

OPTIMAL PILOT-SYMBOL PATTERNS FOR MIMO OFDM SYSTEMS UNDER TIME VARYING CHANNELS

¹MY ABDELKADER YOUSSEFI, ²JAMAL EL ABBADI

Department of Electrical and Communication, EMI School, Mohammed V University Agdal– UM5A

BP 765, avenue Ibn Sina Agdal 10000, Rabat, Morocco

E-mail: ¹ab.youssefi@gmail.com, ²abbadi@gmail.com

ABSTRACT

This paper presents a new approach to achieve optimal training sequences (OTS) in terms of minimizing the mean-square channel estimation error for spectrally efficient MIMO OFDM systems. It is shown that the OTS are equi-powered, equi-spaced and position orthogonal. However, in special cases we can find that required conditions to achieve optimal pilot design are not satisfied, this means the optimum pilot is unachievable. In this paper, an algorithm is proposed to achieve suboptimum pilot design under time varying channels. To offer high throughput gains, we propose an adaptive pilot scheme in order to optimally use pilot tones over time varying channels.

Keywords: *Differential evolution (DE), OFDM, MIMO, Mobile multipath channel.*

1. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) has been widely applied in wireless communication systems due to its high data rate transmission and its robustness to multipath channel delay [1, 2]. Additionally, multiple antenna architecture on the transmitter and receiver side, which is called multiple input multiple output (MIMO) is a suitable technique to improve the OFDM channel capacity [3]. For that reason, OFDM modulation is adopted in a number of standards, e.g IEEE 802.11a/g, IEEE 802.16a/d/e [3], DVB-T, etc.

In OFDM systems, channel estimation is usually performed by sending training pilot symbols on sub-carriers known at the receiver and the quality of the estimation depends on the pilot arrangement. Since the channel's response is a slow varying process, the pilot symbols essentially sample this process and therefore need to have a density that is high enough to reconstruct the channel's response at the receiver side [4].

Two classes of methods are available for pilot arrangements: One is based on regular patterns, where pilot symbols are equally-spaced in time and/or frequency domain, whereas the other relies on irregular patterns.

The optimal spacing design of pilot symbols for OFDM systems has been investigated by several studies over the past ten years. In literature, several methods have been designed for regular pilot lattices that satisfy a suitable Nyquist criterion [3, 5, 6]. These regular patterns are not

acceptable for systems in which pilot tones are not equi-powered or channel is time varying process [7]. Recently, irregular pilot arrangements were shown to be optimal in the mean-square error (MSE) sense for certain classes of time varying channels [8, 9].

In this work we propose an algorithm based on Differential Evolution algorithm for OFDM pilot design optimization. It is specifically tailored to suboptimum pilot arrangements over time varying channels when the optimum pilot design is unachievable. We show how to use an adaptive pilot scheme in order to optimally use pilot tones over time domain.

Recently, Differential Evolution (DE) algorithm has become popular and has been applied to a variety of engineering applications. The effectiveness of DE in tackling challenging optimization problems have now widely been recognized by the computational intelligence community.

This paper is organized as follows. Section 2 presents MIMO OFDM system model. In Section 3, we evaluate optimal pilot sequences for various scenarios, optimality conditions and adaptive pilot design. Suboptimum pilot design is described in Section 4. The system simulation results are presented in Section 5.

2. SYSTEM MODEL

In this section, we briefly introduce a transmission model suitable for our further derivation.

We consider the block diagram of MIMO-OFDM system with N_t transmit antennas, N_r receive antennas, and N subcarriers (Figure 1). Generated OFDM signals are transmitted through a number of antennas in order to achieve diversity.

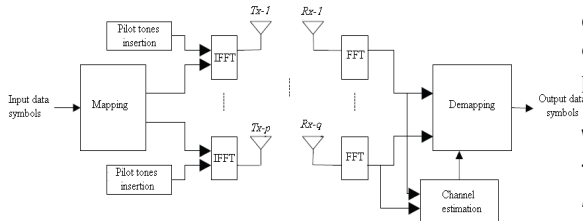


Figure 1: Block Diagram Of MIMO-OFDM System

In MIMO OFDM system shown in Figure 1 (for SISO-OFDM systems we consider $N_t = 1$), we assume that the duration of the cyclic prefix is long enough to avoid inter-symbols interferences (ISI).

