

# TUNING OF NONLINEAR MODEL PREDICTIVE CONTROL FOR QUADRUPLE TANK PROCESS

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## ABSTRACT

Multi Input Multi Output (MIMO) interactive process is more complex than Single Input Single Output (SISO) systems. We consider two inputs and two outputs for four tank process, which is also called Quadruple Tank Process (QTP). It consists of two inputs as voltages fed to motor pumps and water levels in the lower tanks are the two outputs. The inputs are manipulated by setting of valves. It has four tank interactive loop operation consisting of two lower tanks and two upper tanks. The Model Predictive Control (MPC) technique is more suitable and gives optimized operational control in calculating present and future values of output. It tunes output and inputs simultaneously to provide more stable (optimized) output within the permissible limits of tolerance / error. MPC can stabilize all linear processes effectively and efficiently. It can also work with nonlinear processes under extreme conditions. It offers an optimized result under Predictive control “P” and Horizon control “M”, by tuning P and M and its application to QTP as a Nonlinear Model Predictive Control.

**Keywords:** *Nonlinear Model Predictive Control, Optimized, QTP, Predictive Control P, Horizon Control M*

## 1. INTRODUCTION

MPC technique is more applicable for obtaining optimized performance and is attractive as it offers feedback strategy for linear processes. This same method can be applied to nonlinear systems and obtain equally good response or result. This is referred to as moving horizon or receding horizon control. MPC methods use linear or nonlinear models, to calculate present and future values of dynamic systems. Linear MPC theory is quite mature and has wide ranging applications from chemicals to aerospace industries. Most of the physical systems are inherently nonlinear in existence because of economical constraints and quality of product in process industry. This requires

maintaining and operating the system within the admissible operating region which is part of the boundary.

MPC techniques was introduced and implemented in industries since 1960's. Frank Allgower et al [8] provided basic mathematical properties and formulation of Nonlinear Model Predictive Control. Jorge L. Garriage et al[5] provided optimized solution of tuning parameters and applied various new tuning methods based on process response and analysis. Rahul Shridhar et al [6] delivered Strategy of Unconstraint for multiple variable input and output for non linear model. Leonidas G.Bleris et al [4] proposed that

Optimization is part of MPC, where optimal controls are inputs for a process for real time and modeling system. Alberto Beporad [7] discussed about MPC controller design for different set points for evaluation of closed loop performance.

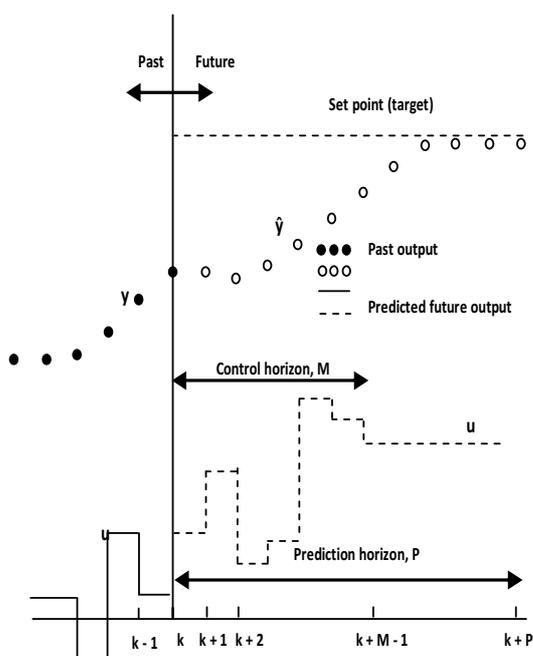


Figure 1: Basic principle of Model Predictive Control

K. H. Johansson [1] motivated the implementation of four tank process with an adjustable zero. Sivakumaran et al [2] explained about neural model predictive control for multi variable process. S.M. Mahadi Alvani et al [3] elaborated feedback network for QTP. This work focuses on the application of MPC techniques to nonlinear model of QTP [1], [2]. The basic principle of MPC is as shown in fig 1.

