INTUITIONISTIC FUZZY SOFT MATRIX THEORY IN MEDICAL DIAGNOSIS USING MAX-MIN AVERAGE COMPOSITION METHOD

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ABSTRACT

In this paper a new technique named as Intuitionistic fuzzy max-min average composition method is proposed to construct the decision method for Medical Diagnosis using different types of Intuitionistic fuzzy soft matrices and its operations. Sanchez’s approach for decision making is studied and the concept is generalized by the application of Intuitionistic fuzzy soft set theory. Through a survey the relations between the symptoms and diseases are discussed and the proposed method is compared with the existing method.

Keywords: Fuzzy Soft Sets, Intuitionistic Fuzzy Soft Sets, Intuitionistic Fuzzy Soft Matrix, Intuitionistic Fuzzy Max Min Average Composition Method.

1. INTRODUCTION


In real life most of the existing mathematical tools for formal modeling, reasoning and computing are crisp, deterministic and precise in nature. The classical crisp mathematical tools are not capable of dealing with problems in uncertainty and imprecision. There are many mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory, interval mathematics etc. Probability theory is applicable only for a stochastically stable system. Interval mathematics is not sufficiently adaptable for problems with different uncertainties. Setting the membership function value is always been a problem in fuzzy set theory. Intuitionistic Fuzzy Soft Set theory (IFSS) may be more applicable in uncertainty and imprecision. The parameterization tool of fuzzy soft set theory enhances the flexibility of its applications.

In this paper, a new approach is proposed to construct the decision method for medical diagnosis by using Intuitionistic fuzzy soft matrices. In order to make this union, intersection and the complement of a Intuitionistic Fuzzy soft matrices are applied. The result is obtained based on the maximum value in the score matrix. We apply Intuitionistic fuzzy soft set theory to develop a
technique through Sanchez’s method [13,14] to diagnose which patient is suffering from what disease.

2. PRELIMINARIES

The basic definitions of Intuitionistic fuzzy soft set theory that are useful for subsequent discussions are given.

Definition 2.1. Suppose that U is an initial Universe of discourse and E is a set of parameters, let P (U) denotes the power set of U. A pair (F, E) is called a soft set over U where F is a mapping given by F: E → P (U). Clearly, a soft set is a mapping from parameters to power set of the Universe.

Definition 2.2. Let U be an initial Universe of discourse and E be the set of parameters. Let A ⊆ E. A pair (F, A) is called a fuzzy soft set over U where F is a mapping given by F: A → I(U), where I(U) denotes the collection of all fuzzy subsets of U.

Definition 2.3. Let U be an initial universe set and E be the set of parameters. Let IF(U) denote the collection of all Intuitionistic fuzzy subsets of U. Let A ⊆ E. A pair (F, A) is called an Intuitionistic fuzzy soft set over U where F is a mapping given by

\[ F: A \rightarrow IF(U) \]

Definition 2.4. Let U = \{c1, c2, c3, ..., cm\} be the Universe set and E be the set of parameters given by E = \{e1, e2, e3, ..., en\}. Let \( A \subseteq E \) and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E). Then fuzzy soft set (F, A) in a matrix form as

\[ A = \left[ \begin{array}{cccc} a_{ij} & a_{i2} & \cdots & a_{in} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array} \right] \]

where

\[ a_{ij} = \left( \begin{array}{c} \mu_j(c_i) \\ \nu_j(c_i) \end{array} \right) \]

if \( e_j \in A \)

\[ a_{ij} = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \]

if \( e_j \notin A \)

\( \mu_j(c_i) \) represents the membership of \( c_i \) in the Intuitionistic fuzzy set \( F(e_j) \).

\( \nu_j(c_i) \) represents the non-membership of \( c_i \) in the Intuitionistic fuzzy set \( F(e_j) \).

