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COMPOSITE TERMINAL SLIDING MODE CONTROL FOR SPACECRAFT WITH COUPLED TRANSLATION AND ATTITUDE DYNAMICS

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ABSTRACT

The composite terminal sliding mode controller design to deal with translation and attitude control of a rigid spacecraft is studied. Based on the terminal sliding mode (TSM) concept, a finite-time controller is designed to achieve translation and attitude maneuvers in the presence of model uncertainties and external disturbances. A finite-time disturbance observer (FTDO) is introduced to estimate the total model uncertainties and external disturbances. The proposed composite terminal sliding mode control consists of a finite-time controller based on TSM concepts and a compensation term based on FTDO. The Lyapunov theory is applied to prove the finite-time stability of the closed-loop system. Numerical simulations on translation and attitude control of a rigid spacecraft are also provided to demonstrate the performance of the proposed controller.

Keywords: *Composite Control, Terminal Sliding Mode Control, Finite-Time Disturbance Observer*

1. INTRODUCTION

Many space missions such as, satellite surveillance, space station docking and installation, spacecraft formation flying ([1], [2]) require spacecraft to perform large angle slew or complicated translational maneuvers. Current research mainly separates attitude motion from translation motion. In practice, the attitude and translation maneuvers are coupled and highly nonlinear. Furthermore, the model parameters of spacecraft cannot be exactly known, and the spacecraft is always subject to environmental perturbations. All of above issues make it difficult to achieve the desired control performance for spacecraft with coupled translation and attitude maneuvers. Various control approaches have been proposed to solve this problem. The sliding mode control algorithm was presented in [3] and the state-dependent Riccati equation method was used in [4] to deal with position and attitude maneuvers. Xu and Balakrishnan [5] combined SDRE with a neural network to design a robust control law under the moment of inertia. Although the control techniques mentioned above have been shown sufficient reliability, they only guaranteed asymptotic stability and convergence. This implies that the control objective can be achieved in infinite time [6]. However, the ability of fast maneuver is highly desirable in many space missions.

To obtain better control performance, many finite time control (FTC) methods have been developed to deal with position and attitude control problem. As is known, terminal sliding mode control (TSMC) is recently proposed based on the concept of a terminal attractor [7]. TSMC can make system states converge to equilibrium in finite time [7], [8].

To improve rejection disturbance performance, the disturbance observers [9]-[10] have been developed for spacecraft attitude control problem. In [11] the total uncertainties have been compensated effectively. Shtessel et al. [12] have employed the disturbance observer to estimate disturbance of missile systems. In [13] Lu et al. applied the differentiator motivated by [14] to design an observer that can provide high-precision estimation outputs.

In this research, a finite-time control law for spacecraft with coupled translation and attitude maneuvers is designed. Furthermore, to gain the improvement of disturbance rejection ability, a FTDO is developed to compensate for the uncertainties and disturbances. Based on the modified differentiator technique proposed in [14], the total model uncertainties and external disturbances will be estimated. Also, a composite control law which consists of a feedback controller based on TSMC concepts and a feed-forward compensation term based on FTDO is designed to

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achieve finite-time stability of the closed-loop system.

The paper is organized as follows. Section 2 contains a description of nonlinear model of spacecraft and the problem formulation. In Section 3 the main results are presented in which the novel position and attitude control law is developed using TSM concepts. In Section 4 a composite finite-time control law is designed by combining the TSM scheme with a feed-forward term based on the FTDO technique. In Section 5 simulation results of translation and attitude control are provided to show the effectiveness of the proposed method. Finally, conclusions are given in Section 6.

2. MODEL DESCRIPTION AND PROBLEM FORMULATION

2.1 Spacecraft Dynamics and Kinematics

The dynamic equations of a six-degree-of-freedom (6-DOF) spacecraft, which performs translation and rotational motion, are modeled as [3]

$$m\dot{v} + m\omega^{\times} v = u_f \,, \tag{1}$$

$$J\dot{\omega} + \omega^{\times}\left(J\omega\right) = \rho^{\times} u_f + u_T \,, \tag{2}$$

where m and J are the mass and the inertia matrix of a rigid spacecraft, $v = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$ and $\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T$ represent the spacecraft's translational and angular velocity, ρ is the distance from the mass center of the spacecraft to point where the force is applied, $u_f = \begin{bmatrix} u_{1f} & u_{2f} & u_{3f} \end{bmatrix}^T$ and $u_T = \begin{bmatrix} u_{1T} & u_{2T} & u_{3T} \end{bmatrix}^T$ denote the control force and control torque. For any vector $a = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^T$, the skew-symmetric matrix $a^{\times}$ is defined by

$$a^{\times} = \begin{bmatrix} 0 & a_3 & -a_2 \\ -a_3 & 0 & a_1 \\ a_2 & -a_1 & 0 \end{bmatrix}. \tag{3}$$