It depicts the dynamic behavior of system over predicted horizon P and control horizon M and determines the predefined open loop performance objective function which has to be optimized. Performance measure is obtained at time t, the controller predicts the future values by tuning for P greater than 1. MPC has two methods of approach,

one is for non linear model of QTP and the other is a linear model of QTP [1], [2]. Linear MPC approaches the operating points by tuning objective function. Nonlinear MPC approaches the problem without consideration to the operating point by tuning of objective function for nominal values.

This is applied for QTP with optimized technique for MPC of two methods.

MPC method has two ways of tuning. The first method is based on simulation of process model [11] or adjusts parameters based on process dynamics, which is approximately adjustable. Similarly second one is by explicitly deriving formulas considering various parameters of the process model with respect to dynamics. The controller design has to set prediction horizon P, control horizon M, weights on the output Q, weights on the rate of change of input  $\lambda$ , the reference trajectory parameter  $\alpha$  and some constant parameters[5]-[7].

These are tuned by two fundamental concepts:

Act of tuning is primary to develop appropriate parameters for process model. If the model exhibits poor performance, then continue tuning till the model satisfies the output requirements. If model is accurate enough, then rest of tuning is not essential.

Substitution between robustness and performance is the second action. Most MPC users in industry design either automatic or detune controller, so that controllers always stay stable at various and different operating points.

This work is mainly focused upon modeling a four tank processor with constant inputs from two pumps however the response is stabilized through manipulation of control valves. Here the inputs  $u_1$ ,  $u_2$  are set for constant flow rate but the valve manipulation is done irrespective of the two phases using MPC controller, through different tuning methods for obtaining an optimized solution. The

main emphasis is to tune the parameters of nonlinear model Quadruple Tank Process through optimized NMPC. It provides for broad treatment of available method and also develops new methods to tune QTP.

This paper is organized as follows: section 2 gives description of four tank process. The explained formulation of MPC and stability is explained in 3 & 4. Optimization and tuning methods are given in 5& 6. The analysis and simulation results are given in section 7. Finally the conclusion is given in 8.

## 2. DESCRIPTION OF FOUR TANK PROCESS

Quadruple-tank process consists of four interconnected water tanks and two pumps as shown in Figure 2. The target is to control the level in the lower two tanks with two pumps. The process inputs are  $v_1$  and  $v_2$  (input voltages to the pumps)

and the outputs are  $y_1 = k_c h_1$  and  $y_2 = k_c h_2$  (voltages from level measurement devices). Mass balances and Bernoulli's law yield the following model [3], [4]:

$$\begin{aligned} \frac{dh_1}{dt} &= -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_1} \sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} u_1 \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_2} \sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} u_2 \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} u_2 \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} u_1 \end{aligned} \quad (1)$$

Where,  $\gamma_i$  is the flow distribution to lower and diagonal upper tank,  $A_i$  is the cross-section area,  $a_i$  is the outlet hole cross section and  $h_i$  is the water level, in tank  $i$  respectively. The voltage applied to pump  $i$  is  $u_i$  and the corresponding flow is  $k_i u_i$ .

The parameters  $\gamma_1, \gamma_2 \in (0,1)$  are determined from how the valves are set prior to an experiment. The acceleration of gravity is denoted by 'g'. The parameter values for the process are given in Table 1.

TABLE 1: Parameter Process Values

parameters	Units	Values
$A_1, A_3$	$[cm^2]$	28
$A_2, A_4$	$[cm^2]$	32
$a_1, a_3$	$[cm^2]$	0.071
$a_2, a_4$	$[cm^2]$	0.057
$k_c$	$[V/cm]$	0.5
g	$[cm/s^2]$	981

The QTP has minimum phase and non minimum phase condition under multivariable zero location at two operating points based on two phases. Normally it works either in minimum phase or non minimum phase at two operating points based on the control valve settings while the flow rate distribution is dependent on control valve settings alone.

