PERFORMANCE COMPARISON OF PARTITIONING PTS BASED PAPR REDUCTION OF OFDM SYSTEMS UNDER DIFFERENT MODULATIONS TECHNIQUES

1,7 ZEYID T. IBRAHEEM, 1,8 MD. MIJANUR RAHMAN, 1,6 S. N. YAAKOB,
1,5 MOHAMMAD SHAHRazel RAZALLI, 1,6 SAMEER A. DAWOOD, 2,8 KAWAKIB K. AHMED

1School of Computer and Communication Engineering, University Malaysia Perlis (UniMAP), Pauh Putra, 02600 Arau, Perlis-Malaysia
2Inter Net Works Research Group, School of Computing, University Utara Malaysia (UUM), Kedah, Malaysia

E-mail: \(^7\) zeyidtariq@yahoo.com, \(^8\) mijanur@unimap.edu.my, \(^6\) shahrulnimizam@unimap.edu.my,
\(^5\) shahrazel@unimap.edu.my, \(^1\) sameerakram_iraq@yahoo.com, \(^8\) Kawakib Khadyair@yahoo.com

ABSTRACT

Partial Transmit Sequence (PTS) technique is widely employed to mitigate the peak-to-average-power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems without any distortion. The crucial step in any PTS system is partitioning of the OFDM frame into disjoint sub-blocks. Adjacent partitioning (AP) is a rather simple partitioning scheme achieving attractive PAPR reduction performance in trade-off between cost and performance. In this paper, Finite Radon Transform (FRAT) is implemented as a modulation technique for data mapping and a comparative analysis of the performance of FRAT is carried out against that of ordinary data mapping techniques such as phase shift keying (PSK) and quadrature amplitude modulation (QAM). The research objective is to find the most appropriate modulation scheme for OFDM system with PTS scheme. In order to perform the comparative analysis, the FRAT as well as PSK and QAM modulation techniques was implemented for the purpose of data mapping. Another dimension for comparative analysis was PTS partition length variability. Adjacent PTS scheme was implemented for both fixed length and variable length partitioning for the aforesaid modulation techniques. The results obtained for all the scenarios were investigated. These modulation schemes were also tested for the case of interleaved PTS scheme. Simulation results with different partitioning scenarios showed that the ordinary mapping for any types of techniques (PSK or QAM) had better PAPR reduction performance compared with FRAT.

Keywords: orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), partial transmit sequences (PTS), sub-block partitioning, Finite Radon Transform (FRAT)

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) system has large advantages over frequency selective channels of having capability of transmitting high data bit rates and simple equalization [1]. OFDM provides a power efficiency, high spectral efficiency, multopath delay spread tolerance, and immunity to the frequency selective fading channels [2]. Thus, OFDM has been proposed in different wireless communication standards such as digital audio/video broadcasting, the ETS1 HIPERLAN/2 standard, IEEE 802.11a standard for wireless Local Area Networks (WLAN), and IEEE 802.16a standard for Wireless Metropolitan Area Networks (WMAN) [3, 4]. However, one major drawback of OFDM is high peak to average power ratio (PAPR) in the transmitted signal. A high PAPR not only limit the application of OFDM transmission systems but also to degrade efficiency of a linear power amplifier [5]. To counteract this well known problem, various PAPR reduction schemes have been proposed in the literatures [6], such as clipping and filtering [7],
block coding [8], selective mapping (SLM) [9], non linear companding methods [10], tone reservation (TR) [11], tone injection (TI) [12], active constellation extension (ACE) [13], and partial transmit sequences (PTS) [14, 15]. Each of these schemes has a different cost for bit error rate (BER) and the reduced PAPR. Among these schemes, PTS scheme is the most efficient approach and a distortion less scheme for PAPR reduction by optimally combining signal sub-blocks [16]. In PTS technique, the input data block is split into disjoint sub-blocks. The sub-blocks are multiplied by phase weighting factors and then added together to produce transmitted signal which containing the same information. The phase weighting factors are chosen such that the resulting PAPR is minimized. In this paper, Finite Radon Transform (FRAT), which is originally used for image processing, is implemented as a modulation technique for data mapping. Then performance of this modulation technique is compared with generally used modulation schemes such as PSK and QAM. For the sake of comprehensiveness in the analysis, both adjacent and interleaved PTS schemes were implemented and tested. The rest of the paper is organized as follows: Section II presents PAPR of OFDM system, PTS technique, and FRAT. Section III presents the simulated partitioning schemes. Section IV shows the simulation results and section V draws a conclusion.