A received symbol vector at a discrete time index n transmitted over a flat and time-variant MIMO channel can be written as

$$Y_k^q(n) = \sum_{p=1}^{N_t} H_{k,k}^p \cdot X_k^p(n) + \underbrace{\sum_{p=1}^{N_t} \sum_{\substack{i=1 \\ i \neq k}}^N H_{k,i}^p \cdot X_i^p(n)}_{ICI} + W_k^p(n)$$

Where $X_k^p(n)$ is the transmitted symbol over the k th subcarrier from the p th antenna at time index n , $Y_k^q(n)$ is the received symbol over the k th subcarrier from the q th antenna at time index n .

$H_{k,i}^p$ denotes a frequency channel response between the k -th and i -th subcarrier.

Inter-carrier interference (ICI) can be neglected for time invariant channels and time varying channels with moderate mobility.

3. OPTIMAL AND ADAPTIVE PILOT DESIGN FOR MIMO OFDM SYSTEMS

There are several criteria used for channel estimation, for reasons of complexity, the estimation can be performed by using either linear MMSE criterion (Wiener filtering) [11, 12] and least squares (LS) criterion. However, in high SNR regimes, LS criterion offers a good compromising between performance and complexity. In this section, optimal pilot design with LS channel estimation scheme is derived for various scenarios.

3.1. Time invariant channels

In this section we evaluate optimal pilot design over time invariant channels or time variant channels with moderate mobility. Therefore, when the channel is frequency-selective invariant over each received block OFDM symbol the orthogonality between subcarriers can be fully preserved and ICI can be neglected.

We suppose the OFDM symbol that is transmitted from the p th antenna at time index n is denoted by the $N \times 1$ vector $X^p(n)$, after removing the cyclic prefix at the q th receive antenna, the received $N \times 1$ vector $Y^q(n)$ at time index n can be written with the following equation [10]

$$Y^q(n) = \sum_{p=1}^{N_t} X_{diag}^p(n) \cdot F \cdot h^{p,q} + W^q(n) \quad (1)$$

Where $h^{p,q}$ is an $L \times 1$ vector representing the length L channel impulse response from the p th transmit antenna to the q th receive antenna. Note that F denotes the $N \times N$ unitary DFT matrix; $W^q(n)$ is additive white Gaussian noise, and $(\cdot)_{diag}$ is a diagonal matrix with column vector (\cdot) .

Knowing that $X^p(n) = D^p(n) + B^p(n)$ [13, 14], where $D^p(n)$ is some arbitrary $N \times 1$ data vector, and $B^p(n)$ is some arbitrary $N \times 1$ pilot sequence vector. Then, from (1) we can write

$$Y^q(n) = \sum_{p=1}^{N_t} D_{diag}^p(n) \cdot F \cdot h^{p,q} + \sum_{p=1}^{N_t} B_{diag}^p(n) \cdot F \cdot h^{p,q} + W^q(n) \quad (2)$$

Therefore, we consider the data model

$$Y^q = G \cdot h^q + A \cdot h^q + W^q \quad (3)$$

where $h^q = [h^{q,1T}, \dots, h^{q,N_tT}]^T$ is a channel impulse

response vector of $N_r \cdot L$ length, $G = [D_{diag}^1 F, \dots, D_{diag}^{N_t} F]$ is a $N \times N_r \cdot L$ matrix, $A = [B_{diag}^1 F, \dots, B_{diag}^{N_t} F]$ is a $N \times N_t \cdot L$ matrix, and $(\cdot)^T$ is the transpose operation.

In this section, LS channel estimation scheme is derived. The LS estimate of h^q can then be obtained as [14]

$$\hat{h}^q = A^t Y^q = h^q + (A^H A)^{-1} A^H W^q \quad (4)$$

$(\cdot)^H$ is the Hermitian matrix and A^t is the pseudo-inverse of A can be written as $A^t = (A^H A)^{-1} A^H$.