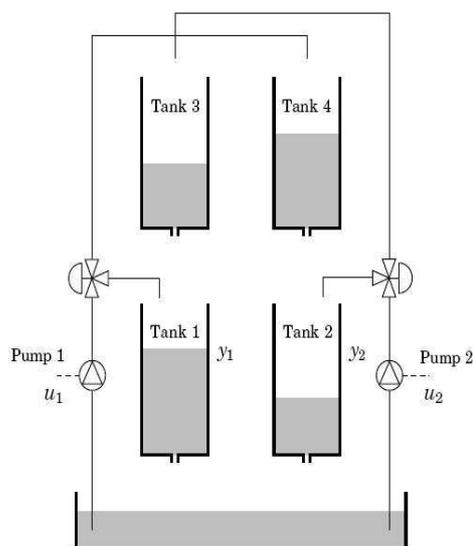


Figure 2: Diagram for Quadruple Tank Process

### 3. FORMULATION OF NMPC

Consider the non linear differential equation for stabilizing the problem [9].

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t_0) = x(0) \quad (2)$$

$$y(t) = g(x(t), u(t)) \quad (3)$$

$$u(t) \in u, \forall t \geq 0, \quad x(t) \in x, \forall t \geq 0$$

$$y(t) \in y, \forall t \geq 0 \quad (4)$$

Where  $x(t) \subseteq R^n$  and  $u(t) \subseteq R^m$  determine the vector of states and inputs. Denotes  $x$  and  $u$  are feasible set of inputs and states and  $y$  is estimated or measured output.

We assume a set of feasible assumptions i.e.,  $x$  and  $y$  as follows:

Assume 1: In its simplest form,  $u$  and  $x$  are given by constraints of the form

$$u_{\min} \leq u \leq u_{\max} \quad (5a)$$

$$x_{\min} \leq x \leq x_{\max} \quad (5b)$$

Assumption 2: The vector field  $f(x(t), u(t))$  is continuous and satisfies  $f(0,0) = 0$  at initial condition.

Assumption 3: Equation 2 has a unique continuous solution for any initial condition in the region of interest and continuous manipulated input function  $u(t) : [0, M] \rightarrow u$  and continuous predicted state function  $x(t) : [0, P] \rightarrow x$

Real systems and models are mainly used for predicting the future values within the limits selected by the controller. The finite horizontal open loop described above is mathematically formulated as follows.  $\bar{u}(t)$  is represented as internal controller

$$\min_{\bar{u}(t)} J(x(t), \bar{u}(t); M, P) \quad (6)$$

$$J(x(t), \bar{u}(t); P, M) := \int_t^{t+P} f(x(t), \bar{u}(t)) dt + \int_t^{t+M} g(\bar{u}(t)) dt$$

Subject to:

$$\bar{x}(t) = f(\bar{x}(t), \bar{u}(\tau)), \quad \bar{x}(t) = x(t) \quad (7a)$$

$$\bar{x}(t) \in x, \quad \forall t \in [t, t+P] \quad (7b)$$

$$\bar{u}(t) \in u, \quad \forall t \in [t, t+M] \quad (7c)$$

$$\bar{u}(\tau) = \bar{u}(\tau + P), \quad \forall t \in [t+M, t+P] \quad (7d)$$

### 4. STABILITY

Comparing the predicted result of open and closed loop behavior is always different. An NMPC strategy that achieves stability independent of the choice of performance measure, cost function and

constraints of model is desirable. We assume that the prediction horizon and control horizon is set such that  $P \neq M$ , and  $P < M$  will result in instability [8]. The one way to achieve stability is the use of an infinite horizon cost function, i.e.,  $P$  in Equation 6 is set at  $\infty$ . Practically as well as theoretically this may not determine the response. More appropriate and feasible condition is  $P > M$ . Similarly examine the model under non feasible conditions, where  $P = M$ ,  $P < M$ . Whereas  $P = M$  exhibits somewhat admissible response, exhibits a more aggressive response where  $P < M$ .