The kinematic equations of a spacecraft in 6-DOF are given as [3]

$$\dot{r} = -\omega^{\times} r + v \,, \tag{4}$$

$$\dot{\bar{q}} = \frac{1}{2} E(\bar{q})\omega \,, \tag{5}$$

where $r = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}^T$ denotes the spacecraft positions, and $\bar{q} = \begin{bmatrix} q_0 & q \end{bmatrix}^T$ with $q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T$ is the unit quaternion, which describes the attitude motion of a spacecraft without singularities [15]. Here, $E(\bar{q}) = \begin{bmatrix} -q^T \\ q^{\times} + q_0 I_3 \end{bmatrix}$ where I_3 is the 3×3 identity matrix. The quaternion $\bar{q}$ consists of the scalar q_0 and the three-dimensional vector q defined as

$$q_0 = \cos\left(\frac{\phi}{2}\right), \quad q = \hat{e}\sin\left(\frac{\phi}{2}\right), \tag{6}$$

where $\hat{e} = \begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \end{bmatrix}^T$ is a unit vector called the Euler axis, ϕ denotes the magnitude of the Euler axis rotation. Note that the quaternion $\bar{q}$ is subject to the constraint

$$q^T q + q_0^2 = 1. \tag{7}$$

2.2 Error-Based Spacecraft Motion Equations

Let us define the desired position $r_d = \begin{bmatrix} r_{1d} & r_{2d} & r_{3d} \end{bmatrix}^T$ and translational velocity $v_d = \begin{bmatrix} v_{1d} & v_{2d} & v_{3d} \end{bmatrix}^T$. Substituting (4) and (5) into (1), one can obtain the relative position dynamics as

$$m\ddot{r}_e + 2mQ\dot{r}_e + mPr_e + m\Sigma = u_f \tag{8}$$

where $r_e = r - r_d$ is the relative position, $Q = \left[T\dot{q}\right]^{\times}$ $T = 2(q_0 I_3 + q^{\times})^{-1}$, $P = \left[[T\dot{q}]^{\times} + [T\dot{q}]^{\times}[T\dot{q}]^{\times}\right]$ and $\Sigma = \left[\dot{v}_d + [T\dot{q}]^{\times} v_d\right]$.

We now briefly explain the use of quaternion for description of the attitude error. We define $\bar{q}_r = \begin{bmatrix} q_{0r} & q_r \end{bmatrix}^T$ with $q_r = \begin{bmatrix} q_{1r} & q_{2r} & q_{3r} \end{bmatrix}^T$ is the desired attitude. The quaternion for attitude error is $\bar{q}_e = \begin{bmatrix} q_{0e} & q_e \end{bmatrix}^T$ with $q_r = \begin{bmatrix} q_{1e} & q_{2e} & q_{3e} \end{bmatrix}^T$. Using the multiplication law for quaternion, we obtain [16]

$$\bar{q}_e = \begin{bmatrix} q_{0r} q - q_0 q_r - q_r^{\times} q \\ q_0 q_{0r} + q^T q_r \end{bmatrix} \tag{9}$$

subject to the constraint

$$\bar{q}_e^T \bar{q}_e = (q^T q + q_0^2)(q_r^T q_r + q_{0r}^2) = 1. \tag{10}$$

The kinematic equation for the attitude error can then be expressed as [10], [13]

$$\dot{\bar{q}}_e = \frac{1}{2} E(\bar{q}_e)\omega_e \,, \tag{11}$$

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where $E(\bar{q}_e)=\begin{bmatrix} -q_e^T \\ q_e^\times + q_{0e}I_3 \end{bmatrix}$ and $\omega_e=\omega-\omega_d$ is the angular velocity tracking error. Substituting (11) into (2), we obtain

$$J^*\ddot{q}_e+\Xi\dot{q}_e+M^TG=M^T(\rho^\times u_f+u_T),\qquad(12)$$

where $M=2E(\bar{q}_e)^{-1},\ G=J\dot{\omega}_d+\omega_d^\times J\omega_d,\ J^*=M^TJM$ and $\Xi=-J^*\dot{M}^TM-M^T[JM\dot{q}_e]^\times M-M^T[J\omega_d]^\times M+M^T\omega_d^\times JM$. It is assumed that $q_{0e}\neq 0$ and M is then invertible.

