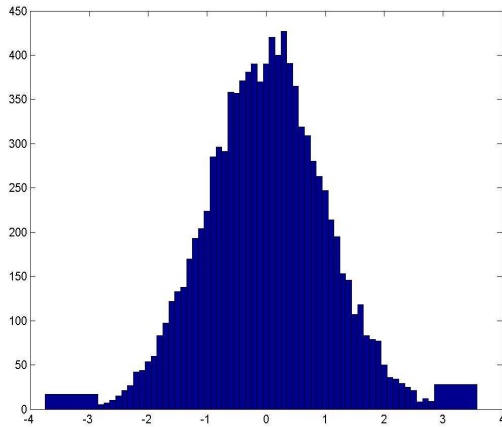


Figure 4: Sobel Operator Histogram

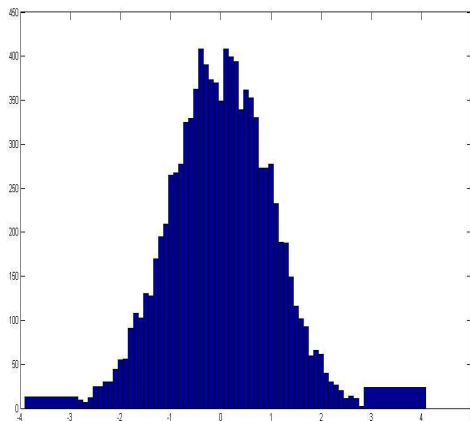


Figure 5: SK Operator Histogram

From the frequency distribution of the pixel value presented in the histogram, we insist that our filtering operator raise to give more information on the edge detection, removal of noise and other parameters pertaining to consequent components of the complete image, better reveal by our operator.

Algorithm to draw histogram and coding for filtering the noise using 1. Sobel operator 2. New operator are as follows:

```
% histogram coding
cd=imread('cameraman source.gif');
cd = -2.9:0.1:2.9;
y = randn(10000,1);
figure(1), hist(y,cd);
```

% filtering the noise

% sobel operator

```
1. ic=imread('cameraman source.GIF');
px=[-1 0 1;-1 0 1;-1 0 1];
icx=filter2(px,ic);
py=px';
icy=filter2(py,ic);
edge_s=sqrt(icx.^2+icy.^2);
figure, imshow(edge_s/255)
```

% new operator

```
2. ic=imread(' cameraman source.GIF');
px=[0 -10 0;-2 0 8;0 4 0];
icx=filter2(px,ic);
py=px';
icy=filter2(py,ic);
edge_s=sqrt(icx.^2+icy.^2);
figure, imshow(edge_s/255)
```

From the table of histogram one can find the distribution of pixel value in three phases of the image reproduction. This enhances edges and bring some objects in the original by appealing to n digit coefficients. We can give this mask for minuet images.

Similarly, we have done with 'lena image' also. The following figures illustrate our work for lena image.



Figure 6: Original Image



Figure 7: SK Operator Image



Figure 8: Sobel Operator Image

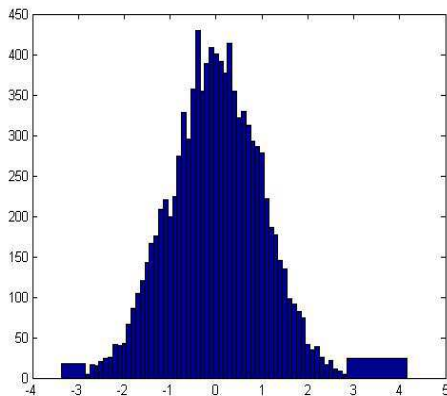


Figure 9: Sobel Operator Histogram

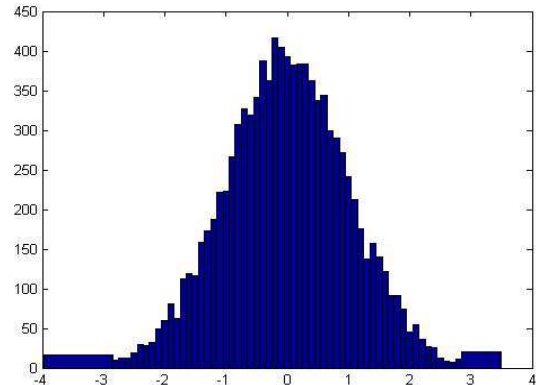


Figure 10: Sobel Operator Histogram

4.2 PSNR Value:

PSNR or peak-to-noise ratio is used to evaluate the quality of the watermarked image after embedding the secret message in the image.

The PSNR value more than 30dB shows that the watermarked image quality is acceptable to human eyes. Basically, the larger the PSNR value, the better the quality of the watermarked image is. It means that the distortions created on the watermarked image is not really perceptible and difficult to be detected by the human visual system.

On the other hand, if the PSNR value is less than 30dB, the watermarked-image is most likely to give some 'noise' on the image. This 'noise' becomes perceptible to human eyes and so the watermarked-image is considered to have lesser quality.

The MATLAB code below leads to compare the original (host) image and the watermarked image and then gives the PSNR value.

```
% matlab code;
clear all; close all; clc;

[filename1,pathname]=uigetfile('*.','Select the
original image');
image1=imread(num2str(filename1));

[filename2,pathname]=uigetfile('*.','Select the
watermarked image');
image2=imread(num2str(filename2));

figure(1);
imshow(image1); title('Original image');
```

```
figure(2);
imshow(image2); title('Watermarked image');

[row,col] = size(image1)
size_host = row*col;

o_double = double(image1);
w_double = double(image2);
s=0;

for j = 1:size_host; % the size of the original image
s = s+(w_double(j) - o_double(j))^2 ;
end

mes=s/size_host;
psnr =10*log10((255)^2/mes);
display 'Value of ,psnr
```

Table 1: Image Quality Tested (PSNR)

S.No	Comparison Images	Figures no	Row	Column	PSNR Value
1	Original image & SK Operator image	Figure 6 and Figure 7	256	768	5.4046
	Original image & Sobel Operator image	Figure 6 and Figure 8	256	768	5.2261
2	Original image & SK Operator image	Figure 1 and Figure 2	664	4098	12.4664
	Original image & Sobel Operator image	Figure 1 and Figure 3	664	4098	12.0173

As the psnr ratio of our operator is greater than sobel operator, we conclude our operator is capable of removing noise in this particular image and we show in general context of blunt images. This we feel may help diagnostic techniques in medical imaging and other video imagery.

5. CONCLUSION

While presenting an illustration for our main result derived on general order difference equation, we could create a new mask which is better in removing additive and multiplicative noises.

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