It is assumed that pilot sequences are designed as $N_p \times N_r \cdot L$ matrix A , which has a full column rank

$N_p L$ that requires $N_p \geq N_t L$ (N_p is the number of pilot tones).

From (4), the MSE of the LS channel estimate is given by

$$\begin{aligned} \text{MSE} &= \frac{1}{LN_t} E \left\{ \left\| \hat{h}^q - h^q \right\|^2 \right\} \\ &= \frac{1}{LN_t} \text{tr} \left\{ A^t E \left\{ N^q N^{qH} \right\} A^{tH} \right\} \end{aligned}$$

For zero-mean white noise, we have

$$E \left\{ N^q N^{qH} \right\} = \sigma^2 I_{N_p} \quad (I_{N_p} \text{ is } N_p \times N_p \text{ identity matrix}),$$

the MSE can be defined as

$$\text{MSE} = \frac{\sigma^2}{LN_t} \text{tr} \left\{ (A^H . A)^{-1} \right\} \quad (5)$$

The minimum MSE can be achieved if $A^H . A = P_p I_{LN_t}$ where P_p is a fixed power dedicated for training [14].

$$\text{MSE}_{\min} = \frac{\sigma^2}{P_p} \quad (6)$$

However, we have the following inequality

$$\text{MSE} \geq \frac{\sigma^2}{P_p} \quad (7)$$

3.2. Time varying channels

The following section illustrates the evaluation of MMSE over time varying channels. It has been shown in [15] that the MSE of the LS channel estimation can be written as

$$\text{MSE} = \frac{\sigma^2}{(Q+1)LN_t} \text{tr} \left\{ (A^H . A)^{-1} \right\} \quad (8)$$

Where $Q = 2 \lceil f_{\max} T \rceil$, f_{\max} is the maximum Doppler frequency, T is the duration of an OFDM packet and A is a $N_p \times N_t L (Q+1)$ matrix. The ceiling of a number is shown by $\lceil \cdot \rceil$.

The least square channel estimation, supposing $A^H A$ has full rank and $N_p \geq N_t L (Q+1)$ (we should have at least $N_t L (Q+1)$ pilot clusters) [15].

1-D case of time invariant channels is obtained from 2-D case by formally setting $Q=0$.

To satisfy condition (6) in SISO-OFDM systems pilots have to be equally-powered and equally-spaced [16, 17]. In MIMO-OFDM systems, all pilot must be equally-powered (all antennas transmit

pilots with the same power), equally-spaced and phase shift orthogonal [14, 15, 18].

For the purpose of comparison, we present different scenarios and the constraints they impose on the optimal pilot sequences in the following table:

Table 1: Optimal Pilot Sequences For Different Scenarios

Configuration	Requirements
SISO	Equally-powered and equally-spaced (N_p is divisor of N) [16, 17]
MIMO flat fading ($L=1$)	Equally-powered, equally-spaced and orthogonal [14, 15]
MIMO frequency selective fading ($L>1$)	Equally-powered, equally-spaced and phase shift orthogonal [14, 15]

Condition (6) may not be necessarily achieved if requirements in table 1 are not satisfied, because the data power distribution is not forced to be uniform if it does not maximize channel capacity or it does not achieve low PAPR to avoid nonlinear distortion of pilots at the transmit power amplifier design or there's no solution to achieve phase shift orthogonal pilots [15]. Indeed, non uniform power loading at the transmitter will provide better overall system performance. In this case the optimum pilot is unachievable and suboptimum pilot design may be an irregular pilot design (condition (6) is not satisfied). In order to minimize MSE channel estimation in Eq. (8) or (5), we propose an algorithm based on differential evolution algorithm in section 4.

