

DECENTRALIZED PI CONTROLLER DESIGN FOR NON LINEAR MULTIVARIABLE SYSTEMS BASED ON IDEAL DECOUPLER

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ABSTRACT

This paper presents a multivariable controller design based on ideal Decoupler with high closed-loop system performance and robustness. The method is effective despite the values and positions of the right half plane zero. The validity of the proposed multivariable control system design and controller tuning is confirmed through simulation results using a benchmark four tank system, which is a Two input Two output (2x2) process and a real time boiler turbine system which is a Three-Input Three-Output (3x3) process.

Keywords: *Multivariable control, Ideal decoupler, Quadruple tank system, Boiler Turbine System, Decentralized PI controller.*

1. INTRODUCTION

Most of the industrial and chemical processes are multivariable in nature. In many cases cross-coupling between inputs and outputs is low and hence conventional controllers can be employed. If multivariable systems exhibit stronger cross-coupling between process inputs and outputs, multivariable controllers should be applied in order to achieve satisfactory performance.

A popular approach to deal with control loop interactions is to design non-interacting or decoupling control schemes. The role of decouplers is to decompose a multi variable process into a series of independent single-loop sub-systems. If such a controller is designed, complete or ideal decoupling occurs and the multivariable process can be controlled using independent loop controllers.[1][2]

In conventional decoupler design, multivariable interactions are handled by designing additional cross controllers along with loop controllers. The block diagram of this structure is shown in fig 1. This decoupling structure addresses the servo problem only. If the loop controllers are tuned on line then the diagonal compensators are to be recalculated. The inter dependence of the decoupling elements becomes the major disadvantage when one of the loop is placed under

manual mode. The decoupling effect will then be lost. [3][4]

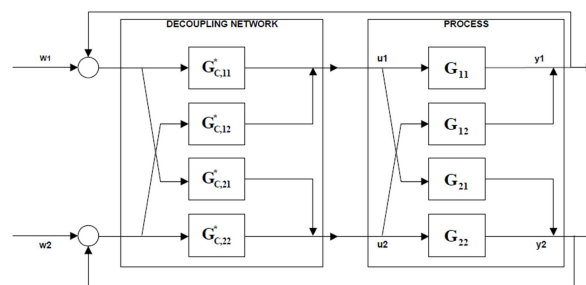


Figure 1 Block Diagram of Conventional Decoupler Control Structure

To address these problems and to eliminate multivariable interactions, ideal or full decoupler is suggested by Zalkind. This control structure is shown in fig.2. The main objective of this study is to design control structure to handle multivariable interactions for complex multivariable systems such as four tank interacting system and boiler turbine system.

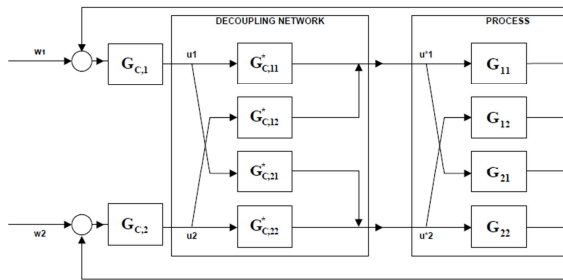


Figure 2 Block Diagram of the Proposed Control Structure

This is done by the following steps: First the mathematical modelling is done for the non linear multivariable system. Next the linearized model is obtained using the steady state conditions. Then decoupler matrix is derived using the model and finally PI ontroller (Z-N tuned) is designed for the decoupled system.[5]

The concepts behind this study are organized as follows: Following section gives the mathematical modeling of Quadruple Tank System (QTS) and the decoupler design for 2x2 system .The mathematical modelling and decoupler design of 3x3 Boiler Turbine System (BTS) is explained in Section III. Results and Discussions are presented in Section IV followed by the conclusion in Section V.

2. QUADRUPLE TANK SYSTEM

A quadruple tank apparatus which was proposed in [6] has been used in chemical engineering laboratories to illustrate the performance limitations for multivariable systems posed by ill-conditioning, right half plane transmission zeros and model uncertainties.

The quadruple tank system consists of four interconnected tanks and two pumps. The schematic of the quadruple tank equipment is presented in fig.3.