The input computed as the solution of NMPC optimization problem is equal to the closed loop trajectory of non-linear system at any given instance of time. Basic steps for infinite horizon proof are based on use of value functions [7], [8]. Feasibility at one sampling instance does impel for next sampling instance for the normal case.

**5. OPTIMIZED THEORY FOR MPC**

The designer needs to optimize control algorithm to minimize cost and maximize performance measure. These depend on the system variables, which are states  $x$ , output  $y$ , tracking error  $e$  and control  $u$ .

Describe the process state equation of nonlinear time invariant [12], [8] and [9]

$$\dot{x}(t) = f(x(t), u(t)) \quad (8)$$

$$y(t) = g(x(t), u(t)) \quad (9)$$

Performance function:

$$J(x(t), u(t), y(t)) = \int_{t_0}^{t_f} w_{xyu}(x, y, u) dt \quad (10)$$

Is minimized to the dynamic system which is represented as maximized performance [12]

measure for determining control law with penalty

$$h(x(t_f)) + \int_{t_0}^{t_f} f(x(t), u(t)) dt \quad (11)$$

$t_f$  =final time,  $t_0$  = initial time;  $t_0 \leq t \leq t_f$

Optimal solution to optimized problem is denoted  $u^*(t)$  and repeatedly solved at sampling instants  $t=k\delta$   $K=0, 1, 2, \dots$  for open loop control problem. Admissible optimal control law is defined for closed loop control for Equation 2 at sampling instants

$$u^*(t) = \bar{u}^*(\tau, x(t), P, M), \tau \in [t, \delta] \quad (12)$$

The optimal value of NMPC open loop optimal control as a function of the state will be denoted in the following as value function

$$V(x, P, M) = J(x(t), \bar{u}^*(t), P, M) \quad (13)$$

In a similar method, we obtain performance measure form, from equations (11) to (13)

$$J(x(t), u(t), P, M) = h(x(P)) + \int_{t_0}^{t_0+P} f(x(t), u(t)) dt + \int_{t_0}^{t_0+M} g(u(t)) dt \quad (14)$$

The admissible controls are constrained to lie in a set  $U$ ; i.e.  $u \in U$ . We first approximate the continuous operation of equation (8) by a discrete system.

$$\frac{x(t+\Delta t) - x(t)}{\Delta t} \approx f(x(t), u(t)) \quad (15)$$

$$x(t+\Delta t) = x(t) + \Delta t f(x(t), u(t)) \quad (16)$$

Shortening the above notion

$$x(k+1) = x(k) + \hat{f}(x(k), u(k)) \quad (17)$$

$$x(k+1) \approx \hat{f}(x(k), u(k)) \quad (18)$$

In a similar manner, we get performance measure form as

$$J = h(x(k)) + \sum_{k=0}^{N-1} (f(x(k), u(k)) + g(u(k))) \quad (19)$$

To minimize the deviation of the final state of system from its desired values, there are more analytical squared terms much more analytically solvable than other types. Because positive & negative deviations are equally undesirable, so absolute value could be used in quadratic form.

Using matrix notation:

$$J = x^T(k) Hx(k) + \sum_{k=0}^{N-1} (x^T(k) Qx(k) + u^T(k) Ru(k)) \quad (20)$$

$$J(x(k), u(k), T_P, T_C) = \min_0^P (x^T(k) Qx(k)) + \sum_0^M (u^T(k) Ru(k)) + x^T(k) Hx(k) \quad (21)$$

Optimized solution for equation (21) with number of intervals

$$J(x(k/k-1), u(k/k-1), P, M) = \min_0^P (x^T(k/k-1) Qx(k/k-1)) + \sum_0^M (u^T(k/k-1) Ru(k/k-1)) + x^T(k/k-1) Hx(k/k-1) \quad (22)$$

Here if, k is present, k-1 is past, if k-1 is present then k is future. Here k is described as discrete or

continuous function Q, H; R is real symmetric positive semi-definite  $n \times n$  matrix. Q is output weighted matrix and R input weighted matrix. H is solution of Ricatti equation from linear standard state space equation (23)

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \quad (23)$$

$$H = A^T H A - A^T H B (B^T H B + R)^{-1} B H A + Q \quad (24)$$

## 6. TUNING METHODOLOGY OF NMPC

MPC optimization is a function of input u and state variable x; these two parameters are tuned externally by P, M and internally by Q,  $\lambda$ , R as per equation (22). All these parameters are however described underneath in the following sections.