In practical situation, the mass and inertia matrix of a spacecraft cannot be exactly known, and only nominal ones are available. Besides, the external disturbances always affect translation and attitude motion. When the uncertainties and disturbances are taken into account, (8) and (12) can be rearranged as [17]

$$m_0\ddot{r}_e+2m_0Q\dot{r}_e+m_0\,\mathrm{P}\,r_e+m_0\Sigma=u_f+\delta,\qquad(13)$$

$$J_0^*\ddot{q}_e+\Xi_0\dot{q}_e+M^TG_0=M^T(\rho^\times u_f+u_T)+\xi,\qquad(14)$$

where $\delta=f_1-\Delta m(\ddot{r}_e+2Q\dot{r}_e+P\,r_e+\Sigma)$, $\xi=M^T(f_2+(\Delta JM\dot{M}^{-1}M-[\Delta JM\dot{q}_e]^\times M-[\Delta J\omega_d]^\times M+\omega_d^\times\Delta JM)\dot{q}_e-\Delta JM\ddot{q}_e-\Delta J\dot{\omega}_d-\omega_d^\times\Delta J\omega_d)$, $m=m_0+\Delta m$, m_0 and Δm denote the nominal and uncertain parts of m, $J=J_0+\Delta J$, J_0 and ΔJ are denote the nominal and uncertain parts of J. f_1 and f_2 are the bound perturbations, and J_0^*, Ξ_0 and G_0 are the nominal functions of J^*, Ξ and G which can be readily obtained by (12). According to (13) and (14), the error-based spacecraft motion equation in the presence of uncertainties and disturbances can be given by [17]

$$C_1\ddot{e}+C_2\dot{e}+C_3e+C_4=Ku+d,\qquad(15)$$

where

$$C_1=\begin{bmatrix} m_0I_3 & 0_3 \\ 0_3 & J_0^* \end{bmatrix},\ C_2=\begin{bmatrix} 2m_0Q & 0_3 \\ 0_3 & \Xi_0 \end{bmatrix},\ C_3=\begin{bmatrix} m_0P & 0_3 \\ 0_3 & 0_3 \end{bmatrix},$$

$$C_4=\begin{bmatrix} m_0\Sigma \\ M^TG_0 \end{bmatrix},\ K=\begin{bmatrix} I_3 & 0_3 \\ M^T\rho^\times & M^T \end{bmatrix},\ e=\begin{bmatrix} r_e \\ q_e \end{bmatrix},\ u=\begin{bmatrix} u_f \\ u_T \end{bmatrix},$$

$d=\begin{bmatrix} \delta \\ \xi \end{bmatrix}$, and 0_3 is the 3×3 zero matrix.

2.3. Terminal Sliding Surface

It is known that the terminal sliding mode (TSM) surface presented in [7], [8] can provide high precision and fast convergence. Using the concepts in [8] the TSM surface is defined as

$$S_i=\dot{e}_i+\beta_i|e_i|^\alpha\,sign(e_i),\quad i=1,\ldots,6\qquad(16)$$

where $S=[S_1\ \ldots\ S_6]^T$ is the sliding vector, $\beta_i>0,\ i=1,\cdots,6,\ \alpha\in(1,2)$, From (16) the first time derivative of S_i is

$$\dot{S}_i=\ddot{e}_i+\alpha\beta_i|e_i|^{\alpha-1}\dot{e}_i,\quad i=1,\ldots,6\qquad(17)$$

Now consider the following reaching law

$$\dot{S}=-\tau s-\rho\,sig^\gamma(S),\qquad(18)$$

where $\tau=diag(\tau_i),\ \rho=diag(\rho_i),\ i=1,\cdots,6$ with positive scalars τ_i and ρ_i. The function $sig^\gamma(S)$ is defined as

$sig^\gamma(S)=\left[sign(S_1)|S_1|^\gamma\ \cdots\ sign(S_6)|S_6|^\gamma\right]^T$, and $0<\gamma<1$.