2.1 Mathematical Model of Quadruple Tank System:

The first principle mathematical model for this process using mass balances and Bernoulli's law is derived from the state equations (1) to (4)

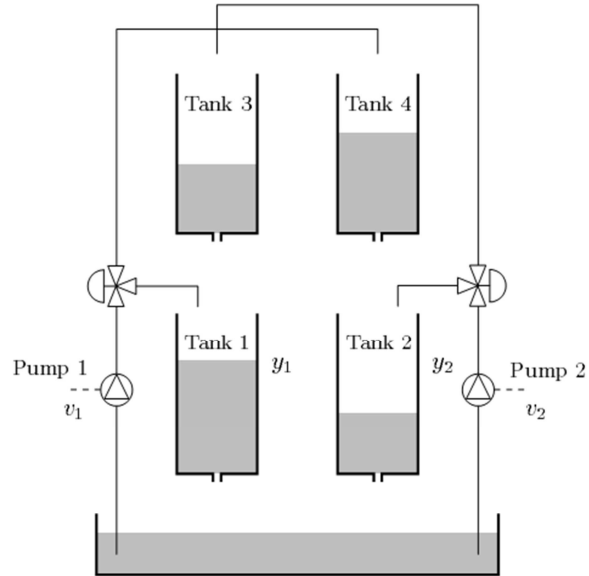


Figure 3 Schematic of Quadruple Tank System

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_1} \sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1 \quad (1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_2} \sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2 \quad (2)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} v_2 \quad (3)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} v_1 \quad (4)$$

For literalizing the model, the Taylor's series equation (5) is applied at steady state operating points,

$$\dot{x} = \dot{h} \approx f(\bar{h}, \bar{v}) + \frac{\partial f(h,v)}{\partial h^T} / h = \bar{h} + \frac{\partial f(h,v)}{\partial v^T} / v = \bar{v} \quad (5)$$

The matrix form of state model is obtained as given in (6)

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \quad (6a)$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x \quad (6b)$$

The transfer function matrix is given in equation (7) for minimum phase and non-minimum phase operating points.

$$G_s = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.5}{1+90s} \end{bmatrix} \quad (7a)$$

$$G_+(s) = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{bmatrix} \quad (7b)$$

The transfer matrix G has two zeros one of them is always in the left half of s-plane, but the other zero can be located either in left half or right half of s-plane based on the position of three way valves. So, the system is in minimum phase, if the values of γ_1 and γ_2 satisfy the condition $0 < \gamma_1 + \gamma_2 < 1$ and in non-minimum phase, if the values of γ_1 and γ_2 satisfy the condition $1 < \gamma_1 + \gamma_2 < 2$.

2.2 Decoupler Design:

The ideal decoupler is designed by the method of Zalkind and Luyben [7]. The Decoupler design equations are

$$G_{c,11}^* = G_{c,22}^* = 1$$

$$G_{c,12}^* = \frac{-G_{12}G_{c,22}^*}{G_{11}} \quad \text{and} \quad G_{c,21}^* = \frac{-G_{21}G_{c,11}^*}{G_{22}}$$

The decoupler matrices designed for minimum phase and non minimum phase systems are given in equation (8a) and (8b) respectively [8].

$$D_-(s) = \begin{bmatrix} 1 & \frac{-0.577}{(1+23s)} \\ \frac{-0.5}{(1+30s)} & 1 \end{bmatrix} \quad (8a)$$

$$D_+(s) = \begin{bmatrix} 1 & \frac{-1.667}{(1+39s)} \\ \frac{-1.5625}{(1+56s)} & 1 \end{bmatrix} \quad (8b)$$

3. BOILER TURBINE SYSTEM

Boiler system is a MIMO system with more loop interaction. To avoid loop interactions, MIMO systems can be decoupled into separate loops known as single input, single output (SISO) systems. The general block diagram for the boiler-turbine system and its whole operation is shown in the fig.4.