### 6.1 Prediction Horizon P

Different outputs will be obtained because of the input values of P as settling time, rise time is quite different. Increasing the value of P tends to minimize controller aggressiveness [6]. This section provides various techniques to tune the prediction horizon by Reference [5], [6]. The final horizon is set to be finite or infinite to ensure stability. In this case, the final horizon is described based on tuning result for closed loop stability of control system or process. List of guide lines are collected from reference [5]

$$P = N + M - 1 \quad (25)$$

$$P = t_{60} + t_{95} / T_s - 1 \quad (26)$$

$$P = t_{80} + t_{90} / 2 / T_s \quad (27)$$

$$P = t_{95} / T_s \quad (28)$$

$$\eta < P \leq t_r / T_s \quad (29)$$

$$P > M + t_d / T_s \quad (30)$$

All the parameters of any SISO or MIMO process are based upon the poles and zeros of transfer function and no of outputs as well as inputs. Stability, of course is a function of the transfer function of process. Tuning the value of P affects the stability value so as to make the system reach the steady state of the reference value, based on values of settling time and rise time. Normally the number of outputs and order are constant while the response stands different for different locations of poles and zeros. Proposed new tuning methods for NMPC of P are elaborated below.

$$P = k\eta + t_r / \eta T_s \quad (31)$$

$$P \geq \text{int}(M+C+\eta k) \pm 1 \quad (32)$$

$$t_r < P < T_p / k(\text{or}) \eta \quad (33)$$

By default  $P = 10$  is probable value of objective function, as per stability criterion P is tuned from the various parameter, like, settling time  $t_s$ , rise time  $t_r$ , no of outputs  $k$ , higher order of process  $\eta$ , no of controllers  $C$ , process response time  $t_p$ , sampling time  $T_s$ , delay time  $t_d$  and response of rise time 60,80,90,95 w.r.t  $t_p$ . P value is calculated as average of number of outputs.

## 6.2 Control Horizon M

Evaluating the value of M, if it increases in value, it tends to become more aggressive over the prediction horizon ( $M > P$ ). This is to monitor and control the response of data from output by adjusting the manipulated variable. This leads to a trade-off between increasing performance and robustness of formulation of control law, as a default control horizon is equal to 1. Formulate control horizon without more aggressiveness and existing robustness of permissible computation load. Collecting tuning methods are from reference5 and exhibit the result of the model. Implement those collected from reference [5] as listed below.

$$M = t_{60} / T_s \quad (34)$$

$$M = \text{int}(P/4) \quad (35)$$

M is different from P and oppositely working. Response of process depends on rise time and settling time. The value of M effectively tunes inputs or manipulated variable of equation (22) and is inversely proportional to the rise time and settling time and is dependent on P as well. A high value of P minimizes the effect of M on the response.

Apply new tuning methods based on the above equations, which are designed mainly based on parameter settling time  $t_s$ , rise time  $t_r$ , number of outputs  $k$ , higher order of process  $\eta$ , sampling time  $T_s$ .

$$M = \min(\text{int}(t_s/2), \text{int}(P/4)) \pm 1 \quad (36)$$

$$M = \text{int}(kn/t_s) \quad (37)$$

$$M \propto k/t_r \quad (38)$$

$$M \propto \eta/t_s \quad (39)$$

## 6.3 Output weighted matrix is represented by Q

The output variables are relatively weighted according to their significance in the process model. It provides individual significance relative to output variable, with the most important variable having a larger weight compared to others. Increasing linearly the weight on the upper limit of output to achieve a smooth response till the desired output is obtained. The elements of Q that correspond to corrected error have nonzero weight to help in relative prediction. Derived expression for the output weight for minimum phase also works for non minimum phase for the closed loop; the bandwidth is made "small" enough as explained in the reference [5] below

$$Q = C^T C \quad (40)$$

Based on above expression and output weights also consider smoothness, expression for both non minimum and minimum phase will be

$$Q < 1 \quad (41)$$

$$Q \leq \det |C^T C| \quad (42)$$

Here C is output matrix of linear state space equation.

#### 6.4 Weights on the magnitude of the inputs R

In similar fashion, R allows to be weighted for input variable according to their relative importance. R is normally considered as diagonal matrix with diagonal elements of  $rM \times rM$  matrix. It is referred as input weighting matrix or move suppression matrix. It is more convenient for tuning parameters based on parameter of  $r_{ij}$  as suppression factor [5], [6] and [10].

#### 6.5 Weights on the rate of change of inputs $\lambda$

This section discusses existing and new approaches for tuning the weights on the rate of change of inputs. Penalizing the rate of change produces a more robust controller but at the cost of the controller becoming more sluggish. Small value adjustments yield a more aggressive controller. We consider tuning guideline from reference [5], [6] as follows.

$$\lambda < 1/mP \quad (43)$$

Based on the above approach and analyzing various guidelines, even a small change exhibits overshoots, but decrease in rise time and settling time. This is compensated by output weights

$$\lambda < 1/\eta P \quad (44)$$

#### 6.6 Reference Trajectory parameters

In MPC application, reference trajectory provides the necessary path to reach final desired set point [10]. It can be specified in several different ways. It is designed between initial value and final value

between  $0 \leq \beta_j < 1$ ,  $j=1 \dots P$ .

$$\beta_j = \text{closedloopt}_s / \text{openloopt}_s \quad (45)$$

$$0 < \beta_j < 1 \quad (46)$$

### 7. SIMULATION ANALYSIS

We discussed extensively the application of non linear processes to QTP for the lower two tanks. By giving appropriate weights to the tuning system we generate conditions that are applicable for representation as for non linear differential equations. Response is plotted for step input for different tuning conditions. Responses are calculated from tuning equations for optimized solutions of NMPC for lower two tanks. It was observed that the responses of different tuning methods are based on different conditions and tuning parameters. Same rise time and settling time w.r.t P, M, Q and  $\lambda$  are obtained

The responses of non linear processes on comparison found to exhibit almost same responses for small deviation, because if P increases, at same time M also increases however the value of P minimizes the effect of aggressiveness of M. If the value of M is kept constant while the values of other parameters are varied slightly- the responses 2, 3, 4, 5 obtained remain almost same as illustrated in fig 3.