Definition 2.5. If \( A = [a_{ij}] \in IFSM_{mxn} \), \( B = [b_{ij}] \in IFSM_{mxn} \), then we define the addition and subtraction of Intuitionistic Fuzzy Soft Matrices of A and B as

\[ A + B = \left[ \begin{array}{c} \max[\mu_j(a_{ij}), \mu_j(b_{ij})] \\ \min[\nu_j(a_{ij}), \nu_j(b_{ij})] \end{array} \right] \forall i,j \]

\[ A - B = \left[ \begin{array}{c} \min[\mu_j(a_{ij}), \mu_j(b_{ij})] \\ \max[\nu_j(a_{ij}), \nu_j(b_{ij})] \end{array} \right] \forall i,j \]

Definition 2.6. Let \( A = [a_{ij}] \in IFSM_{mxn} \), where

\[ a_{ij} = \left( \begin{array}{c} \mu_j(c_i) \\ \nu_j(c_i) \end{array} \right) \]

\( \forall i,j \). Then \( A^c \) is called a Intuitionistic Fuzzy Soft Complement Matrix if

\[ A^c = \left[ d_{ij} \right] \in IFSM_{mxn} \]

where

\[ d_{ij} = \left( \begin{array}{c} \nu_j(c_i) \\ \mu_j(c_i) \end{array} \right) \]

\( \forall i,j \).

Definition 2.7. If \( A = [a_{ij}] \in IFSM_{mxn} \), \( B = [b_{ij}] \in IFSM_{nxp} \), then the max min composition fuzzy soft matrix relation of A and B is defined as

\[ A \odot B = [c_{ik}]_{mxp} \]

where

\[ c_{ik} = \left[ \max\{ \mu_j(a_{ij}), \mu_k(b_{ij}) \}, \min\{ \nu_j(a_{ij}), \nu_k(b_{ij}) \} \right] \]

Definition 2.8. If \( A = [a_{ij}] \in IFSM_{mxn} \), \( B = [b_{ij}] \in IFSM_{nxp} \), then the max min subtraction of Intuitionistic Fuzzy Soft Matrices is defined as

\[ A - B = \left[ \begin{array}{c} \min\{ \mu_j(a_{ij}), \mu_j(b_{ij}) \} \\ \max\{ \nu_j(a_{ij}), \nu_j(b_{ij}) \} \end{array} \right] \]

∀ i,j

Example 2.9. Consider

\[ A = \begin{bmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix} \]

and

\[ B = \begin{bmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{bmatrix} \]

be the two Intuitionistic fuzzy soft matrices, then the addition, subtraction, complement, Max Min Composition and Max Min Average Composition of fuzzy soft matrix relations are

\[ A + B = \begin{bmatrix} (0.8,0.1) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{bmatrix} \]

\[ A - B = \begin{bmatrix} (0.6,0.3) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix} \]
A^c =  
\begin{pmatrix} 
0.1, 0.8 & 0.5, 0.4 \\
0.3, 0.7 & 0.6, 0.4 
\end{pmatrix} 
A \ast B =  
\begin{pmatrix} 
0.6, 0.3 & 0.8, 0.2 \\
0.6, 0.3 & 0.7, 0.3 
\end{pmatrix} 
A^\Phi B =  
\begin{pmatrix} 
0.70, 0.20 & 0.80, 0.15 \\
0.65, 0.3 & 0.75, 0.25 
\end{pmatrix} 

Definition 2.10. If A = [a_{ij}] \in \text{IFSM}_{\text{max}} , B = [b_{ij}] \in \text{IFSM}_{\text{max}} , and A^c , B^c are the complement then the score matrix of A and B is defined as S(A,B) = [V - W] 

where V is the matrix defined as 
V = [\mu(A^\Phi B) - \nu(A^\Phi B)] 
and W is the matrix defined as 
W = [\mu(A^\Phi B^c) - \nu(A^\Phi B^c)] 

3. INTUITIONISTIC FUZZY MAX-MIN AVERAGE COMPOSITION METHOD FOR DECISION MAKING:

In this section an application of Intuitionistic Fuzzy set theory using Max-min average composition method for decision making is presented.

In a given set of system, let P = \{P_1, P_2, ..., P_m\} be the set of m patients and S= \{S_1, S_2, ..., S_n\} be the set of n symptoms and D =\{D_1, D_2, ..., D_k\} be the set of k diseases.

Construct an IFSS relation matrix A called patient symptom matrix (F,S) over P where F is a mapping F : S→IF^p, IF^p is the collection of all Intuitionistic Fuzzy subsets of P.

Then construct another IFSS relation matrix (weighted matrix) B, called symptom-disease matrix, which is a collection of an approximate description of patient symptoms in the hospital (G, D) over S, where G is a mapping G : D→IF^s, IF^s is the collection of all Intuitionistic Fuzzy subsets of S. in which each element denotes the weight of the symptoms for a certain disease.