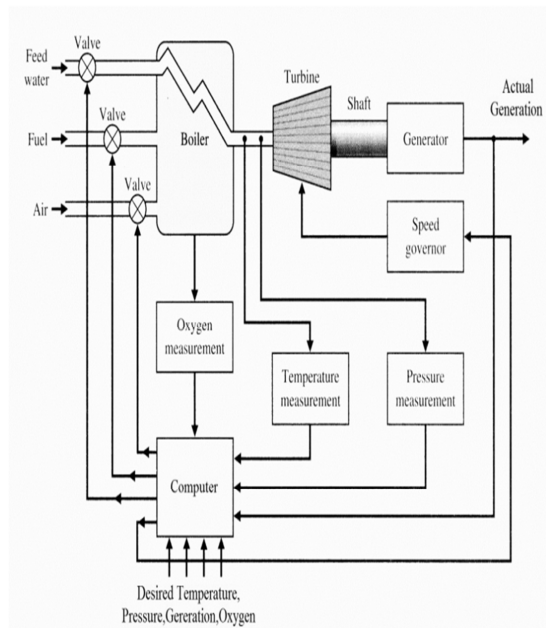


Figure 4 Block Diagram For The Boiler-Turbine System

The two major operations in the boiler-turbine system are generation of steam by heating the water into steam. The steam is heated further to obtain super steam for maximum power generation and to reduce wastage of steam. The steam is used to rotate the turbine for creating mechanical energy. The turbine is rotated in a magnetic field to obtain the magnetic lines of force. The steam which is a conducting device when cuts the magnetic lines of force created in the turbine induces power.

3.1 Modeling Boiler Turbine System:

The model of Bell and Astrom is taken as a real plant among various nonlinear models for the boiler-turbine system. The model represents a boiler-turbine generator for overall wide-range simulations and is described by a third-order MIMO nonlinear state equation. The three state variables are Drum steam pressure, Electric power, Steam water fluid density in the drum, respectively. The three outputs are drum steam pressure, electric power, and drum water-level deviation, respectively. [9]

The nonlinear model is linearized using Taylor series expansion at the operating point, $y_0 = (y_{10}, y_{20}, y_{30})$, $x_0 = (x_{10}, x_{20}, x_{30})$, $u_0 = (u_{10}, u_{20}, u_{30})$. The result of linearization is as follows:

$$\dot{\bar{x}} = A\bar{x}(t) + B\bar{u}(t) \quad (9)$$

$$\bar{y}(t) = C\bar{x}(t) + D\bar{u}(t) \quad (10)$$

Where

$$A = \begin{bmatrix} -\frac{0.0162}{8}u_{20}x_{10}^{1/8} & 0 & 0 \\ \left(\frac{6.57}{80}u_{20} - \frac{1.44}{80}\right)x_{10}^{1/8} & -\frac{1}{10} & 0 \\ \left(\frac{0.19}{85} - \frac{1.1}{85}u_{20}\right) & 0 & 0 \end{bmatrix} \quad (11)$$

$$B = \begin{bmatrix} 0.9 & -0.0018x_{10}^{9/8} & -0.15 \\ 0 & -\frac{0.73}{10}x_{10}^{9/8} & 0 \\ \left(\frac{0.19}{85} - \frac{1.1}{85}u_{20}\right) & -\frac{1.1}{85}x_{10} & \frac{141}{85} \end{bmatrix} \quad (12)$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \left(5\frac{\partial a_{cs}}{\partial x_1} + \frac{1.1}{9}\frac{\partial q_e}{\partial x_1}\right) & 0 & \left(0.065 + 5\frac{\partial a_{cs}}{\partial x_3}\right) \end{bmatrix} \quad (13)$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.2533 & 0.00474x_{10} & -0.014 \end{bmatrix} \quad (14)$$

The variables y , x , and u are the differences of the output, state, and input, respectively from the corresponding operating points. The operating points are determined based on a nominal operation of the plant. Considering that the model represents a 210 MW unit, the operating point for power output y_2 is 210 MW. This yields the corresponding pressure y_1 as 143 kg/cm₂ from the balance of the plant model. The operating point for water level y_3 must be zero in order to keep the water level in the middle of the drum, which is 50% in the drum level. Then, the operating points for other remaining variables can be calculated by neglecting the derivative terms. The resulting operating points are $y_0 = (143, 210, 0)$, $x_0 = (143, 210, 402.759)$, $u_0 = (0.2, 0.7, 0.4)$.