3.3. Adaptive pilot design with optimal sequences

An adaptive pilot scheme is proposed to offer high throughput gains, this dynamic pilot scheme investigate time selectivity of the channel to reduce the number of pilot symbols. When the channel does not change during the transmission of K OFDM blocks, we just use the first OFDM block for the transmission of pilot subcarriers and we keep the same channel estimation during transmission of the following $(K-1)$ blocks. This approach allows varying the amount of the pilot-symbols according to channel time selectivity. For the purpose of comparison, Figure 2 shows a classical regular pattern and proposed irregular pattern.

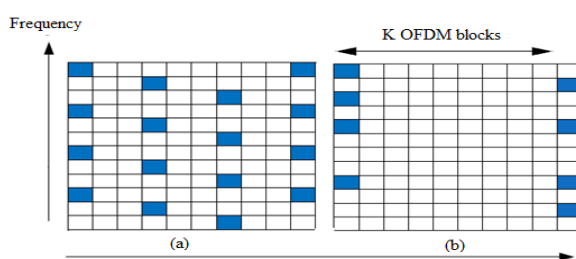


Figure 2: Comparison Between Regular Pattern (A) and Proposed Pattern (B)

Good bit error rate (BER) performance can be achieved by the proposed pattern if K satisfies the following inequality

$$K.T \leq T_{coh}$$

Where T is the time duration of one OFDM symbol and T_{coh} is channel coherence time.

The Doppler spread f_d , and the coherence time T_{coh} , are reciprocally related over Rayleigh fading channel:

$$T_{coh} \approx \frac{9}{16.\pi.f_d}$$

Therefore, K is an integer chosen to satisfy the following inequality

$$K \leq \frac{9}{16.\pi.f_d.T}$$

An optimal choice of K is

$$\begin{cases} K = \left\lceil \frac{9}{16.\pi.f_d.T} \right\rceil & \text{if } Q = 0 \\ K = 1 & \text{if } Q > 0 \end{cases}$$

The ceiling of a number is shown by $\lceil \cdot \rceil$.

We propose a novel concept called useful throughput D_u to characterize throughput available to data transmission by taking into account the number of data subcarriers and pilot subcarriers in a frame of K OFDM symbols.

The pilot symbols causes degradation in terms of useful throughput (data symbols), which can be expressed as

$$D_u = \frac{\text{Number of data tones}}{\text{Number of pilot tones}}.D$$

Where D and D_u are the original throughput (data symbols+pilot symbols) and useful throughput (data symbols), respectively.

Consequently for the purpose of comparison, the ratio between D and D_u is derived for regular pattern and proposed scheme:

$$\frac{D_u}{D} (\text{regular pattern}) = \left(1 - \frac{N_p}{N}\right)$$

$$\frac{D_u}{D} (\text{adaptive pattern}) = \left(1 - \frac{N_p}{K.N}\right)$$

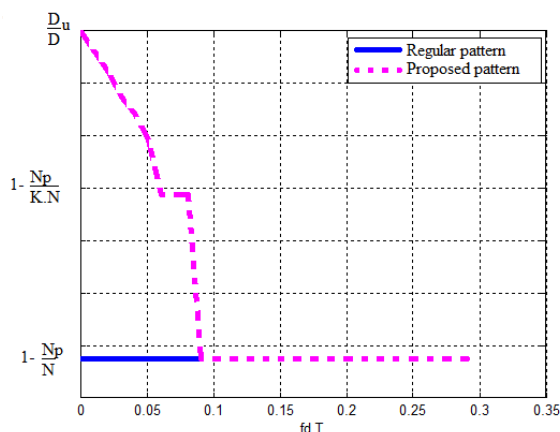


Figure3: Comparison Between Regular Pattern and Proposed Pattern In Terms Of Normalized Useful Throughput

Figure 3. Shows that the throughput gain of our proposed adaptive pattern is significant.

$$\text{throughput gain} = \frac{N_p}{N} \left(1 - \frac{1}{K}\right)D$$

Proposed scheme improved the normalized useful throughput ($f_{d \max} T \leq 0.09$). Especially, throughput gain exceeds 10 % for moderate time selective channels ($f_{d \max} T \leq 0.01$) and moderate pilot density ($N_p/N=4$).