2. PAPR AND ORDINARY PTS SCHEME

A. PAPR IN OFDM SYSTEM

OFDM signal $x_n$ consists of a block of $N$ data symbols, $\{x_n, n = 1, 2, ..., N-1\}$ will be transmitted in parallel. The transmitted signals in discrete time domain are generated by IFFT operation is given as

$$x(n) = \text{IFFT}\{X\}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi kn}{N}}$$

(1)

where $0 \leq n \leq N - 1$

In general, PAPR of OFDM signal is defined as the ratio of its maximum power divided by its average power. It is expressed as [17]

$$PAPR = \frac{\text{max}|x(t)|^2}{E[|x(t)|^2]}$$

(2)

where $E[.]$ denotes the average power.

The PAPR reduction performance is evaluated using the complementary cumulative distribution function (CCDF). The CCDF is defined as the probability that the PAPR of an OFDM symbol exceeds a given threshold $PAPR_0$, which can be expressed as [18].

$$CDF(PAPR_0) = P_r(PAPR > PAPR_0)$$

(3)

B. PTS TECHNIQUE

Partial transmit sequences (PTS) shown in figure 1 is split the input data OFDM symbol of $X$ into $M$ non-overlapping sub-blocks, the size of each sub-block is equal and the location of sub-block that does not have the location of the original OFDM symbol is null value. It can be expressed as $X_m$, where $m = 1, 2, ..., M$. Therefore, is represented as [19]

$$X = \sum_{m=1}^{M} X_m$$

(4)

An Inverse Discrete Fourier Transform (IDFT) is employed for each sub-block to generate time domain candidates, which can expressed as

$$x_m = \sum_{m=1}^{M} \text{IDFT}\{X_m\}$$

(5)

And then each sub-block $x_m$ is rotated by phase factors and combined together to generate a set of candidates. The candidate with minimum PAPR is selected for transmission. Thus, the time domain signal after combination is given by
PTS sub-block partition methods can be classified into three kinds: adjacent sub-block partition, interleaved sub-block partition, and pseudorandom sub-block partition [20]. The interleaved sub-block partition method has the least PAPR reduction and pseudorandom sub-block partition method has the best PAPR reduction. In comparison we used adjacent partitioning due to it is simply implementation as well as it presented PAPR reduction performance very close to pseudorandom partitioning [21].

Figure 1: Block diagram of the traditional PTS technique

C. FINITE RADON TRANSFORM (FRAT)

The Finite Radon Transform (FRAT) is one of the discrete types of the Radon transform that used later as a new data mapping in OFDM systems. This transform is defined for 2-dimensional signals. The size of the matrix in FRAT of a two-dimensional square matrix (A), denoted by p, should be prime which can be obtained firstly by taking the 2-dimensional Fast Fourier Transform (FFT) of matrix A [22, 23]

\[ F(r,s) = \sum_{m=0}^{p-1} \sum_{n=0}^{p-1} A(m,n) e^{-j \left( \frac{2\pi}{p} \right) rm} e^{-j \left( \frac{2\pi}{p} \right) ns} \]  

(7)

After that, an element of matrix F has been re-distributed according to the optimum ordering algorithm given in [23]. Therefore, the dimensions of the resultant matrix will be p × (p + 1) and is denoted by the symbol \( F_{opt} \). Figure 2 shows the optimal ordering of the matrix of Fourier coefficients for matrix dimension p=7.

Finally, the FRAT can be obtained by taking the one dimensional inverse fast Fourier transform (1D-IFFT) for each column of the matrix \( F_{opt} \).

\[ r_i = \text{Re} \left\{ \frac{1}{p} \sum_{m=0}^{p-1} f(i) e^{j \left( \frac{2\pi}{p} \right) km} \right\} \]  

(8)

Now, the matrix \( R \) with the \( r_i \) columns represents the FRAT of A

\[ R = \begin{bmatrix} r_{0,0} & \ldots & r_{0,p} \\ r_{1,0} & \ldots & r_{1,p} \\ \vdots & \ddots & \vdots \\ r_{p-1,0} & \ldots & r_{p-1,p} \end{bmatrix} \]  

(9)

Figure 2: optimal ordering of FRAT coefficients for matrix size: \( p = 7 \)
A modification is made on R which is the matrix of FRAT coefficients, for the purpose of rising the bit per Hertz of the mapping before resizing the mapped data and that’s by construct the complex $\bar{R}$ matrix from the real matrix $R$ according to

$$\bar{r}_{lm} = r_{ij} + ir_{ij+1}, 0 \leq (i,j) \leq p$$

(10)

where $\bar{r}_{lm}$ refers to the elements of matrix $\bar{R}$, and $r_{ij}$ refers to the elements of the matrix $R$.