The constant matrices A , B , C , and D are evaluated at these operating points as follows:

$$A = \begin{bmatrix} -0.0026 & 0 & 0 \\ 0.0735 & -0.1000 & 0 \\ -0.0068 & 0 & 0 \end{bmatrix} \quad (15)$$

$$B = \begin{bmatrix} 0.9000 & -0.4787 & -0.1500 \\ 0 & 19.4120 & 0 \\ 0 & -1.8500 & 1.6588 \end{bmatrix} \quad (16)$$

$$C = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 0.0062 & 0 & 0.0033 \end{bmatrix} \quad (17)$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.2533 & 0.6778 & -0.0140 \end{bmatrix} \quad (18)$$

Then, a simple algebraic operation with Laplace transform gives transfer functions as follows:

$$Y(s) = [C(sI - A)^{-1}B + D]U(s) \quad (19)$$

$$Y(s) = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} U(s) \quad (20)$$

The transfer function matrix is obtained by the MATLAB coding where the A , B , C , D Matrix are used. The output of the coding is obtained as the transfer function matrix of 3x3 matrix.

$$G_{11} = \frac{0.9}{s + 0.002635} \quad (21)$$

$$G_{12} = \frac{-0.4787}{s + 0.002635} \quad (22)$$

$$G_{13} = \frac{-0.15}{s + 0.002635} \quad (23)$$

$$G_{21} = \frac{0.0661}{s^2 + 0.1026s + 0.0002635} \quad (24)$$

$$G_{22} = \frac{19.41s + 0.01599}{s^2 + 0.1026s + 0.0002635} \quad (25)$$

$$G_{23} = \frac{-0.01102}{s^2 + 0.1026s + 0.0002635} \quad (26)$$

$$G_{31} = \frac{0.2533s^2 + 0.006232s - 2.045e^{-005}}{s^2 + 0.002635s} \quad (27)$$

$$G_{32} = \frac{0.6778s^2 - 0.007334s - 5.358e^{-006}}{s^2 + 0.002635s} \quad (28)$$

3.2 Decoupler Design:

For the boiler turbine system the third order decoupler matrix is :

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

According to ideal decoupling procedure,[10],[11] the diagonal elements of the decoupler is taken unity thus $D_{11}=D_{22}=D_{33}=1$. Hence the decoupler matrix D will become

$$D = \begin{bmatrix} 1 & D_{12} & D_{13} \\ D_{21} & 1 & D_{23} \\ D_{31} & D_{32} & 1 \end{bmatrix}$$

The transfer function of decoupled system is given by the equation (29)

$$G^*D = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} 1 & D_{12} & D_{13} \\ D_{21} & 1 & D_{23} \\ D_{31} & D_{32} & 1 \end{bmatrix} \quad (29)$$

For an ideal decoupling, the off diagonal elements of G^*D matrix are equal to zero so by taking the product of the matrices $D * G$ and equating the diagonal elements of the product, the equations for D_{12} , D_{13} , D_{21} , D_{23} , D_{31} , and D_{32} can be obtained as given in (30 to 36)

$$D_{12} = \frac{G_{13}G_{32} - G_{12}G_{33}}{G_{11}G_{33} - G_{31}G_{13}} \quad (30)$$

$$D_{13} = \frac{G_{23}G_{12} - G_{22}G_{13}}{G_{22}G_{11} - G_{21}G_{12}} \quad (31)$$

$$D_{21} = \frac{G_{31}G_{23} - G_{21}G_{33}}{G_{33}G_{22} - G_{23}G_{32}} \quad (32)$$

$$D_{32} = \frac{G_{31}G_{12} - G_{32}G_{11}}{G_{11}G_{33} - G_{31}G_{13}} \quad (33)$$

$$D_{23} = \frac{G_{21}G_{13} - G_{23}G_{11}}{G_{22}G_{11} - G_{21}G_{12}} \quad (34)$$

$$D_{31} = \frac{G_{32}G_{21} - G_{31}G_{22}}{G_{33}G_{22} - G_{23}G_{32}} \quad (35)$$

and the Decoupler Matrix (D) is given in equation (36)

$$D = \begin{bmatrix} 1 & \frac{G_{13}G_{32} - G_{12}G_{33}}{G_{11}G_{33} - G_{31}G_{13}} & \frac{G_{12}G_{23} - G_{13}G_{22}}{G_{22}G_{11} - G_{21}G_{12}} \\ \frac{G_{31}G_{23} - G_{21}G_{33}}{G_{33}G_{22} - G_{23}G_{32}} & 1 & \frac{G_{21}G_{13} - G_{23}G_{11}}{G_{22}G_{11} - G_{21}G_{12}} \\ \frac{G_{32}G_{21} - G_{31}G_{22}}{G_{33}G_{22} - G_{23}G_{32}} & \frac{G_{12}G_{31} - G_{32}G_{11}}{G_{11}G_{33} - G_{31}G_{13}} & 1 \end{bmatrix} \quad (36)$$

Apply the D matrix equation in GD matrix to get the diagonal value of GD matrix which is nothing but the three SISO to control the three output variables.