3. FINITE-TIME ATTITUDE CONTROL

In this section, a novel controller is designed to achieve high-precision attitude tracking and performance.

3.1. Control Objective

In this paper the control objective is to design a control law which forces the states $(e_i,\dot{e}_i)$ of closed-loop system (15) to reach zero in finite time. This can be expressed as

$$\lim_{t\to T}\left(e(t)\right)=0,$$
$$\lim_{t\to T}\left(\dot{e}(t)\right)=0,\qquad(19)$$

where T is a finite time.

3.2. Finite-Time Controller

Before giving the controller design, the following lemmas and assumptions are required.

Lemma 1: (Hardy et al. [18]) If $p\in(0,1)$ then the following inequality holds

$$\sum_{i=1}^{6}|x_i|^{1+p}\geq\left(\sum_{i=1}^{6}|x_i|^2\right)^{\frac{1+p}{2}}.\qquad(20)$$

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Lemma 2: (Du et al. [19]) Suppose $V(x)$ is a smooth positive definite function (defined on $U \subset R^n$) and $\dot{V}(x)+\zeta V^{\theta}(x)$ is negaive semi-definite on $U \subset R^n$ for $\theta \in (0,1)$ and $\xi \in \Re^+$ then there exists an area $U_0 \subset \Re^n$ such that any $V(x)$ which starts from $U_0 \subset \Re^n$ can reach $V(x) \equiv 0$ in finite time. Moreover, if T_r is the time needed to reach $V(x) \equiv 0$ then

$$T_r \le \frac{V^{1-\iota}(x_0)}{\varsigma(1-\iota)}, \tag{21}$$

where $V(x_0)$ is the initial value of $V(x)$.

Lemma 3: (Yu et al. [8]) For any numbers $\lambda_1 > 0,\ \lambda_2 > 0,\ 0 < \varpi < 1,$ an extended Lyapunov condition of finite-time stability can be given in the form of fast terminal sliding mode as

$$\dot{V}(x)+\lambda_1 V(x)+\lambda_2 V^{\varpi}(x) \le 0, \tag{22}$$

where the settling time can be estimated by

$$T_r \le \frac{1}{\lambda_1(1-\varpi)}\ln\frac{\lambda_1 V^{1-\varpi}(x_0)+\lambda_2}{\lambda_2}. \tag{23}$$

Assumption 1: The total uncertainty vector d is assumed to be bounded such that $\left\|C_1^{-1}d\right\| \le L,$ where L is a positive constant.

Lemma 4: Consider the error-based spacecraft motion system (15). Then, the sliding surface (16) satisfying $S(t)=0$ $\{e(t)\equiv 0,\ \dot{e}(t)\equiv 0\}$ can be reached in finite time.

Proof: Next, consider the following candidate Lyapunov function

$$V_1 = \frac{1}{2}\sum_{i=1}^{6} e_i^2 \tag{24}$$

and its derivative is

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{6} e_i\dot{e}_i\\ &= -\sum_{i=1}^{6}\beta_i e_i\left|e_i\right|^{\alpha} sign(e_i)\\ &= -\sum_{i=1}^{6}\beta_i\left|e_i\right|^{\alpha+1},\end{aligned}$$

which can be written as

$$\dot{V}_1 \le -\beta_m\sqrt{2}V^{\frac{\alpha+1}{2}}. \tag{25}$$

where $\beta_m = \min(\beta_i),\ i=1,\cdots,6.$ Then, by Lemma 2 we obtian that $(e_i,\dot{e}_i)$ can converge to the origin along the sliding surface in finite time.

We now consider a terminal sliding mode control law for the error-based spacecraft motion system (15). A finite-time control design based on TSM in the presence of inertia uncertainties and external disturbances is proposed.

