

## AN EFFICIENT MULTIREOLUTION IMAGE DENOISING USING SMOOTHING SPLINE ESTIMATION

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### ABSTRACT

The image data is normally corrupted by additive noise during acquisition. This reduces the accuracy and reliability of any automatic analysis. For this reason, denoising methods are often applied to restore the original image. The effectiveness of image denoising depends on the estimation of noise variance of noisy image. In this paper, the Discrete Wavelet Transform (DWT) is used, which offers simultaneous localization in time and frequency domain and the noise variance is estimated using Smoothing Spline Estimation method. Multiresolution bilateral filter is the extension of bilateral filter. Bilateral filter is a nonlinear filter that does spatial averaging without smoothing out the edges. Multiresolution bilateral filtering is applied to the approximation sub bands of the transformed noisy image and thresholding is applied to the detailed sub bands which uses the estimated noise variance. With this process, a better PSNR is achieved.

**Keywords:** *Denoising, Smoothing Spline Estimation, Multiresolution Bilateral Filter, Noise Variance, Thresholding, Discrete Wavelet Transform.*

### 1. INTRODUCTION

Many denoising methods have been developed over the years, such as the Wiener filter [Wiener 1942], anisotropic diffusion [1] wavelet thresholding [2] bilateral filtering [3], total variation filtering [4], and non-local averaging [5]. Among these strategies, wavelet thresholding has become a preferred approach. In wavelet thresholding signal is rotten into its approximation (low-frequency) and detail (high-frequency) sub bands; as most of the image information is focused in few massive coefficients, the detail sub bands area unit processed with hard or soft thresholding operations. The crucial task in wavelet thresholding is that the threshold choice. Varied threshold choice methods are projected, as an example, VisuShrink [2] Sure Shrink [6] and BayesShrink [7]. Within the VisuShrink approach, a universal threshold that's a perform of the noise variance and also the variety of samples is developed supported the minima error live. The threshold worth within the Sure Shrink approach is perfect within the Stein's unbiased risk computer. The BayesShrink approach determines the threshold worth in Bayesian framework; forward a statistical distribution of the wavelet coefficients. These shrinkage strategies are later improved [8–12] by considering inter-scale and intra-scale correlations of the wavelet coefficients. The tactic in [8] models the neighborhoods of

coefficients at adjacent positions and scales as Gaussian scale mixture and applies Bayesian method of least squares technique to update the wavelet coefficients. In [14] total variance method was proposed. The method, referred to as the BLS-GSM technique, is one in every of the simplest denoising strategies within the literature in terms of the mean sq. error performance. An alternative to the wavelet-based denoising methods is the bilateral filter. In addition to image denoising applications, bilateral filters have also been used in texture removal, tone mapping, image enhancement, volumetric denoising, and exposure correction. It is shown that the bilateral filter is identical to the first iteration of the Jacobi algorithm (diagonal normalized steepest descent)[19] with a specific cost function. Another method [20] describes a fast implementation of the bilateral filter. The technique is based on a piecewise-linear approximation with FFT in the intensity domain and appropriate sub-sampling in the spatial domain. Later derives an improved acceleration scheme for the filter. They express the bilateral filter in a higher-dimensioned space where the signal intensity is added to the original domain dimensions. The bilateral filter can be expressed as simple linear convolutions in this augmented space followed by two simple nonlinearities, so that they can derive simple criteria for down-sampling the key operations and achieve acceleration

## 2. IMAGE DENOISING FRAMEWORK

Multiresolution analysis is an important tool for eliminating noise in signals. It is possible to distinguish between noise (or different frequency components of noise) and image information better at one resolution level than another. This is one of the reasons why wavelet thresholding is so successful in image denoising. Motivated by this fact, we put the bilateral filter in a multiresolution framework.

A signal is decomposed into its frequency sub bands, and bilateral filtering is applied to the approximation subbands. This method turns out to produce better results than the standard bilateral filter does as effective bilateral window size can be increased without losing details of the image. Bilateral filtering works in approximation subbands; however, some noise components can be identified and removed better in detail subbands. Therefore, further improvement can be achieved by including wavelet thresholding into the framework as well.

This denoising framework is illustrated in Fig 1. A signal is decomposed into its low- and high-frequency components through analysis filters. Bilateral filter is applied to low-frequency components; wavelet thresholding is applied to high-frequency components. wherever some noise elements will be known and removed effectively.

