EMBEDDED ZERO TREE WAVELET AND ORTHOGONAL POLYNOMIAL BASED TRANSFORM CODING

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ABSTRACT

In this paper, a new and effective mode for transform coding of images depends on orthogonal polynomials has been recommended. The proposed orthogonal polynomials be contingent on transform coding system has the encoder, comprising of a polynomial transform operation tracked by quantization of transform coefficients and the entropy coding of quantized measurements. Embedded Zerotree of Wavelet transforms is a lossy image compression algorithm. At low bit rates most of the coefficients bent by a subband transform will be zero, or very close to zero. It provides extensive improvement in picture quality at privileged compression ratio. After the encoded bit stream of an input image is transferred over the channel, the decoder rears all the functionalities applied in the encoder and tries to reconstruct a decoded image that looks as close as possible to the inventive input image. The result of the proposed coding are compared with Embedded Zero Tree Wavelet and Discrete Wavelet Transform coding. The new image coding algorithm results in a considerable reduction in computation time and provides better reconstructed picture quality. The experimental results of the proposed technique also show that the efficient bit rate decline is achieved, when equated with existing techniques.

Keywords- Orthogonal Polynomials, Quantized Coefficients, EZW, Lossy Image Compression, Compression Ratio.

1. INTRODUCTION

In recent years many imperative solicitations such as medical imaging, remote sensing, space imaging and image archiving need an efficient image coding scheme. Many Discrete Wavelet Transform based coder have been conveyed in the past years [1-6] for lossy image coding. A two stage lossy coder is presented in [4], wherein the first stage uses DWT for good energy compaction and decorrelation and residual image are coded in second layer using adaptive reversible integer wavelet packet transform. However the main drawback of DWT based coder is that most of the wavelet coefficients are in floating point numbers which are not well suited for lossy image coding schemes. Hence a low complexity Lifting Scheme (LS) based [7] a wavelet compression technique is introduced for lossy image coding [8-12], since a perfect reconstruction is ensured by the structure of LS itself. Embedded Zero tree Wavelet Coding is an efficient universal lossy coding scheme since it attempts to optimally encode a source using no prior knowledge of the source. The EZW algorithm is based on the following key concepts in [26]. Many EZW based coder has been reported in countless literature [13-16]. In [13], the DWT and EZW algorithm are combined for coding of images. The computational complexity of EZW algorithm is compact using quadtree intense and context modelling [14]. In order to increase the compression ratio of EZW coding, the stochastic texture model using Generalized Gaussian Distribution is combined with EZW algorithm [15]. The block based embedded, progressive DWT image compression with EZW algorithm is presented in [16]. In general, the computational complexity of EZW coding algorithm is quite high especially for encoding the lower subband images. In [17], a hybrid coding technique joining the DWT based JPEG coder [18] is employed for reducing the computational complexity of encoding the lower subband signals in EZW algorithm.

Since most of the lossy image coders are based on complex Discrete Wavelet transform coding scheme. It takes the advantages of both wavelet transform based coding scheme and simple Orthogonal Polynomials Transform (OPT) based
coding scheme [19]. This OPT based coding scheme has resulted from our investigations into some low level feature extraction problems such as detection of textures and edges, in monochrome and color images [20-22]. In these works, a point-spread operator M is designed due to a class of orthogonal polynomials and defined a linear two dimensional transformation to analyze the low level primitives of the image under analysis. Normalization and mapping are used in the proposed work in order to obtain the non-linear transformed image data which exploits the subjective redundancy of the image. These non-linear transformed data are rearranged in zero tree structure and are encoded with EZW coding technique.

The rest of the paper is rearranged as follows; the Zero Tree Wavelet based image coding is presented in section 2. The Orthogonal Polynomial Based Image Coding is given in section 3. The measure of performance is given in section 4. The experimental upshots of the proposed coding and their comparison with existing techniques are presented in section 5, and the conclusion is reported in section 6.