Theorem 1: Consider the error-based spacecraft motion system (15) in the presence of the total uncertainty vector d. If the control law is designed as

$$\begin{aligned}u(t) = K^{-1}C_1\Big(&C_1^{-1}C_2\dot{e}+C_1^{-1}C_3 e+C_1^{-1}C_4\\ &-\alpha diag(\beta_i\left|e_i\right|^{\alpha-1})\dot{e}-\tau S-\rho\tanh(S)\Big),\end{aligned} \tag{26}$$

where the function $\tanh(S)$ is defined as $\tanh(S)=\left[\tanh(S_1)\ \cdots\ \tanh(S_6)\right]^T,$ then the system trajectory will converge to the neighborhood of TSM $S=0$ as

$$\left|S_i\right| \le \Delta = \frac{1}{2}\ln\frac{\rho_i+L}{\rho_i-L},\ i=1,\cdots,6, \tag{27}$$

in finite time. Furthermore, the tracking errors e and $\dot{e}$ will converge to the regions

$$\left|e_i\right| \le \varepsilon_1 = \left(\frac{\Delta}{\beta_i}\right)^{\frac{1}{\alpha}} \text{ and } \left|\dot{e}_i\right| \le \varepsilon_2 = 2\Delta \tag{28}$$

in finite time.

Proof: Consider the following candidate Lyapunov function

$$V_2 = \frac{1}{2}S^T(t)S(t). \tag{29}$$

Its time derivative is

$$\begin{aligned}\dot{V}_2 &= S^T\left(\ddot{e}+\alpha diag\left(\beta_i\left|e_i\right|^{\alpha-1}\right)\dot{e}\right)\\ &= S^T C_1^{-1}\big(Ku+d-C_2\dot{e}-C_3 e\\ &\quad -C_4+\alpha C_1^{-1}diag\left(\beta_i\left|e_i\right|^{\alpha-1}\right)\dot{e}\big)\end{aligned} \tag{30}$$

Substituting (26) into (30), gives

$$\begin{aligned}\dot{V}_2 &= \tau S^T S-\rho S^T\tanh(S)+S^T C_1^{-1}d\\ &\le -\sum_{i=1}^{6}\tau_i S_i^2-\sum_{i=1}^{6}\left(\rho_i\tanh\left(\left|S_i\right|\right)-L\right)\left|S_i\right|\end{aligned} \tag{31}$$

If $\rho_i\tanh\left(\left|S_i\right|\right) > L$ which means

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$$|S_i| > \frac{1}{2}\ln\frac{\rho_i + L}{\rho_i - L}, \tag{32}$$

then the following inequality holds

$$\dot{V}_2 \le -\tau_m \sum_{i=1}^{6} S_i^2 - \gamma_m \sum_{i=1}^{6} |S_i| \tag{33}$$

where $\tau_m = \min(\tau_i)$, $\gamma_m = \min(\rho_i \tanh(|S_i|) - L)$, $i = 1,\cdots,6$. By Lemma 1, one can obtain

$$\dot{V}_2 \le -\bar{\gamma} V_2^{\frac{1}{2}} - \bar{\tau} V_2. \tag{34}$$

where $\bar{\gamma} = \sqrt{2}\gamma_m$ and $\bar{\tau} = 2\tau$. Obviously, both $\bar{\gamma}$ and $\bar{\tau}$ are positve real numbers. Therefore, by Lemma 3, the finite-time stability can be ensured, and the region

$$|S_i| \le \frac{1}{2}\ln\frac{\rho_i + L}{\rho_i - L} \tag{35}$$

can be reached in fnite time.

After S_i reaching the boundary layer $|S_i| \le \Delta$, one can obtain

$$\dot{e}_i + \beta_i |e_i|^\alpha\, sign(e_i) = \Omega_i,\ i = 1,\cdots,6 \tag{36}$$

where Ω_i satisfies $|\Omega_i| \le \Delta$. We can also rewrite (36) as

$$\dot{e}_i + \left(\beta_i - \frac{\Omega_i}{|e_i|^\alpha\, sign(e_i)}\right)|e_i|^\alpha\, sign(e_i) = 0 \tag{37}$$

Then, when $\beta_i - \dfrac{\Omega_i}{|e_i|^\alpha\, sign(e_i)} > 0$, (37) is still kept in the form of terminal sliding mode, which also means that the tracking errors will converge to the region

$$|e_i| \le \varepsilon_1 = \left(\frac{\Delta}{\beta_i}\right)^{\frac{1}{\alpha}} \tag{38}$$

in finite time. Moreover, from will converge to

$$|\dot{e}_i| \le \beta_i |e_i|^\alpha + |\Omega_i| \le 2\Delta \tag{39}$$

in finite time. This completes the proof.

4. DISTURBANCES OBSERVER-BASED FINITE-TIME CONTROL

Next we present another approach to solve position and attitude control problem in the presence of uncertainties and disturbances. A TSMC will be designed to drive the state variables to converge to the desired states and totally compensates for the uncertainties and disturbances via a disturbance estimator.

Let $y_1 = e$ and $y_2 = \dot{e}$ then the error-based spacecraft motion equation (15) becomes

$$\begin{aligned}\dot{y}_1 &= y_2,\\ \dot{y}_2 &= \tau_u + \tilde{d} + F(y_1, y_2),\end{aligned} \tag{40}$$

where

$F(y_1, y_2) = -C_1^{-1}C_2 y_2 - C_1^{-1}C_3 y_1 - C_1^{-1}C_4$,

$\tau_u = C_1^{-1}Ku$ and $\tilde{d} = C_1^{-1}d$.

Before describing the controller design, the following assumption and lemma are recalled.

Assumption 2: The total uncertainty vector $\tilde{d} \in C^2$ satisfies

$$|\ddot{\tilde{d}}_i| < l_i,\ \ i = 1,\cdots,6, \tag{41}$$

where l_i are positive constants.

Lemma 4: (Levant [14]) With Assumption 2, consider the system (40), where F is a sufficiently smooth function. The second-order differentiator proposed for the estimate of the total disturbance vector $\tilde{d}$ is

$$\begin{aligned}
\dot{z}_0 &= v_0 + \tau_u + F\\
v_0 &= -\lambda_0 diag\left(l_i^{1/3}|z_{0i} - e_{2i}|^{2/3}\right) sign(z_0 - e_2) + z_1,\\
\dot{z}_1 &= v_1,\\
v_1 &= -\lambda_1 diag\left(l_i^{1/2}|z_{1i} - v_{0i}|^{1/2}\right) sign(z_1 - v_0) + z_2,\\
\dot{z}_2 &= -\lambda_2 diag(l_i) sign(z_2 - v_1),\quad i = 1,\cdots 6.
\end{aligned} \tag{42}$$

where z_0, z_1 and z_2 are the estimates of e_2, $\tilde{d}$ and $\dot{\tilde{d}}$ respectively. Here, for any $s = [s_1\ \ s_2\ \cdots s_n]^T$, the function $sign(s)$ is defined as

$sign(s) = [sign(s_1)\quad sign(s_2)\quad \ldots\quad sign(s_n)]^T$.

Proof: Let us define $E_1 = z_0 - e_2$, $E_2 = z_1 - \tilde{d}$, $E_2 = z_2 - \dot{\tilde{d}}$. The observer error dynamics can be obtained as

$$\begin{aligned}
\dot{E}_1 &= -\lambda_0 diag\left(l_i^{1/3}|E_{1i}|^{2/3}\right) sign(E_1) + E_2\\
\dot{E}_2 &= -\lambda_1 diag\left(l_i^{1/2}|E_{2i} - \dot{E}_{1i}|^{1/2}\right) sign(E_2 - \dot{E}_1) + E_3,\\
\dot{E}_3 &= -\lambda_2 diag(l_i)\, sign(z_2 - v_1) - \ddot{\tilde{d}}_i\\
&\in -\lambda_2 diag(l_i)\, sign(E_3 - \dot{E}_2) + [-l_i, l_i].
\end{aligned} \tag{43}$$

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Based on concepts in [14], (43) is globally finite-time stable. Hence, there exists a fixed constant T^* such that $E_i = 0 \quad (i = 1, \cdots, 6)$ for $t \geq T^*$.

Theorem 3: With Assumption 2, Consider the spacecraft system (15) in the presence of the total uncertainty vector $\tilde{d}$ and FTDO (42). If the control law is designed as

$$u(t) = K^{-1}C_1\Big(C_1^{-1}C_2\dot{e} + C_1^{-1}C_3 e + C_1^{-1}C_4 - z_1 \\ -\alpha\, diag(\beta_i |e_i|^{\alpha-1})\dot{e} - \tau S - \rho \tanh(S)\Big) \quad (44)$$

then the tracking errors e and $\dot{e}$ can converge to zero in finite time.

Proof: The proof is similar to that of Theorem 1, since the term $\tilde{d} - z_1$ is close to zero due to the convergence of FTDO (42). Thus, it is easy to obtain the same conclusion.