The diagonal elements of G^*D matrix are given in equations (37) to (39)

$$GD_{11} = G_{11} + G_{12}D_{21} + G_{13}D_{31} \quad (37)$$

$$GD_{22} = G_{21} + G_{21}D_{12} + G_{23}D_{32} \quad (38)$$

$$GD_{33} = G_{33} + G_{31}D_{13} + G_{32}D_{23} \quad (39)$$

Applying equations (30) to (35) the diagonal elements are evaluated as specified in equations (40), (41) and (42)

$$GD_{11} = \frac{0.49291s^3 + 0.09907s^2 + 0.17153s + 0.12128}{0.27174s^4 + 0.09502s^3 + 0.2352s^2 + 0.000666s} \quad (40)$$

$$GD_{22} = \frac{0.49291s^3 + 0.09938s^2 + 0.17153s + 0.21}{0.0254s^4 + 0.0578s^3 + 0.03627s^2 + 0.00369s} \quad (41)$$

$$GD_{33} = \frac{0.49291s^3 + 0.0628s^2 + 0.17205s + 0.00151}{17.469s^3 + 0.09206s^2 + 0.0012s} \quad (42)$$

The above equations (40) to (42) give the decoupled boiler turbine system as three independent SISO systems.

4. RESULTS AND DISCUSSIONS

The designed controllers for the multivariable processes are simulated using MATLAB / SIMULINK toolbox presented in [12] and the following results are obtained for 2x2 QTS and 3x3 BTS. The responses of the quadruple tank process for both minimum phase and non minimum phase operating points are obtained using ZN tuned PI controller and ZN tuned decoupled PI controller. The minimum phase responses are shown in fig. 5 and the non minimum phase responses are shown in fig. 6. The responses for boiler turbine system with and without decoupler are presented in fig. 8 and fig. 7 respectively.

It has been observed from the responses that simple PI controller with conventional tuning is not sufficient to handle multivariable interactions. Using decoupler, the performance gets improved because the system is entirely decoupled and the variables are independent of each other.

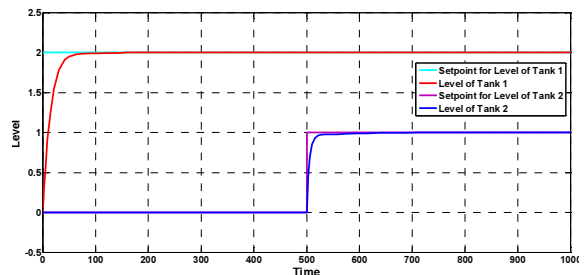
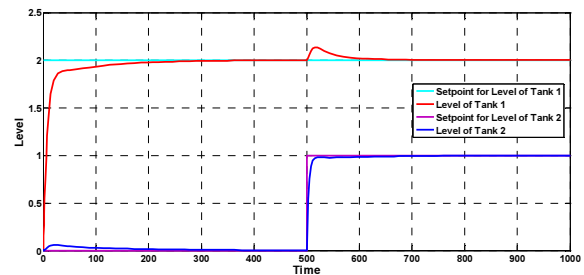


Figure 5 Response of QTS in Min Phase for a) PI Controller without Decoupler and b). PI Controller with Decoupler

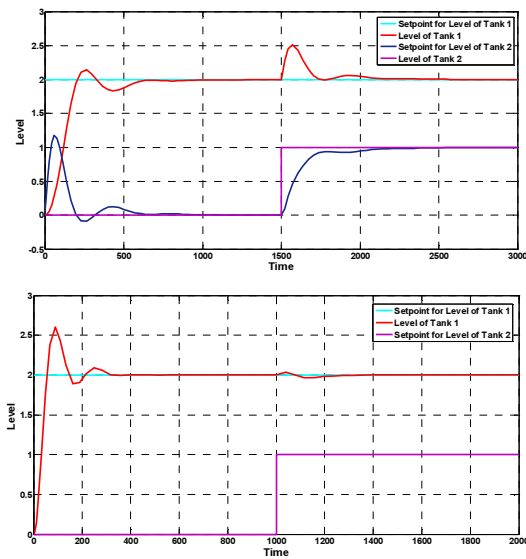


Figure 6 Responses of QTS in Non Minimum Phase for a) PI Controller without Decoupler and b). PI Controller with Decoupler