In this section, we deal with designing adaptive pilot symbol pattern for MIMO OFDM systems. We demonstrated that an optimal adaptive pattern depends on maximum delay spread τ_{\max} and maximum Doppler spread $f_{d, \max}$. In order to allow the transmitter to update the pilot pattern, a feedback is required between the receiver and transmitter. The main idea is the usage of existing feedback, for example in LTE there is a so-called Channel Quality Indicator (CQI) that is reported by the user equipment back to an eNodeB.



4. SUBOPTIMUM PILOTS DESIGN SCHEME

In this section, we describe an algorithm for pilot design optimization to think out a suboptimum pilot design scheme. In order to optimize the positions of pilot tones, we use DE algorithm to find pilot design minimizing the MSE cost function given by Eq. (8) or (5). Our goal is to search for the suboptimal pilot positions that will minimize the MSE. The optimal pilot design position can be derived from an extensive matching of all possible positions. But the exhaustive search will be extremely time consuming and thus we will employ the optimization process on the cost function defined in Eq. (8) or (5) and we will reduce the computational complexity using Gershgorin theorem. DE optimization process for discrete variable is discussed in [7].

4.1. Fitness function of differential evolution algorithm

Proposed algorithm seeks the most appropriate frequency pilot space that minimizes the mean square error of channel estimate. We use the MSE function as an objective function for the DE algorithm. The matrix inversion in Eq. (5) will increase the computational load of the optimizer; we can reduce this computational complexity using Gershgorin theorem.

We will employ the optimization process on the cost function defined in Eq. (5) as

$$MSE = \frac{\sigma^2}{LN_t} \text{tr}\{(A^H .A)^{-1}\}$$

Let $\lambda_1, \lambda_2, \dots, \lambda_L$ the eigenvalues of $A^H .A$, then the eigenvalues of $(A^H .A)^{-1}$ are $1/\lambda_1, 1/\lambda_2, \dots, 1/\lambda_L$.

Since the trace of a matrix is the sum of its eigenvalues, we have $\text{tr}\{(A^H .A)^{-1}\} = \sum_{p=1}^{N_t L} \frac{1}{\lambda_p}$

$A^H .A$ can be expressed as

$$A^H .A = \begin{pmatrix} P_p & x & x & x \\ x & P_p & x & x \\ x & x & \dots & x \\ x & x & x & P_p \end{pmatrix}$$

where P_p is the diagonal element of matrix $A^H .A$. P_p is a fixed power dedicated for training.

Let a_{ij} ($i=1, \dots, N_t L ; j=1, \dots, N_t L$) the elements of matrix $A^H .A$ and $R_{max} = \max(R_i)$ the maximum radius of the Gershgorin disk, where $R_i = \sum_{\substack{j=1 \\ j \neq i}}^{N_t L} |a_{ij}|$.

According to Gershgorin theorem [22] we have

$$|P_p - \lambda_i| \leq R_{max}, i=1, \dots, N_t L$$

Therefore, if $P_p > R_{max}$ we have the following inequality

$$\text{tr}\{(A^H .A)^{-1}\} = \sum_{p=1}^L \frac{1}{\lambda_p} \leq \frac{N_t L}{P_p - R_{max}} \tag{9}$$

From (5) and (8), the upper bound of MSE that we can use as an objective function for DE is obtained by:

$$MSE \leq \frac{\sigma^2}{P_p - R_{max}} \tag{10}$$

Knowing that the power dedicated for the pilot is fixed, the value of diagonal element P_p does not change during the optimization process. Therefore, we will use R_{max} as the fitness function for differential evolution algorithm.

4.2. Proposed algorithm

Based on the discussion above, the proposed algorithm is described by the following steps.