5. SIMULATIONS

In this section, numerical simulations are carried out to demonstrate the performance of the proposed control law (44). The model of the spacecraft is taken from [17] where the nominal inertia matrix and the parameter uncertainties of inertia matrix are

$$J_0 = \begin{bmatrix} 1000 & -50 & -10 \\ -30 & 1000 & -40 \\ -20 & -40 & 800 \end{bmatrix} kg \cdot m^2$$

$$m_0 = 1000\ kg,$$

$$\Delta J \leq 0.1 J_0, \quad \Delta m \leq 0.1 m_0.$$

The initial conditions of position and attitude vector are

$$r(0) = \begin{bmatrix} 25 & -20 & 18 \end{bmatrix}^T m, \quad v(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T m/s,$$

$$\bar{q}(0) = [0.93 \quad 0.22 \quad -0.21 \quad 0.19]^T,$$

$$\omega(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} rad/s, \quad v(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T m.$$

The desired position and attitude are given by

$$r_d = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T m, \quad v_d = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T m/s,$$

$$\bar{q}_d = [1 \quad 0 \quad 0 \quad 0]^T, \quad \omega_d = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} rad/s.$$

In the simulations, the parameters of controller () are set as

$$\gamma = \frac{5}{7}, \quad \alpha = 0.5, \ \beta = 2, \ \tau = 2I_3, \quad \rho = 0.5 I_3$$

and the external disturbances are considered which are provided as

$$f_1 = \begin{bmatrix} 0.01\sin(0.025t) \\ 0.02\sin(0.025t) \\ 0.03\sin(0.025t) \end{bmatrix} m/\sec^2$$

and

$$f_2 = \begin{bmatrix} 0.001\sin(0.025t) \\ 0.002\sin(0.025t) \\ 0.003\sin(0.025t) \end{bmatrix} N \cdot m.$$

Simulation results for the controller (44) are presented in Figures 1-7. Figures 1 and 2 show that the transient behaviors of quaternion and angular velocity tracking errors are smooth and converge to zero after 40 seconds. We can see that high attitude tracking performance is achieved. As shown in Figures 3 and 4, the responses of position and velocity errors converge to zero in about 70 seconds. The responses of control toques and control forces are plotted in Figures 5 and 6 where the chattering problem is effectively attenuated. The observed value error of $\delta_e = \delta - \hat{\delta}$ is given in Figure 7. The estimate δ_e rapidly reaches zero.

6. CONCLUSIONS

The composite terminal sliding mode controller has been successfully designed to solve the problems of translation and attitude control of a rigid spacecraft. Based on the TSM concept, a finite-time controller is developed to achieve translation and attitude maneuvers in the presence of model uncertainties and external disturbances. A finite-time observer is designed to estimate the total model uncertainties and external disturbances. The proposed composite terminal sliding mode control consists of a finite-time controller based on TSMC and compensation term based on FTDO. The Lyapunov theory is employed to prove the finite-

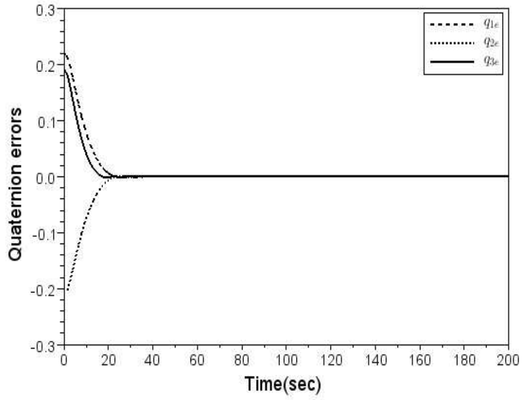

Figure 1: Quaternion errors.

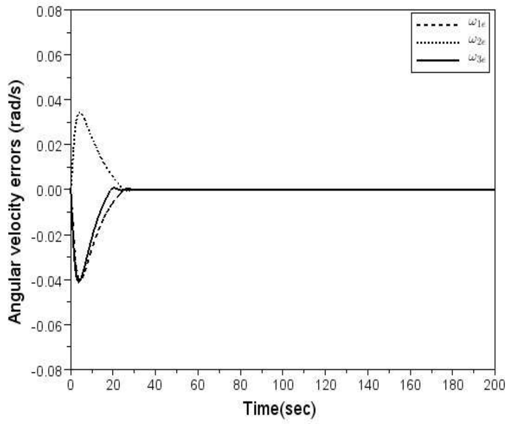

Figure 2: Angular velocity errors.