To reversing the above procedure gives the recovered matrix $A$, and that is by taking one dimensional fast Fourier transform (1D˗FFT), retrieving the original Fourier coefficients ordering, and then taking the two dimensional inverse fast Fourier transform (2D˗IFFT). Figure 3 shows the main procedure of taking the FRAT and IFRAT.

B. ADJACENT PARTITIONING WITH VARIABLE LENGTH

In this section, we present the framework for the analysis. Overall methodology and implementation details for different PTS techniques are discussed.

A. Methodology

In order to carry out the comparative analysis, we implemented Finite Radon Transform (FRAT) as a modulation technique for data mapping. We developed a comprehensive simulation framework which facilitated us to simulate OFDM transmission with PTS under different mapping techniques including FRAT, PSK and QAM. The simulation was repeated statistically significant number of times for each data mapping technique. CCDFs were calculated for different PTS partitioning schemes under different data mapping techniques. Finally, these CCDF measures were carefully analyzed to find the combination with minimum PAPR.

B. ADJACENT PARTITIONING WITH VARIABLE LENGTH

In this scheme, the input data OFDM symbol is first partitioned into $M$ disjoint with equal variable sub-block size. Then IFFT is separately computed for each sub-block. The output of IFFT in each sub-block is phase rotated by the rotation factors and combined to achieve PAPR as low as possible. The transmitted signal in this scheme can be represented by,

$$x(n) = \sum_{i=0}^{M-1} x_i^{(r_i)}(n)$$

(11)

where $x_i^{(r_i)}(n)$ is the phase rotated of the time domain signal for partition $P_i$ with length $L_i$. As mentioned, first variable length partitions are generated as follows.

$$X = [P_0, P_1, P_2, ..., P_{M-1}]$$

(12)

$$P_0 = [0,00000 ... 0, P_0]$$

(13a)

$$P_1 = [00000 ..., 0, P_1, 00000 ... 0]$$

(13b)

$$P_{M-1} = [00000 ..., 0, P_{M-1}]$$

(13c)

Then time domain signal is obtained by taking IFFT of these partitions as shown by Eq. (14).

$$x_i(n) = IFFT(P_i), i = 0, 1, 2, ..., M - 1$$

(14)

Phase rotated $x_i^{(r_i)}(n)$ of the time domain signals are obtained simply through multiplication by
phase factors, \( \varphi(r_i) = e^{i\theta_i} \) as in Eq. (15) where \( \theta_i \) are the rotation angles.

\[
 x_i^{(r_i)}(n) = \varphi(r_i)x_i(n) \tag{15}
\]

C. Adjacent Partitioning Scheme With Fixed Length

We repeated the procedure in step (a) with fixed length size of sub-blocks partitioning for the same input data OFDM symbol.

D. Interleaved Partitioning

In interleaved partitioning, the N subcarriers are first divided into M groups with each group having \( L = N/M \) contiguous subcarriers. Then the i-th interleaved partition is formed by assigning i-th subcarrier of each group to the i-th interleaved partition. The partitions can be represented by the following equations,

\[
P_0 = \begin{bmatrix} p_0^{(0)} & 0 & \ldots & 0 \\ p_0^{(1)} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_0^{(M)} & 0 & \ldots & 0 \end{bmatrix} \tag{16a}
\]

\[
P_1 = \begin{bmatrix} 0 & p_1^{(0)} & \ldots & 0 \\ 0 & p_1^{(1)} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p_1^{(M)} \end{bmatrix} \tag{16b}
\]

\[
P_L = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p_L^{(M)} \end{bmatrix} \tag{16c}
\]

where \( p_i^{(j)} \) is the j-th element of the i-th interleaved partitioning. Remaining steps to generate transmitted signal are similar to those of adjacent partitioning.