- 1) We first specify the channel parameters, the impulse response length $N_t L$ and Doppler spread as well as maximum number of iterations ($N_i = 1$ in SISO-OFDM schemes).
- 2) The total number of pilot tones in one OFDM frame is chosen greater or equal to $(Q+1)N_t L$. Then, we define pilot power allocation and initial population for DE (initial pilot design). The initial pilot positions are initialized at equally spaced random values. The use of equally spaced pilot tones as initial population is a good choice for fast convergence.
- 3) In order to get the best positions of the pilots, we use modified DE algorithm mentioned in [7]. The pilot positions are improved by the mutation, crossover, and selection operators. The optimization procedure is repeated until we find a solution for the optimization problem defined in Eq. (6) or until the end of maximum number of iterations.

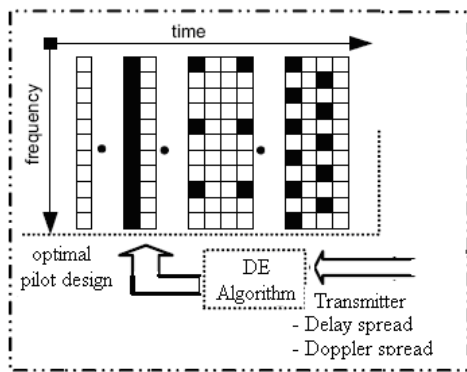


Figure 4: Description Of Transmitter Adaptive Pilot Arrangement

any random irregular pilot sequences in terms of MSE and BER.

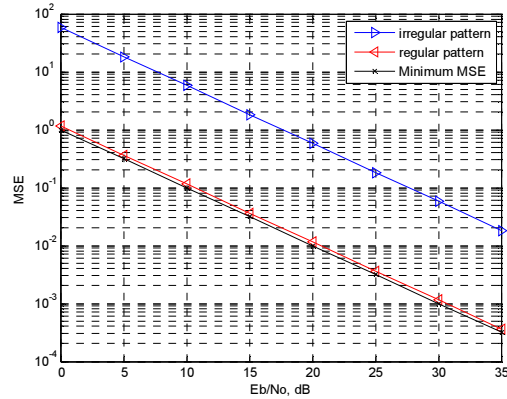


Figure 6: MSE Versus SNR

5. SIMULATION RESULTS

5.1. Suboptimum Pilots Design Scheme

We consider an OFDM system with $N = 256$ subcarriers of which 8 serve as pilot tones ($N_p = 8$), and an invariant multipath channel model ($Q=0$) with 4 paths according to Jackes model ($L=4$ and $N_p > L$) and 4-QAM modulation. For optimizing the pilot placement, we use the differential evolution parameters of a population size of 10, scale factor of 0.8 and crossover of 0.9.

In order to evaluate the performance of the proposed method, we deal with two pilot configurations: equi-powered and unequi-powered pilots.

- **Equi-powered pilots**

We simulate an OFDM system with equi-powered pilot tones (we use pilot symbols $2+2i$, in this case pilot power is the same for all pilot tones).

Figure 5 shows the designed optimal set for pilot tones using our method. As it can be seen, equi-spaced pilots are the optimal pilot design as it is shown in [16, 17] with equi-powered pilot tones. The use of equally spaced pilot tones as initial population is a good choice for fast convergence (number of iterations=1 and $MSE=MSE_{min}$).

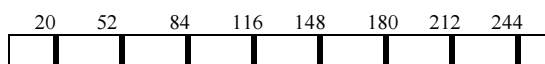


Figure 5: The Optimal Placement For Equi-Powered Pilot Tones (Regular Pattern)

As shown in Figs. 6 and 7, using equi-spaced pilot sequences (regular pattern) outperforms the use of

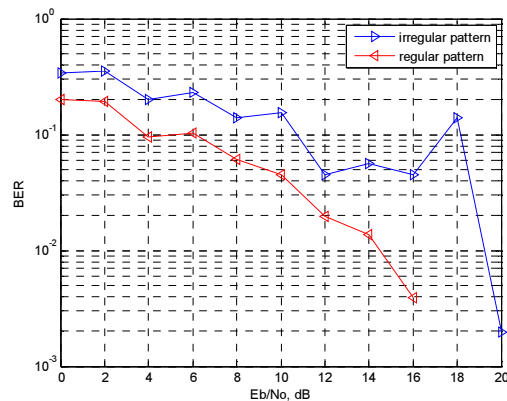


Figure 7: BER Versus SNR

- **Unequal-powered pilots**

We simulate an OFDM system with unequal-powered pilot tones. We use as pilot symbols $2+2i$, $2+2i$, $2+2i$, $1+i$, $1+i$, $1+i$, $1+i$ and $2-2i$. Pilot symbols are randomly chosen.