Form the matrices A and B corresponding to the Intuitionistic Fuzzy soft sets (F,E) and (G,E) and compute the complements (F,E)^c and (G,E)^c and their matrices A^c and B^c corresponding to (F,E)^c and (G,E)^c respectively.

Compute A^\Phi B and A^\Phi B^c which is the maximum membership and minimum non membership of Symptoms of the diseases using definition (2.8),

Compute A^\Phi B , A^\Phi B^c and the Score matrix S (A^\Phi B , A^\Phi B^c) using Definition 2.10.

Finally find the maximum score for each student P_i in the score matrix, and then conclude that the patient P_i is suffering from disease D_j.

3.1 ALGORITHM

Step1: Input the Intuitionistic fuzzy soft set (F,S) , (G,D) and obtain the Intuitionistic fuzzy soft matrices A, B corresponding to (F,S) and (G,D) respectively.

Step2: Using Definition 2.6, obtain the Intuitionistic fuzzy soft complement matrices A^c , B^c.

Step3: Using Definition 2.8, compute the Intuitionistic fuzzy max-min average composition
A^\Phi B and A^\Phi B^c.

Step4: Compute the matrices V, W and obtain the score matrix S (A^\Phi B , A^\Phi B^c) using Definition 2.10.

Step5: Identify the maximum score S_i,j for each patient P_i, Then we conclude that the patient P_i is suffering from disease D_j.

4. CASE STUDY

Suppose the test results of four patients P = \{P_1, P_2, P_3, P_4\} as the universal set where P_1, P_2, P_3 and P_4 represents patients Amity, John, Peter, and Ram with symptoms S = \{s_1, s_2, s_3, s_4, s_5\} as the set of symptoms where s_1, s_2, s_3, s_4, s_5 represents symptoms temperature, headache, cough, stomach problem and body pain respectively for the case study. Let the possible diseases relating to the above symptoms D = \{D_1, D_2, D_3\} be viral fever, typhoid and malaria.

Suppose that IFSS (F, S) over P, where F is a mapping F : S→IF^p, gives a collection of an approximate description of patient symptoms in the hospital.
Then the Intuitionistic fuzzy soft complement matrix

\[
A^c = \begin{bmatrix}
(0.8, 0.1), (0.8, 0.1) \\
(0.0, 0.8), (0.8, 0.0) \\
(0.0, 0.8), (0.8, 0.0) \\
(0.0, 0.8), (0.8, 0.0)
\end{bmatrix}
\]

Then the max-min average composition method matrices are (Using Definition 2.8)

\[
D_1 = \begin{bmatrix} 0.2, 0.6 \\ 0.5, 0.3 \\ 0.5, 0.4 \\ 0.7, 0.1 \\ 0.7, 0.1 \end{bmatrix},
D_2 = \begin{bmatrix} 0.2, 0.6 \\ 0.6, 0.2 \\ 0.6, 0.2 \\ 0.7, 0.2 \\ 0.7, 0.2 \end{bmatrix},
D_3 = \begin{bmatrix} 0.4, 0.5 \\ 0.7, 0.2 \\ 0.3, 0.4 \\ 0.8, 0.1 \\ 0.8, 0.2 \end{bmatrix}
\]

Intuitionistic fuzzy max-min average composition method is (Using 2.10)

\[
V = \begin{bmatrix} 0.55 \\ 0.10 \\ 0.55 \\ 0.45 \end{bmatrix},
W = \begin{bmatrix} 0.70 \\ 0.65 \\ 0.65 \\ 0.40 \end{bmatrix}
\]
It is clear from the above matrix that patients Amity, John, Peter (p1, p2, and p3) is suffering from malaria (D3) and p4 is suffering from typhoid (D2).

5. CONCLUSION

It is seen that the max-min average composition method and max min composition method [10, 11] gives the same maximum score in the score matrix of the patients and the diseases. The doctors agree that Amity, John and Peter are suffered from malaria (D3) whereas the max score of John is 0.55, p4 is suffering from typhoid. Compared with conventional techniques, the proposed approach in medical diagnosis effectively reduces the repetition. For example repetition occurs in the fourth row of the score matrix when the membership value of a symptom in A is 0.6, but in proposed method it does not occurs. As a result, our approach makes it possible to introduce weights for all symptoms and reduces the confusion about the possibility of two diseases in a patient and also it is an efficient tool for decision making problem.

REFERENCES: