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A NOVEL DIFFERENCE EQUATION REPRESENTATION FOR AUTOREGRESSIVE TIME SERIES

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ABSTRACT

The major components of the time series are the long term trend, the short term trend, cyclic variation and irregular fluctuations. Various attempts have been made to give necessary conditions for processing the specific components. Here we take necessary conditions to predict the asymptotic behavior of the time series using second order difference of the combinations of observations obtained from a general time series. Specific illustrations are given to authenticate our claim.
Keywords: *Secular Trend, Time Series, Difference Equations, Second Order Forward Difference, Asymptotic Behavior, Numerical Data.*
Subject classification: 37M10, 39A10.

1. INTRODUCTION

J.Neyman[10]observed "currently in the period of dynamic indeterminism in science, there is hardly a serious piece of research, which, if treated realistically, does not involve operations on stochastic processes". Deterministic and probabilistic models have been studied to real time data of time series by M.G.Kendall[5] and average processes which help one to predict long time trend of the time series. This connects a linear combination of any n consecutive observations of a given time series. In this paper, the model such linear combinations as a result of second order difference of any two linear combination of a given series. This is an outcome of a decay study of the asymptotic behavior of sequence under suitable combination of second order functional difference. This is an extension of the papers studied by Mei-RongXu and et al.[15], and Yu-Ping Zhao and Xi-Lan Liu[17]. The relation suggested by this study paves a way to fit even order first type reciprocal equation and their solutions, which explains claims made in our lemma. Specific illustrations on thedifference equation suggest the valid models for the auto regressive processes. Here we consider the second order neutral delay difference equationwith new conditions. R.P.Agarwal[1], R.P.Agarwal and et al[2]. discussed the general theory of difference equations. Many references to some applications of the difference equations discussed by Walter G.Kelley and Allan C.Peterson[4].

This paper is organized as follows: In section 2, we give basic concepts and results. Models on time series and our novel results in the asymptotic behavior are given in section 3. Section 4 deals with illustration for time series model and difference equations. Last section gives our contribution and future work in this direction.

2. BASIC CONCEPTS AND RESULTS

We consider the second order neutral delay difference equations of the form

$$\Delta^2\left(x_n + p_i x_{n-\tau_i} - q_j x_{n-\sigma_j}\right) + f\left(n, x_{n-l}\right) = 0, \quad (1)$$

where $p_i \geq 0, \tau_i \geq 1$, $q_j \geq 0$, $\sigma_j \geq 1$, for i, j, n $\in$N = {0, 1, 2, ...}, l $\in${−s, ..., 0}, s = max {τ, σ}, $\tau = \max_{0\leq i<\infty}\{\tau_i\}$, $\sigma = \max_{0\leq j<\infty}\{\sigma_j\}$, Δ is the forward difference operator defined by $\Delta x_n = x_{n+1} - x_n$ and the continuous function *f*: N − {0} × R → R is increasing or decreasing in x and y and *f*(x, y) y >0, for y ≠ 0.

We use the following notations throughout, N = {0, 1, 2, ...}, the set of natural numbers including zero;

N (a) = {a, a + 1, a + 2, ...}, where a $\in$N.

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Many authors [6, 12, 13, 14, 16] have studied the cases of $p_i \equiv 0$ and f is increasing, the author [11] has studied the cases of $q_j \equiv 0$. Few authors [6, 17] have studied the cases of $p_i \neq 0$ in the first order difference equations. Here we consider the equation (1) with the following assumptions:
(C1) $p_i = 0$, $0 < q_j \leq 1$,
(C2) $p_i = 0$, $q_j > 1$,
(C3) $p_i > 0$, $0 < q_j \leq 1$,
(C4) $p_i > 0$, $q_j > 1$,
(C5) $\liminf_{n\to\infty} \sum_{s=M}^{n-1}\sum_{t=s}^{n-1} f\left(t, \frac{\varepsilon}{2}\right) = 0$, for M >0.

Definition 2.1: By a solution of equation (1), we mean a real sequence $\{x_n\}$ which is defined for all k $\geq \min_{k\in N(1)}\{\tau_k, \sigma_k\}$ and satisfies equation (1) for sufficiently large k ∈N (a), a ∈N. A nontrivial solution $\{x_n\}$ of equation (1) is saidto be nonoscillatory if it is either eventually positive or eventually negative, and otherwise it is oscillatory.An equation is oscillatory if all its solutions are oscillatory.

Definition 2.2:A series of observations x(t), t∈T made sequentially in time t constitutes a time series. Examples of data taken over a period of time are found in abundance in diverse fields such as meterology, geophysics, biophysics, economics, commerce, communication engineering systems analysis etc. Daily records of rainfall data, prices of a commodity etc. constitute time series. The variate t denotes time, i.e., changes occur in time. But this need not always be so. For example, the records of measurements of the diameter of a nylon fibre along its length (distance) t also give a time series. Here t denotes length.

Definition 2.3: The essential fact which distinguishes time series data from other statistical data in the specific order in which observations are taken. While observations from areas other than time series are statistically independent, the successive observations from a time series may be dependent, the dependence based on the position of the observation in the series. The time t may be a continuous or a discrete variable. A general mathematical model of the time series Y(t), t∈T is given as Y(t)=f(t)+X(t).Here f(t) represents the systematic part and X(t) represents the random part. These two components are also known as signal and noise respectively. The model is theoretical: f(t) and X(t) are not separately observable. While the model for Y(t) gives the structure of the generating process, a set of observations(or time series data) is a realization of a sample function of the process. The effect of time may be in both the systematic and the random parts.

We shall use the following propositionsfor model I.

Proposition 2.1: Consider the difference equation
$\Delta^2 (x_n + p_n x_{n-k}) + q_n \max_{[n-l,n]} x_s = 0.$ (2)
Let $z_n = x_n + p_n x_{n-k}$. Let the following conditions hold:
(H1) k and l are nonnegative integers,
(H2) $\{p_n\}$ is a real sequence,
(H3) $\{q_n\}$ is a sequence of nonnegative real numbers,
(H4) $\sum_{s=n_0}^{\infty} q_s = \infty$, and
$p_1 \leq p_n \leq p_2 \leq -1$. Then the following assertions are valid:
(a) If $x_n > 0$ eventually, then either $z_n < 0, \Delta z_n < 0$ and $\Delta^2 z_n \leq 0$,eventually and $\lim_{n\to\infty} z_n = \lim_{n\to\infty}\Delta z_n = -\infty$ or $z_n < 0$, $\Delta z_n > 0$ and $\Delta^2 z_n \leq 0$, eventually and $\lim_{n\to\infty} z_n = \lim_{n\to\infty}\Delta z_n = 0$.
(b) If $x_n < 0$ eventually, then either $z_n > 0$, $\Delta z_n > 0$ and $\Delta^2 z_n \geq 0$,eventually and $\lim_{n\to\infty} z_n = \lim_{n\to\infty}\Delta z_n = \infty$ or $z_n > 0, z_n < 0$ and $\Delta^2 z_n \leq 0$,eventually and $\lim_{n\to\infty} z_n = \lim_{n\to\infty}\Delta z_n = 0$.

Proposition 2.2: Let conditions H in proposition 1.1 hold and $-1 < p \leq p_n \leq 0$. If $\{x_n\}$is a nonoscillatory solution of equation (2), then $\lim_{n\to\infty} x_n = 0$.

3. ASYMPTOTIC BEHAVIOR OF DIFFERENCE EQUATION IN TIME SERIES

3.1. Asymptotic Behavior of Difference Equation

Theorem 3.1.1: If one of the conditions (C1) and (C3) is satisfied alongwith (C4), then every nonoscillatory solution of the equation (1) tends tozero as n → ∞. If the condition (C2) is satisfied, then every nonoscillatorysolution of equation (1) tends to ∞ or −∞ as n → ∞.
Proof: Without loss of generality we may assume that $\{x_n\}$ be an eventually positive solution of equation (1). Then there exists n_1∈N (1) such that $x_n > 0$, for n ∈N (n_1). It follows that $x_{n-\tau_i}$, $x_{n-\sigma_j}$ and $x_{n-l} > 0$, for n ∈N(n_2), i,j ∈N, where $n_2 = n_1 + s$, s = max {τ, σ}.

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Let $z_n = x_n + p_i x_{n-\tau_i} - q_j x_{n-\sigma_j}$, for n ∈N ($n_2$). (3)

First we consider the condition (C1). Then from equation (3), we have

$z_n = x_n - q_j x_{n-\sigma_j}$, for n ∈N ($n_2$) . (4)

It follows from equation (1.1) that

$\Delta^2 z_n = -f(n, x_{n-l}) < 0$, for n ∈N ($n_2$). (5)

By proposition2.1, we have z_n<0. We claim $\lim_{n\to\infty} x_n = 0$.

Case(i): Suppose $f$is increasing. Then by proposition 2.2, we attain our claim.

Case(ii) Suppose f>0 is decreasing. We assume that $\lim_{n\to\infty} x_n \neq 0$. Then there exists aninfinite subsequence $\{n^{(i)}\} \subset \{n\}$ such that $\lim_{n\to\infty} x_{n^{(i)}-l} = \varepsilon > 0$. Therefore we can find a sequence of subsets N (i) ⊂N such that $n^{(i)}$∈N(i), fori ∈N (1). So, there exists i_1∈N (i) such that $x_{n-l} > \frac{\varepsilon}{2}$, for n ∈N ($i_1$) .Thus

$f(n, x_{n-l}) < f\left(n, \frac{\varepsilon}{2}\right)$, for n ∈N ($i_1$). (6)

Now, inequalities (5) and (6) implies that

$$\Delta^2 z_n > -f\left(n, \frac{\varepsilon}{2}\right).$$

Summing the above inequality from M ∈N (n_2) to n − 1, we obtain

$$\Delta z_n > -\sum_{s=M}^{n-1} f\left(s, \frac{\varepsilon}{2}\right).$$

Again summing from M ∈N (n_2) to n − 1, we have

$$z_n > -\sum_{s=M}^{n-1}\sum_{t=s}^{n-1} f\left(t, \frac{\varepsilon}{2}\right).$$

By the condition (C5), we see that z_n>0 as n → ∞. This is a contradiction to the proposition 2.1. Thus $\lim_{n\to\infty} x_n = 0$.

Next, we consider the condition (C3). In this case $z_n$is in equation (3) and consequently from equation (1), we have

$\Delta^2 z_n = -f(n, x_{n-l}) < 0$, for n ∈N ($n_2$).

To prove z_n<0, for n ∈N (n_2). Suppose that $z_n \geq 0$, for n ∈N (n_2).

Then there exists n_3∈N (n_2) and k >0 such that $z_n \geq$ k. Therefore, fromequation (3), we have

$x_n \geq k - p_i x_{n-\tau_i} + q_j x_{n-\sigma_j}$, for n ∈N ($n_3$). (7)

Now two cases arise.

Suppose $\{x_n\}$ is unbounded.i.e., $\limsup_{n\to\infty} x_n = \infty$.

Then there exists a subsequence $\{n_u\}_{u=1}^{\infty}$ ⊂N such that $n_u \geq n_3 + \sigma$ and$n_u \to \infty$ as u → ∞ and $x_{n_u} = \max_{l}\{x_{n_u - l}\}$. In view of the inequality(7), we have

$x_{n_u} \geq k - p_i x_{n_u-\tau_i} + q_j x_{n_u-\sigma_j}$.

$\Rightarrow -x_{n_u} \leq -k + p_i x_{n_u-\tau_i} - q_j x_{n_u-\sigma_j} \leq p_i x_{n_u}$.

⇒$p_i \geq -1$, which leads to a contradiction.

Suppose $\{x_n\}$ is bounded, i.e., $\limsup_{n\to\infty} x_n = \varepsilon < \infty$.

Then there exists a subsequence $\{n_u^*\}_{u=1}^{\infty}$ ⊂Nsuch that $n_u^* \to \infty$ and $x_{n_u^*} \to \varepsilon$as u → ∞ and $x_{n_u^*} = \max_{l^*}\{x_{n_u^*-l^*}\}$. Then, we have $\limsup_{u\to\infty} x_{n_u^*} \leq \varepsilon$.

In view of the inequality (7), we have

$x_{n_u^*} \geq k - p_i x_{n_u^*-\tau_i} + q_j x_{n_u^*-\sigma_j}$.

Taking the superior limit as u → ∞, we obtain ε≥ k − p_iε+ q_jε.

⇒ −ε≤ −k + p_iε− q_jε≤ p_iε.

⇒$p_i \geq -1$, which also leads a contradiction.

In both cases we obtain thecontradiction to the assumption $z_n \geq 0$. Thus z_n<0. The proof of theremaining is the same as in the first part and hence we omit it.

Finally, we consider the condition (C2). Then from the first part of theproof, we see that the equation (4), inequality (5) and hence z_n<0 hold.

Now we prove the claim $\lim_{n\to\infty} x_n = \infty$by the following contradiction that it is impossible that $\lim_{n\to\infty} x_n = 0$. Suppose that $\lim_{n\to\infty} x_n = 0$. Then from equation (4), we obtain $\lim_{n\to\infty} z_n = 0$. Therefore from equation (4), we have

$x_n - q_j x_{n-\sigma_j} < 0$, for n ∈N ($n_2$).

Now, let us define $x_{n_\gamma} = \min_{0\leq j<\infty}\{x_{n-\sigma_j}\}$.

Therefore we obtain $x_n \leq q_j x_{n_\gamma}$. Taking limit as n → ∞, we obtain q_j>0.This leads a contradiction to q_j>1.

Next, we shall show that it is possible that $\lim_{n\to\infty} x_n = \infty$. Suppose that $\lim_{n\to\infty} x_n \neq \infty$. Then there exists a subsequence $\{n^{(i)}\}$⊂N such that $0 < \lim_{n\to\infty} x_{n^{(i)}-l} = \varepsilon < \infty$. Thus there exists n_3∈N (n_2) and i_1∈N (1) such that

n(i) ∈N (n_3) and $x_{n^{(i)}-l} > \frac{\varepsilon}{2}$, for i∈N ($i_1$). Now, we define $x_{n-\alpha} = \max_{0\leq j<\infty}\{x_{n-\sigma_j}\}$. Then from the equation (4), we obtain

$$\frac{z_n}{x_{n-\alpha}} = \frac{x_n}{x_{n-\alpha}} - \frac{q_j x_{n-\sigma_j}}{x_{n-\alpha}}.$$

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$\Rightarrow q_j \geq -\frac{z_n}{x_{n-\alpha}}$.⇒$q_j \geq 0$.This leads to a contradiction to the fact that $q_j \geq 1$.
Thus in both cases,we proved that the solution of equation (1) tends to ∞ as n → ∞.The similar arguments are used for proving the result while $\{x_n\}$ is eventually negative and hence we omit it. Hence the theorem is completely proved.

Corollary 3.1.1: Assume that $f(n, v) \geq r_n v^{\beta}$,for all v >0, where β is the ratio of odd positive integers such that
0 < β <1 and r_n>0, for all n ∈N. Suppose that for any sequence of subsets N (i) ⊂N , $\sum_{n\in N(i)} r_n = \infty$ .In addition to the condition (C4), every nonoscillatory solution $\{x_n\}$ of equation (1) satisfies either $\liminf_{n\to\infty} x_n = 0$, or $\limsup_{n\to\infty} x_n = \infty$.
Proof: Without loss of generality we may assume that $\{x_n\}$ be an eventually positive solution of equation (1). Then there exists n_1∈N (1) such that x_n>0, for n ∈N (n_1). It follows that $x_{n-\tau_i}$, $x_{n-\sigma_j}$ and

$x_{n-l} > 0$, for n ∈N (n_2),
i,j ∈N, where $n_2 = n_1 + s$, s = max {τ, σ}. Consider the condition (C4), and then from equation (3), we have the same equation (3) with the condition (C4). It follows from equation (1) that the same inequality (5) with condition (C4).Summing the new inequality (5) from M >0 to n − 1, we obtain

$\Delta z_n < \Delta z_M - \sum_{s=M}^{n-1} r_s x_{s-l}^{\alpha} < 0$, for n ∈N ($n_2$).

There exists ς >0 such that $\Delta z_M \leq \varsigma$, for n ∈N ($n_2$). Again summing the above inequality from M >0 to n − 1, we obtain

$z_n < \sum_{s=M}^{n-1}\left(\varsigma - \sum_{t=s}^{n-1} r_t x_{t-l}^{\alpha}\right) < 0$, for n ∈N ($n_2$).

From the above two inequalities, we have $\Delta z_n<0$ and $z_n<0$ respectively,for n ∈N (n_2). Thus $\liminf_{n\to\infty} x_n = 0$ and $\limsup_{n\to\infty} x_n = \infty$ are follows from the proposition 2.1 and proposition2.2. Hence the corollary is completely proved.

3.2. Auto Regressive Process (AR Process)

The process {X(t)} given by

$X_t+b_1X_{t-1}+b_2X_{t-2}+\ldots+b_nX_{t-n}=e_t,\ b_n\neq 0,$ (8)

where$\{e_t\}$is purely random process, with mean 0, is called an autoregressive process of order n.X_t can be obtained as a solution of the linear stochastic difference equation

$g(B)X_t=e_t$, where $g(B)=\sum_{r=0}^{n} b_r B^r$, b_0=1. (9)

Suppose that $g(B)= \prod(1-z_i B)$, $z_i \neq z_j$, i.e., $z_1^{-1},\ldots,z_n^{-1}$ are the distinct roots of the equation g(z)=0. Further suppose that $|z_i| < 1$ for all i, i.e., all the roots of g(z)=0 lie outside the unit circle; the roots z_i of the characteristic equation $f(z) \equiv \sum_{r=0}^{n} b_r z^{n-r} = 0$ (where $f(z) = z^{-n} g(z^{-1})$) all lie within the unit circle. The complete solution of (9) can be written as

$X_i = \sum_{r=1}^{n} A_r z_r^t + \frac{1}{g(B)} e_t$, where A_r's are constants.

Now

$$\frac{1}{g(B)} e_t = \prod_{i=1}^{n} (1 - z_i B)^{-1} e_t$$
$$= \sum_{r=0}^{\infty} b'_r B^r e_t = \sum_{r=0}^{\infty} b'_r e_{t-r},\ b_0 = 1,$$

where b'_r are constants involving z_i's.

If the process is considered as begun long time ago, then the contribution of $\sum_{r=0}^{n} A_r z_r^t$ damps out of existence.X_t is then given by

$X_t = \sum_{r=0}^{\infty} b'_r e_{t-r}$. (10)

Thus an AR process can be represented by an MA process of infinite order.

The coefficients b'_r of e_{t-r} in the right hand side of (10) can also be obtained as follows. Using the expressions for X_t as given in (10), for t, t-1,…,t-n and then substituting in (8), we get

$$\sum b'_r e_{t-r} + b_1 \sum b'_r e_{t-1-r} + b_2 \sum b'_r e_{t-2-r} + \ldots + b_n \sum b'_r e_{t-n-r}.$$

Equating the coefficients of e_t, e_{t-1},... from both sides, we get.

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$$\left.\begin{aligned} b'_0 &= 1 \\ b'_1 + b_1 b'_0 &= 0 \\ &\dots \dots \\ b'_{n-1} + b_1 b'_{n-2} + \dots + b_{n-1} b'_0 &= 0 \end{aligned}\right\} \qquad (11)$$

and

$$b'_r + b_1 b'_{r-1} + \dots + b_n b'_{r-n} = 0, \text{ r=n, n+1,\dots} \qquad (12)$$

In otherwords, b'_r satisfy the difference equation (12) together with the initial conditions (11). Thus, b'_r can be obtained by solving (12) with the help of (11).

We can put the result in the form of a theorem as follows:

Theorem 3.2.1: If the roots of the equation $f(z) = z^n + b_1 z^{n-1} + \dots + b_n$ all lie within the unit circle, then the autoregressive process

$X_t+b_1X_{t-1}+b_2X_{t-2}+\dots+b_nX_{t-n}=0$, can be represented as an infinite moving average $X_t = \sum_{r=0}^{\infty} b'_r e_{t-r}$., where b'_r are the roots of the difference equation $b'_r + b_1 b'_{r-1} + \dots + b_n b'_{r-n} = 0$, r=n, n+1,…. subject to the initial conditions $b'_0 = 1$, $b'_1 + b_1 b'_0 = 0$,…, $b'_{n-1} + b_1 b'_{n-2} + \dots + b_{n-1} b'_0 = 0$.

4. ILLUSTRATIONS

We give the following illustrations to authenticate our claim.

Illustration 4.1: Consider the difference equation

$$\Delta^2\left(x_n - \frac{1}{3}x_{n-3}\right) + \frac{10}{3}x_{n+3} = 0. \qquad (13)$$

Here p_i= 0, 0 <$q_j=\frac{1}{3}$<1, σ_j= 3, $f(n, x_{n-l}) = \frac{10}{3}x_{n-l}$, l = −3.

Here equation (13) can be written as

$$b_5 x_{n+5} + b_4 x_{n+4} + b_3 x_{n+3} + b_2 x_{n+2} + b_1 x_{n+1} + b_0 x_n = c \qquad (14)$$

with the conditions $-b_1 = \frac{b_0}{2} = -c = \frac{1}{3}, b_2 = \frac{-b_3}{2} = b_4 = 1, \text{and } b_5 = \frac{10}{3}$.

This is the representation of the difference equation in autoregressive process.

Conditions (C1), (C5) of theorem 3.1.1 are satisfied. Hence all solutions of equation (13) are nonoscillatory.

In fact, $\{x_n\} = \left\{\frac{1}{2^n}\right\}$ is one such solution of equation (13).

Illustration 4.2: Consider the difference equation

$$\Delta^2\left(x_n + \frac{1}{2}x_{n-1} - \frac{3}{5}x_{n-2}\right) + \frac{(1-e)^2(6e^2 - 5e - 10)}{10}x_{n+2} = 0 \qquad (15)$$

Here $p_i=\frac{1}{2}$, 0 <$q_j=\frac{3}{5}$<1, τ_i= 1, σ_j= 2, $f(n, x_{n-l}) = \frac{(1-e)^2(6e^2 - 5e - 10)}{10}x_{n-l}$, 2.5 < e <2.9, l = −2.

Condition (C3) of theorem 3.1.1 is satisfied.Hence all solutions of equation (15) are nonoscillatory.

In fact, $\{x_n\} = \left\{\frac{1}{e^n}\right\}$ is one such solution of equation (15).

Illustration 4.3: Consider the difference equation

$$\Delta^2(x_n - 12x_{n-2}) + \frac{1}{2}x_{n+2} = 0. \qquad (16)$$

Here p_i= 0, q_j= 12>1, σ_j= 2, $f(n, x_{n-l}) = \frac{1}{2}x_{n-l}$, l = −2.

Condition (C2) of theorem 3.1.1 is satisfied. Hence all solutions of equation (16) are nonoscillatory.

In fact, $\{x_n\}$ = $\{2^n\}$ is one such solution of equation (16).

Illustration 4.4: Let {X(t)} be an MA process of order n given by

$X_r=a_0e_t+a_1e_{t-1}+\dots+a_ne_{t-n}$, $a_n\neq0$,

where $\{e_t\}$ is a purely random process. If the roots of the characteristic equation $z^n+a_1z^{n-1}+\dots+a_n=0$all lie within the unit circle, then X_t can be represented as an autoregressive process of infinite order $\sum_{r=0}^{\infty} c_r X_{t-r} = e_t$,where the coefficients c_r satisfy the difference equation $c_r + a_1 c_{r-1} + \dots + a_n c_{r-n} = 0$,

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r=n, n+1,….and the initial conditions $c_0 = 1$. $c_1 + a_1 = 0$, $c_1 + a_1 c_1 + a_2 = 0$,.., $c_{n-1} + a_1 c_{n-2} + ... + a_{n-1} = 0$.

5. CONCLUTION AND FUTURE WORK

In this paper, we have presented necessary conditions to predict the asymptotic behavior of the time series using second order difference of the combinations of observations obtained from a general time series. Specific illustrations are given to authenticate our claim. In future, we will consider the generalization this model to higher order difference equation to study time series analysis.

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