2. ZERO TREE WAVELET BASED IMAGE CODING

Wavelet based image coding techniques [24] afford substantial improvements in picture worth at sophisticated compression ratios. The main idea in using transformation is to compact the energy of signals in much less samples than in time domain, so we can discard small transform coefficients. Wavelet transform has a worthy location property in time and frequency domain and is exactly in the direction of transform compression idea. The discrete wavelet transforms states to wavelet transforms for which the wavelets are disjointly appraised. A transform which limits a function both in space and scaling and has some necessary properties compared to the Fourier transform. The transform is centred on a wavelet matrix, which can be figured more quickly than the analogous Fourier matrix. Most notably, the DWT is used for signal coding, where the assets of the transform are exploited to signify a discrete signal in an extra fired form, regularly as a preconditioning for data compression. The discrete wavelet transform has a vast quantity of uses in Computer Science, Engineering, Mathematics, and Science, and. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. The basic awareness of the wavelet transform is to exemplify any arbitrary signal “X” as a superposition of a regular of such wavelets or basis functions. These basis functions are gained from a single photo type wavelet called the mother wavelet by dilation (scaling) and translation (shifts). The discrete wavelet transform for two dimensional signals can be defined as follows.

\[ w(a_1, a_2, b_1, b_2) = \frac{1}{\sqrt{a}} \psi \left( \frac{X - b_1, Y - b_2}{a_1, a_2} \right) \]

Where, \( a = a_1a_2 \)

The indexes equation (1) \( w (a_1, a_2, b_1, b_2) \) are called wavelet coefficients of signal X plus \( a_1, a_2 \) are dilation & \( b_1, b_2 \) are translation, \( \psi \) is the transforming function, which is well-known as mother wavelet. Low frequencies are surveyed with low temporal resolution while high frequencies with more temporal resolution. A wavelet transform pools both low pass and high pass filtering in spectral decomposition of signals. In case of discrete wavelet, the image is decomposed into a discrete set of wavelet coefficients using an orthogonal set of basic functions. These sets are divided into four parts such as approximation, horizontal facts, vertical facts and diagonal facts. Discrete Wavelet transform [23] provide substantial improvement in picture quality at higher compression ratio.

The Embedded Zero tree Wavelet coding is a simple, effective progressive image coding algorithm and can be worn for both lossless and lossy compression systems. This algorithm works well with the proposed coding scheme because the zero tree structure is effective in describing the significance map of the transform coefficients, as it exploits the inherent self-similarity of the subband image over the range of scales, and the positioning of majority of zero valued coefficients in the higher frequency subbands. The EZW algorithm applies Successive Approximation Quantization in order to provide multi-precision representation of the transformed coefficients and to facilitate the embedded coding. The algorithm codes the transformed coefficients in decreasing order in several scans. Each scan of the algorithm consists of two passes: significant map encoding and refinement pass. The dominant pass scans the subband structure in zigzag, right-to-left and then top-to-bottom within each scale, before proceeding to the next higher scale of subband structure as presented in Figure 1. For each and every pass, a threshold (T) is chosen against which all the
coefficients are measured and encoded as one of the following four symbols,

- Significant positive – If the coefficient value is greater than threshold T
- Significant negative – If the magnitude of the coefficient value is greater than threshold T
- Zero tree root – A coefficient is encoded as zero tree root if the coefficient and all its descendants are insignificant with respect to threshold T
- Isolated zero – If the coefficient is insignificant but some of its descendants are significant.

\[ T_0 = 2^{\log_2 C_{max}} \]

where equation (2) \( C_{max} \) is the maximum coefficient in the subband structure. The successive approximation quantization uses a monotonically decreasing set of thresholds and encodes the transformed coefficients as one of the above four labels with respect to any given threshold. For successive significant pass encoding, the threshold is lowered as \( T_{k} = \frac{T_{k-1}}{2} \) and only those coefficients not yet found to be significant in the previous pass are scanned for encoding, and the process is repeated until the threshold reaches zero, and results in complete encoded bit streams.

In the embedded zero tree wavelet coding strategy, developed by Shapiro, a wavelet/subband decomposition of the image is performed. The wavelet coefficients/pixels are then grouped into Spatial Orientation Trees. The magnitude of each wavelet coefficients/pixels in a tree, starting with the root of the tree, is then compared to a particular threshold T. If the magnitude of all the wavelet coefficients/pixels in the tree are smaller than T, the entire tree structure (that is the root and all its descendant nodes) is coded by one symbol, the zerotree symbol ZTR. If however, there exist significant wavelet coefficients/pixels, then the tree root is coded as being significant or insignificant, if its magnitude is larger than or smaller than T, respectively. The descendant nodes are then examined in turn to determine whether each is the root of a possible sub zerotree structure, or not. This process is carried out such that all the nodes in all the trees are examined for possible sub zerotree structures.

The significant wavelet coefficients/pixels in a tree are coded by one of two symbols, POS or NEG, depending on whether their actual values are positive or negative, respectively. The process of classifying the pixels as being ZTR, IZ, POS, or NEG is referred to as the dominant pass in [9]. This is then followed by the subordinate pass in which the significant wavelet coefficients/pixels in the image are refined by determining whether their magnitudes lie within the intervals \((T, 3T/2)\) and \((3T/2, 2T)\). Those wavelet coefficients/pixels whose magnitudes lie in the interval \((3T/2,2T)\) are represented by a 0 (LOW), whereas those with magnitudes lying in the interval \((3T/2,2T)\) are represented by a 1 (HIGH). Subsequent to the completion of both the dominant and subordinate passes, the threshold value T is reduced by a factor of 2, and the entire process repeated. This coding strategy, consisting of the dominant and subordinate passes followed by the reduction in the threshold value, is iterated until a target bit rate is achieved.

The root node of each tree is located at the highest level of the decomposition pyramid, and all its descendants are located in different spatial frequency bands at the same pyramid level, or clustered in groups of 2 X 2 at lower levels of the decomposition pyramid. An EZW decoder reconstructs the image by progressively updating the values of each wavelet coefficient/pixel in a tree as it receives the data. The decoder's decisions are always synchronized to those of the encoder.

2.1 EZW Algorithm

1) Initialization: Set the threshold T to the minimum power of that is greater than max \((i,j)\) \( C_{ij} \), where \( C_{ij} \) are the wavelet coefficients.
2) Significance map coding: Scan all the coefficients in a predefined way and output a symbol when \( |C_{ij}| > T \). When the decoder inputs this symbol, it sets \( C_{ij} = \pm 1.5T \).
3) Refinement: Refine each significant coefficient by sending one more bit of its binary representation. When the decoder obtains this, it boosts the current coefficient significance by \( \pm 0.25T \).
4) Set \( T_k = T_{k-1}/2 \), and go to step 2 if more iterations are needed.

3. ORTHOGONAL POLYNOMIAL BASED IMAGE CODING

Orthogonal Polynomial Based Image Coding [25] of 2-D monochrome images has been discussed. In order to devise a transform coding, a linear 2-D
image formation system is considered around a Cartesian coordinate separable, distorting, point spread operator in which the image $I$ results in the superposition of the point source of impulse weighted by the value of the object function $f$. Articulating the object function $f$ in terms of derivatives of the image function $I$ relative to its Cartesian coordinates is very useful for analysing the image. The point spread function $M(x, y)$ can be painstaking to be real valued function defined for $(x, y) \in X \times Y$, where $X$ and $Y$ are ordered subsets of real values. In case of gray-level image of size $(m \times n)$ where $X$ (rows) consists of a finite set, which for convenience can be labelled as $\{0, 1, \ldots, n-1\}$, the function $M(x, y)$ reduces to a sequence of functions.

$$M(i, t) = u_i(t), \quad i, t = 0, 1, \ldots, n-1$$

The linear two dimensional transformation can be defined by the point spread operator $M(x, y)$ ($M(i, t) = u_i(t)$) as shown in Eq.(2).

$$\beta(\zeta, \eta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(\zeta, x)M(\eta, y)f(x, y)dx\,dy$$

Considering both $X$ and $Y$ to be a finite set of values $\{0, 1, 2, \ldots, n-1\}$, equation.(2) can be written in matrix notation as follows,

$$\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} = \begin{bmatrix} |M| \otimes |M|' \end{bmatrix} \begin{bmatrix} I \end{bmatrix}$$

Where $\otimes$ is the outer product, $|\beta_j|$ are $n^2$ matrices arranged in the dictionary sequence, $|I|$ is the image, $|\beta_j|$ are the coefficients of transformation and the point spread operator $|M|$ is

$$\begin{bmatrix} u_0(t_0) & u_1(t_1) & \ldots & u_n(t_n) \\ u_0(t_2) & u_1(t_2) & \ldots & u_n(t_2) \\ \vdots \\ u_0(t_n) & u_1(t_n) & \ldots & u_n(t_n) \end{bmatrix}$$

Consider a set of orthogonal polynomials $u_0(t), u_1(t), \ldots, u_{n-1}(t)$ of degrees $0, 1, 2, \ldots, n-1$ respectively to construct the polynomial operators of different sizes from equation.(4) for $n \geq 2$ and $i = i$. The generating formula for the polynomials is as follows,

$$u_{i+1}(t) = (t - \mu_i(t) - b_i(n))u_{i+1}(t) \text{ for } i \geq 1$$

Where, $b_i(n) = \frac{\sum_{i=0}^{n} u_i(t)}{\sum_{i=1}^{n} u_i^2(t)}$ and $\mu = \frac{1}{n} \sum_{i=1}^{n} t_i$

Considering the range of values of $t$ to be $t_i = i, i = 1, 2, 3\ldots n$,

$$b_i(n) = \frac{i^2(n^2 - i^2)}{4(i^2 - 1)}, \mu = \frac{1}{n} \sum_{i=1}^{n} t = \frac{n+1}{2}$$

The point-spread operators $|M|$ of different size can be constructed from equation.(5) using the above orthogonal polynomials for $n \geq 2$ and $i = i$. For the convenience of point-spread operations, the elements of $|M|$ are scaled to make them integers.

The proposed OPT coding is basically a symmetrical algorithm since it compresses and decompresses an image. The block diagram of OPT encoder is presented in Figure.2. The orthogonal polynomials based transform is first applied to the source image data as explained in section II. The transform coefficients $\beta_i$ are then quantized and entropy coded, before forming the output code-stream. The code-stream is first entropy decoded, de-quantized and inverse transformed, thus resulting in the reconstructed image data. The encoder is composed of three main parts: Forward Transform based on proposed orthogonal polynomials, quantization and entropy encoder. The decoder also consists of three main parts i.e. Entropy decoder, de-quantization, and inverse polynomial transform using basis function, which are all of have inverse functionality to their corresponding parts in the encoder.

The proposed coding scheme is performed on block by block basis and each block has a size of $(n \times n)$ pixels. The fact that the proposed scheme goes on petite blocks is motivated by both computational or memorial considerations and the need to account for the non-stationery of the image. A quality measure defines the quantization step for each of the $n^2$ transformed coefficients $b_i$. The quantized transform coefficients of each block are then crisscross scanned into one dimensional vector that goes through a run length coding of the zero series, thereby clustering long insignificant stumpy energy coefficients into short and compact descriptors. Finally the run-length string is fed to an entropy coder that can be coded with a known dictionary,
extracted from the specific measurements of the given image. The DC component of the proposed orthogonal polynomials based transform coding, $\beta_{ij}$ of each block is coded in a differential pulse code modulation format i.e. the difference between the DC component of this block and that of the previous block is coded. All the AC coefficients are coded in the Pulse Code Modulation. As too few bits are allocated, only the DC coefficients are implicit and the resultant image after decompression contains of essentially constant valued blocks. The first stage in the decoder part is entropy decoder. This stage takes the compressed image data as input and uses the inverse procedure of coding and produces the output equivalent to the output of the quantizer in the encoder. The next stage takes the output of the entropy encoder as input and applies inverse function of the quantization called dequantization. The dequantization output is obtained by multiplying entropy output with corresponding quantization table elements. The final stage is the inverse transform using the basic functions of the proposed scheme. The output of this stage results in an image which is very similar to the original image.

4. MEASURE OF PERFORMANCE

The performance of the proposed scheme with orthogonal polynomials, Embedded Zero Tree Wavelet, and Discrete Wavelet transform coding is measured by computing the peak signal-to-noise ratio (PSNR), which is defined as

$$PSNR = 10 \log_{10} \frac{(225)^2}{e_{ms}^2}$$

Where the average mean square error, $e_{ms}$ is,

$$e_{ms}^2 = \frac{1}{PQ} \sum_{i=1}^{P} \sum_{j=1}^{Q} (u_{i,j} - \hat{u}_{i,j})^2$$

Where $u_{i,j}$ and $\hat{u}_{i,j}$ represent the ($P \times Q$) original and reproduced images respectively.