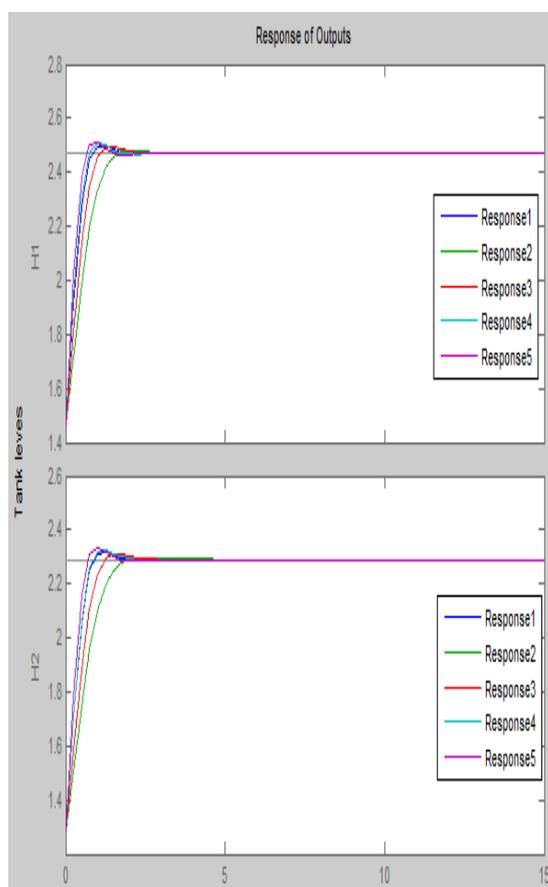


Figure 3: Response Of Lower Tanks H1, H2

Response 1 is calculated for P from equation (27), for M equation (35), for Q from equation (41), and  $\lambda$  from equation (43). Response 2 is calculated for P from equation (26), for M equation (35), for Q from equation (41), and  $\lambda$  from equation (43). Response 3 is calculated for P from equation (28), for M equation (35), for Q from equation (41), and  $\lambda$  from equation (43). Response 4 is calculated for P from equation (31), for M equation (37), for Q from equation (41), and  $\lambda$  from equation (44). Response 5 is calculated for P from equation (32), for M equation (37), for Q from equation (41), and  $\lambda$  from equation (44).

Values for responses as follows and plots response as shown in figure 3, tuning of input as shown in figure 4.

Response 1 is defined by  $P=16, M=4, C= 0.25, Q= 0.9, \lambda= 0.03125$ . Response 2 is defined by  $P=9, M=2, C= 0.25, Q= 0.8, \lambda= 0.0554$ . Response 3 is defined by  $P=11, M=2, C= 0.25, Q= 0.85, \lambda= 0.045$ . Response 4 is defined by  $P=10, M=2, C= 0.25, Q= 0.67, \lambda=0.025$ . Response 5 is defined by  $P=11, M=2, C= 0.25, Q= 0.7, \lambda=0.02273$ .

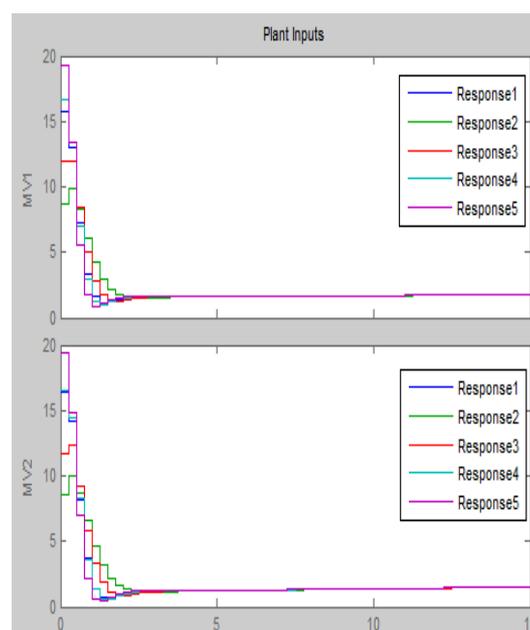


Figure 4: Tuning Of Input Parameters

As per stability criteria from section (4), the way of response for NMPC of QTP is illustrated in figure 5. Results of simulation provided very aggressive response as shown in fig 5 for unstable condition also. Because of optimality for MPC can provide somewhat aggressive response when compared with remaining conditions.

Examined the QTP for stability conditions,  $P > M$ ,  $P = M$ , and  $P < M$  is plotted for step response. Response 1, 2 & 3 is calculated for P from equation (32), M from equation (37), Q from equation (41), and  $\lambda$  from equation (44).