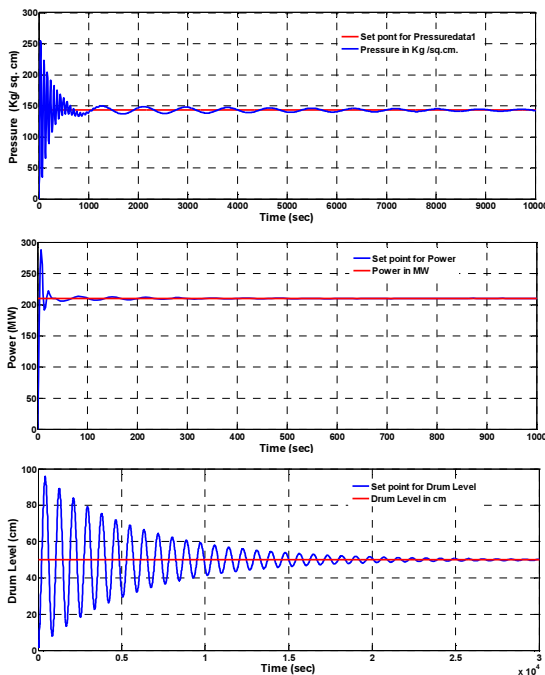


Figure 7 Responses of BTS a) Pressure b). Power and c) Drum level for PI Controller without Decoupler

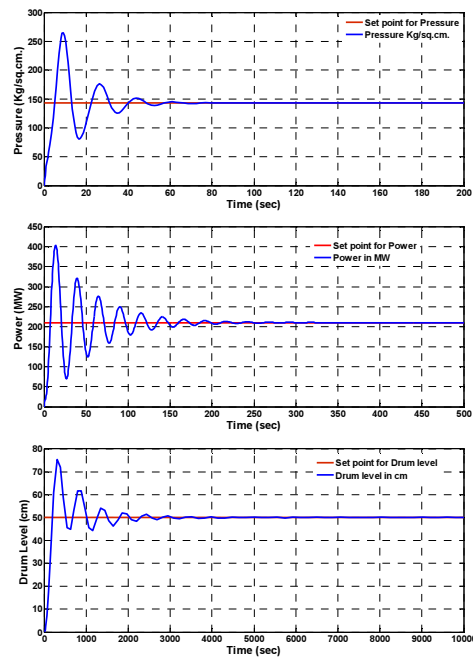


Figure 8 Responses of BTS a) Pressure b). Power & c) Drum Level for PI Controller with Decoupler

The quantitative comparison of various parameters for QTS is shown in Table 1 and that of BTS is given in Table 2. From the tables it is shown that the decoupler based design produces improved performance in the aspect of settling time and rise time. However the overshoots are slightly higher in this case.

5. CONCLUSION

In this paper, ideal decoupler design for a 2x2 MIMO system and a 3x3 MIMO system is discussed. Simulation results show that the decoupling elements are independent of the loop controllers and hence the on line tuning of the controllers will not result in redesigning of decoupling elements. Controller modes can be changed manually without loss of decoupling. This technique is not limited to servo problem alone. But the limitation is it requires process model and for non minimum phase system on line decoupling is not assured and hence inverse decoupling is suggested.



Table 1 Quantitative Comparison Of Controller Performance For QTS

Controller	Parameters	Minimum Phase		Non-Minimum Phase	
		Level 1	Level 2	Level 1	Level 2
PI Controller without decoupler	Settling Time (sec)	250	150	1380	1380
	Peak Overshoot (%)	1	2	10	25
	Rise Time (sec)	15	10	240	210
PI Controller with decoupler	Settling Time (sec)	90	50	600	800
	Peak Overshoot (%)	1	2	30	20
	Rise Time (sec)	12	10	80	65

Table 2 Quantitative Comparison Of Controller Performance For Boiler Turbine System

Controller	Parameters	Pressure	Power	Drum Level
PI Controller without decoupler	Settling time(sec)	1,000	500	25,000
	Peak overshoot (%)	67	88	80
	Rise time(sec)	80	30	500
PI Controller with decoupler	Settling time(sec)	80	260	4,500
	Peak overshoot (%)	78	95	50
	Rise time(sec)	7	10	120

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