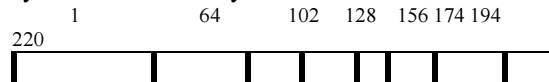


Figure 8: The Optimal Placement For Unequi-Powered Pilot Tones (Irregular Arrangement)

Figure 8 shows that the optimal set for pilot tones is an irregular arrangement. In this case of unequal-powered pilot tones, we have numerically evaluated the MSE for regular and irregular pilot arrangements, it has been found that irregular pilot arrangement evaluate MSE and BER better than equi-spaced pilot arrangement (Figure 9 and 10). Therefore, regular pilot arrangements are not always optimal.

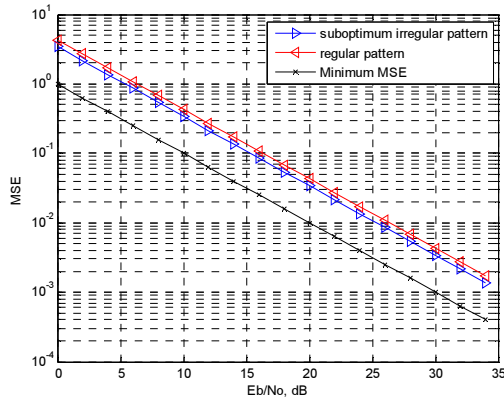


Figure 9: MSE Versus SNR

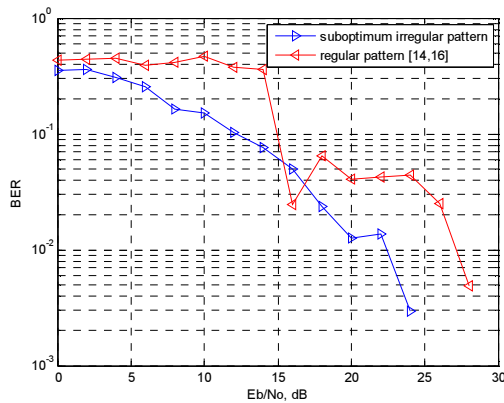


Figure 10: BER Versus SNR

5.2. Performance of the proposed pattern

In this section, we present simulation result of the proposed pattern and we compare the throughput gain of a system using proposed adaptive pilot pattern, against a system using fixed pilot pattern defined by LTE standard

In LTE standard, pilot symbols are transmitted during the first and fifth OFDM symbols of each slot when the short cyclic prefix (CP) is used and during the first and fourth OFDM symbols when the long CP is used [24]. The frequency spacing between two successive pilot symbols is 6 subcarriers. The following figure illustrates the pilot scattering for this case:

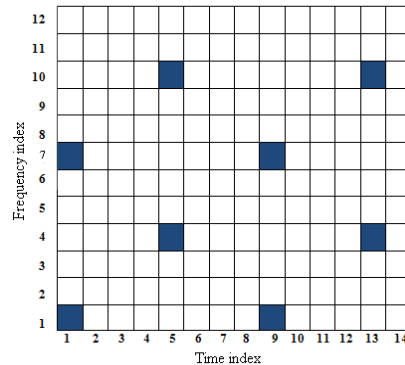


Figure 11: Pilot Structure For LTE System

Consequently for the purpose of comparison, the ratio between D and D_u is derived for LTE regular pattern and proposed scheme:

$$\frac{D_u}{D} (\text{LTE regular pattern}) = \left(1 - \frac{N_p}{4N}\right) \approx 0,958$$

$$\frac{D_u}{D} (\text{adaptive pattern}) = \left(1 - \frac{N_p}{K \cdot N}\right) = 1 - \frac{1}{6K}$$

A typical LTE system shall support users moving with velocities up to 500km/h, which corresponds a Doppler frequency of approximately 1150 Hz at a carrier frequency of 2.5GHz, the duration of one OFDM symbol is $T=72\mu s$. OFDM System is simulated using the parameters on downLink LTE system.