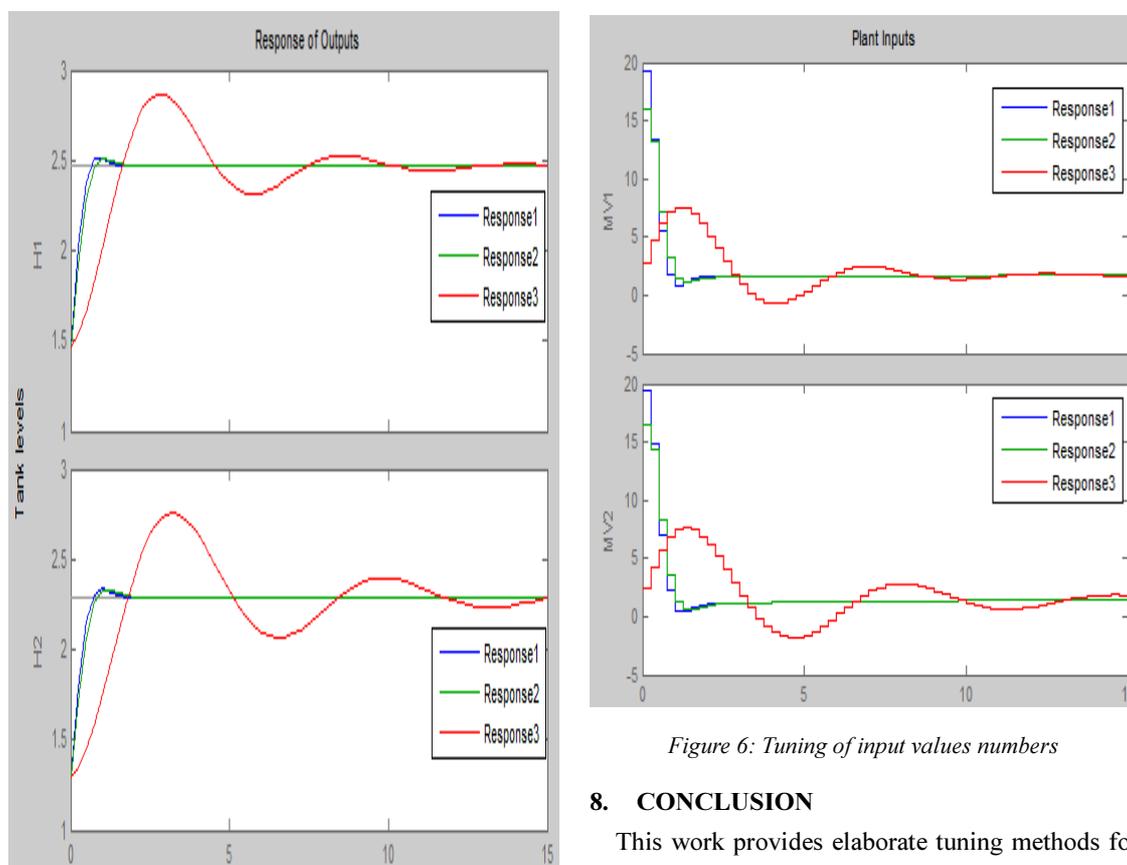


Figure 5: Response of lower tanks H1, H2.

Values for responses as follows and plotted response as shown in fig 5, tuning of input is shown in fig 6.

Response 1 is defined by  $P=11$ ,  $M=2$ ,  $C=0.25$ ,  $Q=0.7$ ,  $\lambda=0.02273$ . Response 2 is defined by  $P=11$ ,  $M=11$ ,  $C=0.25$ ,  $Q=0.7$ ,  $\lambda=0.02273$ . Response 3 is defined by  $P=2$ ,  $M=11$ ,  $C=0.25$ ,  $Q=0.7$ ,  $\lambda=0.125$ .

Figure 6: Tuning of input values numbers

## 8. CONCLUSION

This work provides elaborate tuning methods for Quadruple Tank Process through Nonlinear Model Predictive Control with several conditions and constraints. We generated different responses with minute deviations for obtaining steady state conditions. Verified all possible stability conditions based on control and predicted horizon. When control horizon is more than prediction horizon, it exhibits aggressive response. Proposed new tuning methods for NMPC, provides stable response. The usage of MPC regularly exhibits smooth response under all tuning parameters. Because optimality of MPC can be provide somewhat aggressiveness response for unstable comparing of remaining conditions.

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