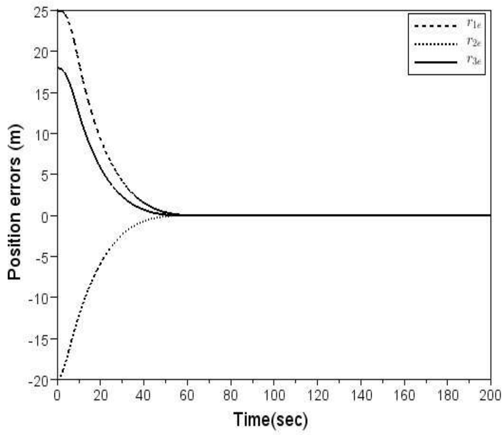

Figure 3: Position errors

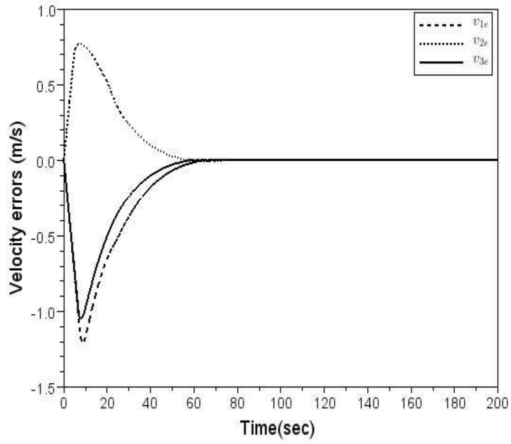

Figure 4: Velocity errors

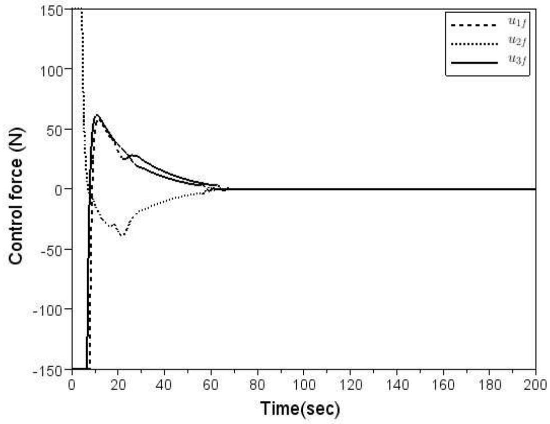

Figure 5: Control force

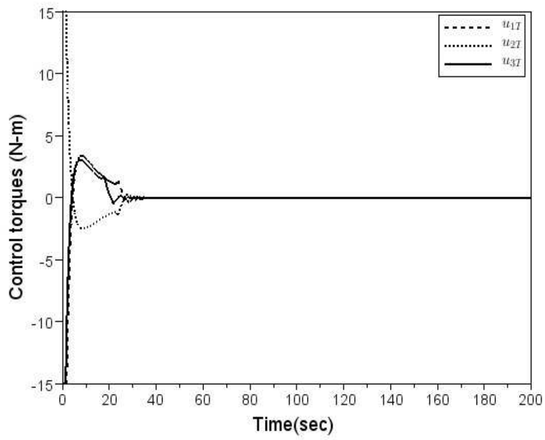

Figure 6: Control torques

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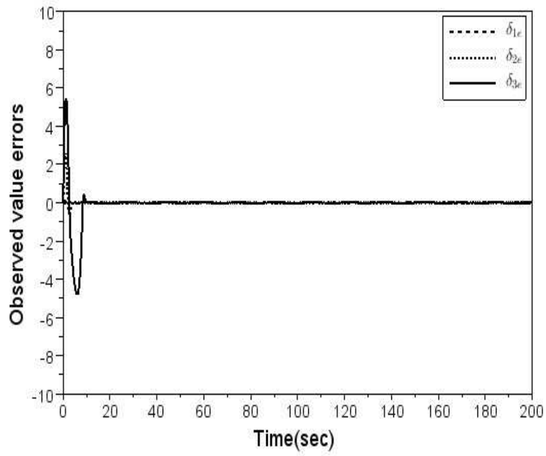

Figure 7: Observed value errors

time stability of the closed-loop system. Numerical simulations on translation and attitude control of a rigid spacecraft are also provided to demonstrate the performance of the proposed controller.

7. ACKNOWLEDGMENTS

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