For the purpose of comparison between fixed pilot pattern in LTE and proposed pattern, in Table 2 we present useful throughput for two different velocities:

Table 2: Comparison between LTE pattern and proposed pattern in terms of normalized useful throughput

	LTE pattern	Proposed pattern
$v=30\text{km/h}, f_d=70\text{Hz}$ ($f_d \cdot T=0.01$)	$\frac{D_u}{D} = 0.958$	$\frac{D_u}{D} = 0.992$ ($K=17$)
$v=300\text{km/h}, f_d=695\text{Hz}$ ($f_d \cdot T=0.1$)	$\frac{D_u}{D} = 0.958$	$\frac{D_u}{D} = 0.875$ ($K=1$)

According to previous results (Table 2), throughput gain of the proposed pattern shall achieve more than 4 % over time varying channels with moderate mobility.

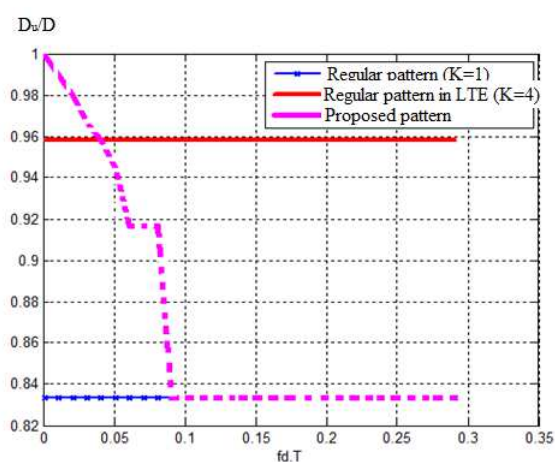


Figure 12: Normalized Throughput Versus $F_d.T$ For Different Pattern Configurations

It is shown in Figure 12 that the proposed adaptive pattern performs better than LTE regular pattern in terms of throughput gain when $f_d.T$ is less than 0.04. When $f_d.T$ is greater than 0.04, LTE regular pattern seems performing better than adaptive pattern in terms of throughput gain, but it has inferior performance in terms of channel estimation because channel is time varying inside one OFDM block. It can be seen in Figure 13 that the proposed adaptive pattern has the same performance as regular pattern ($K=1$).

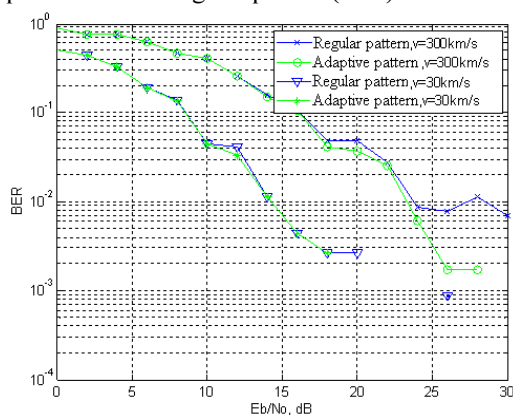


Figure 13: BER Versus SNR

6. CONCLUSION

The proposed DE method for OFDM pilot design optimization is specifically tailored to optimal irregular pilot arrangements over time varying channels when the optimality of regular patterns is not achievable. The algorithm can be

implemented in a computationally efficient manner using the upper bound of MSE for the fitness function instead of using MSE directly. We introduced adaptive pilot patterns that adjust density of the pilot symbols in time domain to time selectivity of channel.

This study has demonstrated the effectiveness of the DE method as a design tool for irregular pilot arrangements in SISO and MIMO-OFDM systems. Furthermore, the proposed adaptive patterns improve throughput gain compared to fixed pilot patterns and LTE pattern.

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