4. SIMULATION RESULTS

In this section, numerical simulation show the comparative performance of PTS sub-blocks partition schemes (interleaved and adjacent) for numbers of subcarriers \( N=64 \) number of sub-blocks \( M=8 \), and various types of modulation techniques including QPSK, 8PSK, 16QAM, and 64QAM that compared with another type of data mapping namely, Finite Radon Transform (FRAT). The performance will be analyzed using MATLAB version 7.8.

Figure 4, show the performance of fixed length ordinary PTS schemes and variable length adjacent scheme PTS (VL-AP) by using FRAT as a mapping technique which is compared with the conventional QPSK modulation technique based OFDM system. At \( \text{CCDF}=10^{-3} \), it can be seen that the PAPR\(_0 \) of original OFDM (signal without PTS) under FRAT mapping is 11.1 dB, original OFDM (signal without PTS) in QPSK modulation is 10.3 dB, (VL-AP) in FRAT mapping is 7.9 dB, interleaved partitioning (IP) with QPSK modulation is 7.3 dB, (VL-AP) in QPSK modulation is 7.1 dB, and adjacent partitioning fixed length (AP) with QPSK modulation is 6.9, respectively. Therefore, AP with QPSK modulation reduced PAPR by 4.2 dB, VL-AP with QPSK modulation by 4 dB, IP with QPSK modulation by 3.8 dB, VL-AP with FRAT by 3.2 dB, and original OFDM with QPSK modulation by 0.8 dB from the original signal under FRAT mapping technique. The conventional modulation QPSK for ordinary PST with fixed and variable length sub-blocks partitioning is better in PAPR reduction performance than the FRAT mapping technique.

In figure 5, the original OFDM signal for 8PSK modulation obtains much 0.2 dB of PAPR reduction than the original OFDM signal under FRAT mapping when \( \text{CCDF}=10^{-3} \). The partitioning fixed length (AP) with 8PSK modulation compared with VL-AP in 8PSK modulation, IP-PTS in 8PSK modulation, and VL-AP with FRAT reduced PAPR by 0.1 dB, 0.4 dB, 0.6, respectively.

Also, in figure 6, presents the PAPR reduction performance of FRAT and 16QAM modulation based on OFDM system when \( \text{CCDF}=10^{-3} \). The AP-PTS with 16QAM modulation scheme can obtain PAPR reduction about 0.1 dB, 0.4 dB, 1.3 dB, 3.3 dB, and 3.6 dB better than VL-AP, IP in ordinary 16QAM modulation, VL-AP with FRAT, original OFDM with 16QAM modulation, and original OFDM with FRAT, respectively. In this figure, the ordinary 16QAM modulation gives PAPR reduction performance better than FRAT based on OFDM at the same CCDF.

Figure 7, shows the PAPR reduction performance of partition schemes for using ordinary 64QAM against FRAT based on OFDM when \( \text{CCDF}=10^{-3} \). Obviously, the AP with 64QAM modulation reduced PAPR by around 4.1 dB, VL-AP with 64QAM modulation by 3.8, IP with 64QAM modulation by 3.5 dB, VL-AP with FRAT by 3 dB, and original OFDM by 0.7 dB from the original signal in FRAT.
Finally, the PAPR reduction performance for any type of ordinary modulation techniques is better than FART.

Figure 4: comparison of PAPR reduction performance of ordinary QPSK modulation with Finite Radon Transform (FRAT) based on PTS scheme

Figure 5: comparison of PAPR reduction performance of ordinary 8PSK modulation with Finite Radon Transform (FRAT) based on PTS scheme

Figure 6: comparison of PAPR reduction performance of ordinary 16QAM modulation with Finite Radon Transform (FRAT) based on PTS scheme

Figure 7: comparison of PAPR reduction performance of ordinary 64QAM modulation with Finite Radon Transform (FRAT) based on PTS scheme

5. CONCLUSION

In this paper, Finite Radon Transform (FRAT) based on OFDM is implemented as a modulation technique for data mapping and compare with the various types of conventional modulation technique such as (PSK and QAM) in order to realization of reduce the PAPR performances. We investigate and analyzed the effects disjoint with (fixed and variable length) sub-blocks partitioning in PTS technique for the performance of PAPR reduction. Through comprehensive simulation and analysis we showed that the conventional modulation techniques has better performance in PAPR reduction as compared to FRAT and with low computational complexity because of the FRAT-OFDM used two dimensional of FFT in the data mapping and in the sub-carrier modulation. Sub-blocks partitioning PTS with fixed and variable length in conventional modulations techniques have better performance compared to FRAT.

REFERENCES:


