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ALGORITHMS FOR MAGIC LABELING ON GRAPHS

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ABSTRACT

This article deals with the labeling of vertices and edges of a graph. The kind of labeling we have applied is Magic Labeling. Again Magic Labeling is expressed in-terms of Vertex Magic Total Labeling (VMTL), Edge Total Magic Labeling (EMTL) and Total Magic Labeling (TML). We have studied existing approaches for Magic Labeling and we found some improvements can be done over existing VMTL algorithms and we design algorithm to find EMTLs. We propose new and enhanced algorithms for VMTL, EMTL and TML. We applied these algorithms on different kinds of graphs like cycles, wheels, fans and friendship graphs. We found the number of such distinct labelings. Also we made comparative study of these algorithms over existing.

Keywords: *Magic Labeling, Vertex Magic Total Labeling, Edge Total Magic Labeling, Total Magic Labeling, Cycles, Wheels, Fan, Friendship graphs.*

1. INTRODUCTION

The Graphs we have considered here are finite, simple and undirected. . Let G be a graph with vertex set V and edge set E, where $V \in V(G)$ and edge set $E \in E(G)$. We denote $e = |E|$ and $v = |V|$ the standard graph theoretic notation is followed. A general reference for graph theoretic notations is [3].

General definitions of cycles, wheels, fans, Friendship graphs, magic labeling, vertex magic total labeling, edge magic total labeling, total magic labeling are as follows. Cycle is a graph where there is an edge between the adjacent vertices only and the vertex is adjacent to last one (Fig1.a). Wheel is a Cycle with central hub, where all vertices of cycle are adjacent to it (Fig1.b).Fans and Friendship Graphs are sub classes of wheels. If a path is connected to central hub it is a Fan Graph (Fig1.c). A Friendship Graph consists of n triangles with one common vertex called as hub where n is size of Friendship Graph (Fig 1.d).

Labeled graphs are becoming an increasingly useful family of Mathematical Models for a broad range of applications such as Conflict resolution in social psychology, electrical circuit theory and energy crisis, Coding Theory problems, including the design of good Radar location codes, Synch-set codes; Missile guidance codes and convolution codes with optimal autocorrelation properties and in determining ambiguities in X-ray Crystallographic analysis, to Design Communication Network addressing Systems, in determining Optimal Circuit Layouts and Radio Astronomy., etc[9].

Labeled graphs are playing vital role especially in the field of computer science. Our investigation showed its importance in networking channels [13], data mining, cryptography, SQL query solving, etc,.. This vast range of applications motivated us towards labeled graphs. Hence we focused to design algorithms for various kinds of magic labelings.

Labeling is the process of assigning integers to graph elements under some constraint. If the constraint is applied on only vertex set V then it is called vertex magic, if it is on the edge set E then it is called edge magic, if it is applied on both vertices and edges it leads to the total magic labeling. In other words labeling of a graph is a map that takes graph elements such as vertices and edges to numbers (usually non-negative integers). The most complete recent survey of graph labeling

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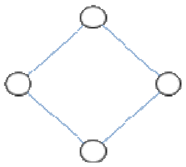
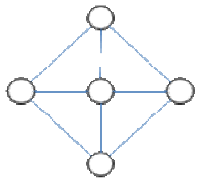
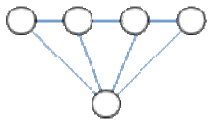
is by [2]. Different kinds of labels are discussed in [12]. Sedlacek introduced the magic labeling concept in 1963.



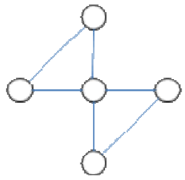
Fib1.a Cycle



Fib1.b Wheel



Fib1.c Fan Graph



Fib1.d Friend Graph

These are expressed in terms of two bijection methods. If A bijection λ_1:V∪E → {1, 2… |V| + |E|} is called a Vertex-Magic Total Labeling (VMTL) if there is a vertex magic constant v_k such that the weight of vertex m, $\lambda_1(m) + \Sigma_{n\in A(m)} \lambda_1(mn) = vk, \forall m \in V$ Where A(m) is the set of vertices adjacent to x. This labeling was introduced by McDougall et al. [4] in 2002. Another bijection λ_2:V∪E → {1, 2… |V| + |E|} is called Edge-Magic Total Labeling (EMTL) if there is a edge magic constant e_k such that the weight of an edge e_{mn}, $\lambda_2(m)+\lambda_2(n)+\lambda_2(emn)=ek, \forall e\in E$ This is described in [5,8]. By assigning different combinations of labels to vertices and edges, it is possible to construct magic labelings with different magic constants on the same graph. A lower bound for a VMTL is obtained by applying the largest |V| labels to the vertices, while an upper bound is found by applying the smallest |V| labels to the vertices. The following formula gives lower and upper bound for vertex magic constant without taking into account the structure of the graph [6].

$$\frac{13n^2+11n+1}{2(n+1)} \leq k \leq \frac{27n^2+15n+2}{2(n+1)} \quad (1)$$

This is a vertex magic constant *k* limit equation. Once the structure of the graph is taken into account, additional limits may be found. The set of integers which are delimited by these upper and lower bounds is the feasible range. The values which are the magic constant for some VMTL of a graph form the graph's spectrum. Therefore the spectrum is a subset of the feasible range. For a Cycle, these limits are given by H.R. Andersen et al. [7] in 2002 as given below.

$$\frac{5n+3}{2} \leq k \leq \frac{7n+3}{2} \quad (2)$$

Computational methods for solving labeling problem are described in [10][11]. In [09], wheel graphs, fan graphs, t-fold wheels, and friendship graphs are all investigated, and in all cases an upper limit is found for the size of such graphs which permit a vertex-magic total labeling. As well, the spectrum of all wheels is investigated, and a complete enumeration of all distinct labelings of wheels with less than six rim vertices is provided.

In order to develop this theory strong Baker and Sawada [1] gave algorithms that generate all non-isomorphic VMTLs for cycles and wheels. We studied those and proposed an enhanced algorithm for VMTL. Also we designed algorithms for generating all non-isomorphic EMTL's. By using results of these two, number of TML's can be found for each graph structure.

2. BACKGROUND

A cycle graph C_n is a connected graph where from n vertices every vertex is adjacent to exactly two other distinct vertices. A wheel graph W_n is a cycle graph C_n with central vertex called hub where all vertices if cycle are connected to hub.i.e. W_n=C_n+hub. The Fan F_n and Friendship T_nGraphs belong to wheel class where F_n=Wn-any one edge and Tn=W2*n-all even/odd edges.

Vertex Total Magic Labeling (VTML) is the assignment of labels (integers) in the range {1,2…... v+e} to components of graph such that each vertex weight ($\lambda 1$) is same and a constant generally referred as vertex magic constant (vk). Here the weight of vertex refers to the label applied to that vertex and all edges connected to it. The function A() consist all adjacent vertices. Example of VMTL on C4 is given in Fig 2.a. The expression for vertex m as follows.

$$\lambda_1(m) + \Sigma_{n\in A(m)} \lambda_1(mn) = vk, \forall m \in V$$

To find all non isomorphic VMTL's baker and scoda designed a trial and error method with back tracking. This algorithm fixes each number to a graph component and check for possible assignments of remaining numbers to other graph components. We have reduced the work by identifying and fixing a set of suitable values from

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domain. For this we used the concepts of variations and sumset sequences.

Edge Total Magic Labeling (ETML) is an assignment of labels in the range {1,2… v+e} to components of graph such that each edge weight ($\lambda 2$) is same and a edge magic constant (ek). The weight of a vertex refers the label applied to that vertex and all edges connected to it. Here is the expression for an edge (m,n) where m and n are end vertices. Example of EMTL on W4 is given in Fig.2.b.

$$\lambda_2(m)+\lambda_2(n)+\lambda_2(emn)=ek, \forall e \in E$$

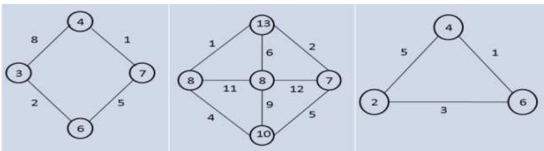

Fig 2 (a) VMTL on C_4 (b) EMTL on W_4 (c) TML on C_3

By using same concepts the variations and sumset sequences we proposed a new algorithm to find number of non-isomorphic EMTL's on different structures of graphs. A graph which consists of both VMTL and EMTL is said to be Total Magic Labeling (TML). Example of TML on C4 is given in fig.2c.

A variation which is permutation without repetition. The variations of size r chosen from a set of n different objects are the permutations of combinations of r. The equation for variation is as follows

$$V(n,r) = C(n,r).r! = \frac{n!}{(n-r)!}$$

Permutation without repetition generally referred as variations. We are also using trial and error method with back tracking method with added filters to get better results for existing and we found many other. These are explained in next session.

3. MAGIC LABELING ALGORITHMS ON CYCLES

Here in this session we deal with cycles. 3.1 give details of algorithm description to generate VMTL/EMTL on cycle. Next in 3.2 we have a module which uses above results to identify TMLs. Each algorithm is followed with results.

Cycle is a closed path/circle. Each of our solution can be VMTL/EMTL. This always depends on starting point. If the starting point is a vertex, the result produces VMTL or if the starting point is an edge, it results EMTL. Magic constant values can be counted with Equation (2).

3.1 Algorithm for VMTL on cycles

Algorithm to generate VMTL for given cycle

Input : Graph (Cycle) size n and magic constant k

Output : Generates all non-isomorphic VMTLs

Other Variables used:

Availability[x]: An array which decides whether a label is available or already used.

nP_x : results a variation nP_x, with available unused labels, whose sum is a magic constant k.

Initial assumptions: All labels are available.

Algorithm:

1. Set labels range as {1, 2……, 2*n} and domain consist all graph components..
2. for i:1 to 2*n
   if there is a (2*n)P_3
   set them as $\lambda 1$(e_n), $\lambda 1$(v_1) and $\lambda 1$(e_1)
   previous vertex=1
   current vertex=2
   for i:2 to 2*n
   if there is (2*n)p2
   set them as $\lambda 1$(vi) and $\lambda 1$(ei).
   previous vertex=current vertex
   current vertex++
   if current vertex=n and
   (available label + $\lambda 1$(vi)+ $\lambda 1$(en) =k)
   then assign label to $\lambda 1$(vi)
   display Solution.
   otherwise make the labels assigned to previous vertex as available.
   previous vertex - -
   current vertex--
   else display "Work Finished".
3. Stop.

Results This algorithm results same as sedwick.ea.al@ with less computation burden. At the beginning stage itself the total number of branches to be examined and reduced to half. So, the computation burden becomes half.

3.2 Algorithm for TML on cycle

For the solution from 3.1, which is a VMTL, let us check whether this is an EMTL for a different magic constant ek or vice versa. If this condition satisfies this is a TML.

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Input: G(V,E) with VMTL (from 3.1)

Output: prints TML if exists.

Algorithm:

1. ek= λ1(vn)+ λl(e1)+ λ1(v1)
2. count=1;
3. for i:2 to n
   if λ1(vi-1)+ λ1(ei) + λ1(vi) = ek
   count++
4. if count=n them display "TML exist"
   else display "TML not exist".

From the above module we checked the cycles, which are of different sizes (n=3to10) and we found that there is only TML with n=3 vertex magic constant as 10 and edge magic constant as 11.

4. MAGIC LABELING ALGORITHMS ON WHEELS

Wheel is a cyclic in structure which performs as central hub. The edges from hub to all vertices are termed as spokes. The total number of graph elements are 3*n+1. Here we need different algorithms for VMTL and EMTL. We designed different algorithms for these two and TML's always depends on result of these two. Magic constant values are counted with (1).

4.1 Algorithm for VMTL on Wheels

Input **:** Graph (Wheel) size n and vertex magic constant vk

Output **:** Generates all VMTLs for given graph size and magic constant.

Other Variables used:

Availability[x]: An array which decides whether a label is available or already used.

nP_x : results a variation nP_x, with available unused labels, whose sum is a magic constant vk.

Initial assumptions: All labels are available.

Algorithm:

1. set labels range as {1, 2……, 3*n+1} and domain consist all graph components.
2. for i: 1to3*n+1
   if there is a variation (3*n+1)P_n
   (it should not be an isomorphic set)
   set them as - λ1 of spokes.
   for j:1 to 3*n+1
   if there is (3*n+1)P_3, whose sum is a k-λ1 (sj).
   set them as λ1(en), λ1(v1) and λ1(e1).
   previous vertex=1
   current vertex=2
   for k:2 to n
   if there is (3*n+1)P_2,
   whose sum is k- λ1(spoke-k).
   set them as λ1(vk), λ1(ek).
   previous vertex=current vertex
   current vertex= current_vertex+1
   if current vertex=n and
   available label+ λ1(vk)+ λ1(sk)+ λ1(en)=k
   display "Got Solution"
   Otherwise
   make the labels assigned to previous vertex as available.
   previous vertex=previous_vertex-1
   current vertex= current_vertex-1
   otherwise display "Finished work".
3. Stop.

This algorithm resulted as sedwick.ea.al@ with less computation burden. The first step reduces it by n(no. of spokes). Later the procedure for this is same as the cycle which has given 50% reduction in computation effort. So this algorithm reduces total work by 2n.

4.2 Algorithm for EMTL on Wheels

Input **:** Graph (Wheel) size n and edge magic constant ek

Output: Generates EMTLs for given graph size and magic constant.

Other Variables used:

Availability[x]: An array which decides whether a label is available or already used.

nP_x : results a variation nP_x, with available unused labels, whose sum is a magic constant vk.

Initial assumptions: All labels are available.

Algorithm:

1. set labels range as {1, 2……, 3*n+1}.

2. for i: 1 to 3*n+1
   generate all variation sets (3*n+1)p3 and all those sets must start with common number.
   Set setcount=number of such sets generated
   if setcount>=n
   for each variation set setcount P_n

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prepare matrix of size n*3
call method chkforsoluiton (matrix)
otherwise report as "Work Finished".
3. Stop.

chkforsolution (matrix)

1. fix the common first number as λ2(hub).
2. Assign second column λ2(spokes) and third column to λ2(edges).
3. now for i=1 to n
   λ2(ei)- λ2(e(i+1)%n) is available label
   display "got solution".
4. Rearrange any one row of matrix elements (2, 3 columns only) again call chkforsolution(matrix) until there is no possible re-arrangement. This is a bit tricky process. Here We used DFS Traversal of Binary tree which is created based on size of graph.
5. Stop.

Here the matrix prepared consist same element in each row. We fix this as label of hub. The remaining two elements applied to spoke and spoke vertex. We exchange each set of labels assigned to spoke and spoke hub by fixing others. Now directly the labels for rim are calculated, checked to identify EMTL. All such results are given in table 1.

4.3 Algorithm for TML on Wheel

For all VMTL/ EMTLs from 3.2.1and 3.2.2 we checked whether this is EMTL/VMTL for a different magic constant ek/vk. If this condition satisfies we can print the solution as TMLs for given VMTL. The same work continues for EMTL also by interchanging vertices and edges.

Input: G(V,E) with VMTL

Output: prints TML if exists.

Algorithm:

1. Initialize ek, ek= λ2(en)+ λ2(v1)+ λ2(en1)
2. count=1;
3. for i:2 to n
   if λ2(ei-1)+ λ2 (vi) + λ2(ei) = ek
   count=count+1
4. for i:1 to n
   if λ2(vi)+ λ2 (si) + λ2 (hub) = ek
   count=count+1
5. if count=2*n them display "Given is TML"
   else display "Given is not TML".

With this module we can get number of TML's for given graph size and any of vertex magic constant or edge magic constant.