5. EXPERIMENTS AND RESULTS

The proposed orthogonal polynomials based image preserving compression has been experimented with various test images. Two sample standard images viz Boat and Girl images, which are of size (256 x 256) with pixel values in the range (0 – 255) are shown in figure 3.1 and figure 3.2 respectively. The input images are partitioned into various non-overlapping sub-images of size (4 x 4), and are subjected to the proposed orthogonal polynomials based transformation to obtain the transform coefficients. A compression ratio of 87.86% with a PSNR value 31.6dB is achieved when the quality factor is 5 for Boat image and for the same quality factor, a compression ratio of 87.22% with a PSNR value 41.24dB is achieved when the quality factor is 50 for Boat image and for the same quality factor is presented in chart Fig 4.1(a) (b) and Fig 4.2(a) (b).

The proposed transform coding is compared with the DWT based JPEG and EZW. Here, the transform coefficients obtained after the proposed transformation with different quality factors and the bit allocated using variable length code. For the Boat image with quality factor of 5, a compression of 87.16% and 86.11% with PSNR value 30.9dB and 30.2dB is obtained. With regard to image, a compression of 87.22% and 87.12% with PSNR value 40.81dB and 40.38dB is achieved when the quality factor of 50. For the quality factor of 5, on the same Girl image, the wavelet-based scheme gives a compression of 39.34dB and 39.9dB is resulted. For the quality factor of 50, on the same image, the wavelet-based scheme gives a compression of 87.63% and 87.11% with a PSNR value 39.34dB and 38.88dB is resulted. The experiment is repeated for the quality factors 5 and 50 for all standard images and corresponding results are tabulated in the tables. It is evident from the table 1 that the proposed transform coding is giving better compression ratio than the DWT and EZW results and their corresponding PSNR values against different quality factor of image based transform coding are plotted on the graph for Boat and Girl images are presented in Table 1. These outputs are presented in Fig 3.2(a) (b) and 3.3(a) (b). The experiments with different step sizes for OPT with DWT and EZW is carried out for both Boat and Girl images and are incorporated in the Table 1 and 2.

6. CONCLUSION

In this paper, a new transform coding technique for low bit rate image compression with orthogonal polynomials has been presented. The proposed scheme is configured from a set of orthogonal polynomials. The responses of these polynomial operators are uncorrelated, linear contrasts and hence unbiased statistical estimates. As the proposed transform coding is configured as integer, it has low computational complexity. A quantization scheme that tunes the compression ratio and the quality of the reconstructed image after compression and decompression process is also implemented. The quantized transform coefficients are then subjected to entropy coding as in JPEG.
compression standard. The performance of the proposed integer transform coding is reported by computing peak signal to noise ratio and is compared with DWT based international compression standard JPEG and EZW image compression techniques. During the Experimental results, at the level of Compression ratio 5%, OPT is 100% best. At the level of Compression ratio is 50%, OPT is 60% best when compared to DWT is 30% and EZW is 10%. Even though the proposed orthogonal polynomials based coding gives good compression ratio, it can be observed that at low-bit rates, visible artifacts are observed.

REFERENCES


**Figure 1:** EZW subband structure scanning order

**Figure 2:** Block diagram of the Proposed Transform Coding Scheme

**Figure 3.1:** Original Test Images

(a) Boat  (b) Girl

**Figure 3.2:** Result of OPT When Quality factor is 5%

(a) Boat  (b) Girl

**Figure 3.3:** Result of OPT When Quality factor is 50%

(a) Boat  (b) Girl

**Figure 4.1(a):** 5% CR Value

(a) Boat  (b) Girl

**Figure 4.2(a):** 50% PSNR Value

(a) Boat  (b) Girl

**Figure 4 1(a):** 50% CR Value

(a) Boat  (b) Girl

**Figure 4 2(a):** 5% PSNR Value

(a) Boat  (b) Girl
Figure 4.2(b): 5% PSNR Value

![5% PSNR Value](image)

<table>
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<th>50% Compression Ratio</th>
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<td>DWT</td>
</tr>
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<tr>
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</tr>
<tr>
<td>Mandrill</td>
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<tr>
<td>Barbara</td>
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<td>87.6</td>
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Table 1: Compression Ratio results by the proposed OPT approach with DWT and EZW

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Table 2: PSNR Value results by the proposed OPT approach with DWT and EZW