Table 1: Number Of EMTL's For Given Wheel Graph Size And Magic Constant

Wheel size=4		Wheel size=5		Wheel size=6	
Magic constant	#EMTLs	Magic constant	#EMTLs	Magic constant	#EMTLs
16	2	19	2	22	3
18	2	20	2	23	2
19	1	21	4	24	6
20	4	22	1	25	6
21	6	23	3	26	4
22	3	24	6	27	11
23	1	25	4	28	5
24	1	26	4	29	8
26	1	27	6	30	0
		28	3	31	8
		29	1	32	5
		30	4	33	11
		31	1	34	4
		32	2	35	6
				36	6
				37	2
				38	4
Total #EMTL's	21		43		91

5. MAGIC LABELING ALGORITHMS ON FAN GRAPH

Fan Graph is a wheel with out an edge. So here also we need different algorithms for VMTL and EMTL. The possible values of k is given as

$$\frac{13n^2+n}{2(n+1)} \le k \le \frac{17n^2+3n-2}{2(n+1)} \cap \frac{(n+1)(n+2)}{2} \le k \le \frac{7n^2-2}{n}$$

5.1 Algorithm For VMTL On Fan Graphs

We have applied the algorithm of VMTL on wheels with following changes.

1. Set labels range as {1, 2……, 3*n}
2. After setting label to spokes, for the first variation $(3*n)P_2$, whose sum is a k-λ1(sj).
   set them as λ1(v1) and λ1(e1).
3. λ1(en)=0

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The results are given with table 2.

Table 2: Number Of VMTL's For Given Fan Graph Size And Magnetic Constant Mc.

Fan size=3		Fan size=4		Fan size=5		Fan size=6	
mc	#V	mc	#V	mc	#V	mc	#V
15	0	22	25	28	105	34	53
16	2	23	59	29	475	35	2073
17	3	24	93	30	773	36	9652
18	0	25	62	31	1283	37	17946
19	1	26	30	32	914	38	17489
20	0	27	4	33	473	39	13012
				34	86	40	6018
						41	674
Total #V	6		273		4109		66917

Table 3: Number Of EMTL's For Given Fan Graph Size And Magic Constant Mc.

Fan size=4		Fan size=5		Fan size=6	
mc	#E	mc	#E	mc	#E
16	2	19	2	22	3
18	2	20	2	23	2
19	1	21	4	24	6
20	4	22	1	25	6
21	6	23	3	26	4
22	3	24	6	27	11
23	1	25	4	28	5
24	1	26	4	29	8
26	1	27	6	30	0
		28	3	31	8
		29	1	32	5
		30	4	33	11
		31	1	34	4
		32	2	35	6
				36	6
				37	2
				38	4
Total #EMTL's	21		43		91

5.2 Algorithm for EMTL on Fan Graphs

We have applied the algorithm of EMTL on wheels with following changes.

1. Set labels range as {1, 2……, 3*n}
2. When preparing matrix there must be one zero in third column. Since these values will be assigned to edges.

The results are given with table 2.

6. MAGIC LABELING ALGORITHMS ON FRIENDSHIP GRAPH

A friendship F_n consists of n copies of C_3 with a common vertex. In other words Friendship Graph is a wheel with even size whose alternate edges are removed. So here also we need different algorithms for VMTL and EMTL. The possible values of k for friendship Graph are

$$\frac{17n^2+9n+1}{2(n+1)} \leq k \leq \frac{23n^2+13n+1}{2(n+1)} \cap (2n+1)(n+1) \leq k \leq \frac{17n+5}{2}$$

6.1 Algorithm For VMTL On Friendship Graphs

We have applied the algorithm of VMTL on wheels with following changes. A Fan of size m will be a wheel with rim vertices 2*m whose alternate edges removed. Here we have taken friendship graph size as n=2*m.

1. Set labels range as {1, 2……, 3*n+1-(n/2)}
2. After setting label to spokes,
   for all remaining odd vertices get variation (3*n+1-(n/2))P_2, whose sum is a k- $\lambda 1(sj)$
   set them as $\lambda 1(vj)$ and $\lambda 1(ej)$.
   for all remaining even vertices get variation (3*n+1-(n/2))P_1, whose sum is a k- $\lambda 1(sj)$
   set them as $\lambda 1(vj)$ and $\lambda 1(ej)$=0.

This results the table 4.

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Table 4: Number Of VMTL's For Given FriendshipGraph Size And Magic Constant

Friendship graph size=2		Friendship graph size=3	
mc	#v	mc	#v
18	1	28	30
19	1	29	9
20	4	30	6
21	1		
Total #VMTL's	7		45

Table 5: Number Of EMTL's For Given Friendship Graph Size And Magic Constant

Friendship graph size=2		Friendship graph size=3	
mc	#E	mc	#E
15	1	21	2
16	1	22	13
17	1	23	25
18	5	24	90
19	2	25	33
20	2	26	98
		27	109
		28	100
		29	71
		30	32
		31	9
Total #EMTL's	12		573

6.2 Algorithm for EMTL on Friendship Graphs

We have applied the algorithm of EMTL on wheels with following changes.

1. Set labels range as {1, 2……, 3*n+1-(n/2)}.
2. Prepare Matrix and call method *chkforsolution (matrix).*
3. In *chkforsolution (matrix)* see the possibility of getting alternate zeros .If possible display solution.

This results the table 5.

7. CONCLUSION

In this paper, we give algorithms to enumerate all non-isomorphic VMTL, EMTL on cycle graphs, wheels, Fan Graphs and Friendship graphs. The idea of the algorithms can be applied to other classes of graphs or adopted to develop algorithms for other type of labeling. In the mean time, we are still working on algorithm for other type of labeling such as edge magic total, vertex anti magic total, harmonious, and graceful.

We also present the number of non-isomorphic VMTLs on each graph for some small size graphs. As the number of non-isomorphic VMTLs for friendship has been enumerated, the number of non-isomorphic labeling on larger size of the remaining graphs is still an open problem.

REFERENCES:

[1] Baker A., J. Sawada, 2008, Magic labelings on cycles and wheels, COCOA LNCS, 361-373.
[2] J.A.Gallion,"A dynamic survey of graph labeling", Electronic J.Combinations, 14 (2007), DS6.
[3] D. B. West. An introduction to graph theory, Prentice-Hall., 2004.
[4] J. A. MacDougall, M. Miller, Slamin and W. D. Wallis, "Vertex Magic Total Labelings of Graphs", Utilitas Math, 61 (2002), 3-21.
[5] I. Gray, J.McDougall, and W.D.Wallis, On Vertex labeling of complete Graphs, Bull, Inst, Combin, Appl., 38 (2003) 42-44.
[6] MacDougall, J.A., Miller, M., Wallis, W.D.: Vertex-magic total labelings of wheels and related graphs. Utilitas Mathematica 62 (2002) 175{183}
[7] H.R. Andersen ,H. Hulgaard, "Boolean Expression Diagrams," Information and Computation, vol. 179, pp. 194-212, 2002.
[8] [8] E.T. Baskoro, M. Miler, Slamin and W.D. Wallis," Edge Magic *Total Labelings".*
[9] Narsingh Deo, "Graph theory with applications to Engineering and Computer Science", 56 (2000).
[10] Exoo, G.,Ling, A.C.H, McSorley, J.P., Philips, N.C., N.C., Wails, W.D : "Totally Magic Graphs".*Discrete Mathematics 254,*103-113(2002).
[11] Kalantari, B., Khosrovshahi, G.B. :"Magic Labeling in Graphs: Bounds, Complexity and an Application to a Variant of TSP". *Networks28,*211-219(1996).
[12] Wallis, W.D.: "Magic Graphs" Birkhauser(2001).
[13] N. Lakshmi Prasanna, K. Sravanthi And NagallaSudhakar,"Applications Of Graph Labeling In Communication Networks" Oriental Journal Of Computer Science And Technology, Volume 7 No. 01 Page No. 139-145 Apr 2014.