For the wavelet thresholding of detail sub bands, we used the BayesShrink soft-thresholding technique, which uses noise variances by means of smoothing spline estimation determine threshold values. This image denoising frameworks combines bilateral filtering and wavelet thresholding.

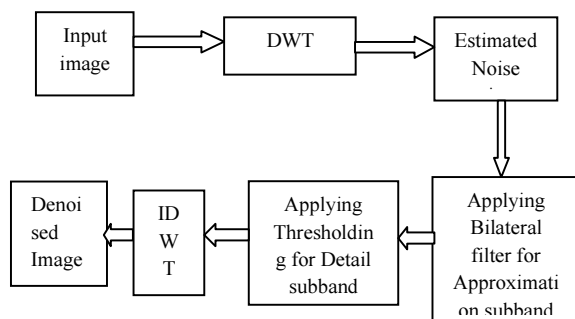


Figure 1 Block Diagram For Multiresolution Denoising Framework

### 2.1. Wavelet Decomposition

Wavelet is a mathematical function used to

divide a given function or continuous-time signal into different scale components. One can assign a frequency range to each scale component. Each scale component can then be studied with resolution that matches its scale. Thus the Wavelet is a multi resolution representation function. Wavelet transform is the discrete sampling of the wavelets. Based on the recurrence relations property of wavelet, the most common wavelet transforms, such as Daubechies wavelet transform, generate progressively finer discrete samplings of an implicit mother wavelet function; each resolution is twice that of the previous scale down-sampled by 2.

Therefore, using the one level wavelet transform, the input signal can be decomposed into two frequency coefficients, the approximation coefficients as the low frequency part and the detail coefficients as the high frequency part. This is the so called wavelet decomposition. With higher level decompositions, multi resolution representation of the signal can be achieved.



Figure 2 Right One Shows The Wavelet Decomposition Of The Left Picture.

Figure 2 shows the wavelet decomposition of an image. The left picture is the original image and the right one using 1-level wavelet decomposition. We can see from the right image that the top left small picture is the low frequency part which keeps the energy mostly while the others are detail information. Figure 2 Right one shows the wavelet decomposition of the left picture. It uses 1 level wavelet decomposition in the Matlab.

### 2.2. Noise Variance Estimation

#### 2.2.1. Donoho's Method:

Decomposed the noisy image into three sub bands with levels using discretewavelet transform; the noise variance of the image is estimated

$$\sigma = \frac{\text{median}(|y_i|)}{0.6745}(1)$$

Where  $y_i$  are wavelet coefficients in the first level in the diagonal direction of 2D wavelet



**2.2.2. Smoothing Spline Estimation with Discrete Wavelet Transform**

In this work uses a robust and efficient noise variation estimation scheme based on the wavelet transform and smoothing spline estimation[21]. Consider the following regression model.

$$y_i = x_i^T \beta + \varepsilon_i, i = 1, \dots, n(2)$$

Or  $y_i = f(t_i) + \varepsilon_i, t_i = \frac{i}{n}, i = 1, \dots, n$

It is often reasonable to assume that f is a smooth function of  $t \in [0,1]$ .

Define the following infinite dimensional space [16]

$$W_2(\text{per}) = \{f: f \text{ and } f' \text{ are absolutely continuous, } f(0) = f(1), f'(0) = f'(1), \int_0^1 (f''(t))^2 dt < \infty\} \quad (3)$$

As the model space for f. A smoothing spline estimate of f is the minimizer of the following penalized least square

$$\min_{f \in W_2(\text{per})} \left\{ \frac{1}{n} \sum_{i=1}^n \|y_i - f(t_i)\|^2 + \lambda \int_0^1 \|f''(t)\|^2 dt \right\} \quad (4)$$

Where the first part measures the goodness-of-fit, the second part is a penalty to the roughness of the estimate, and  $\lambda(0 \leq \lambda \leq \infty)$  is the so called smoothing parameter.

Define the following Space  $M_\alpha = \text{span}\{1, \sqrt{2}\sin 2\pi vt, \sqrt{2}\cos 2\pi vt, v = 1, \dots, \alpha\} \quad (5)$

Where the order  $\alpha$  is unknown and need to be selected in  $\{0,1, \dots, N\}$ . This work uses  $\alpha=2$ . In the infinite dimensional space, the following exact solution for (4) in which substitute  $W_2(\text{per})$  with  $M_\alpha$  holds ,

$$\hat{f}_\lambda(t) = \beta_1 + \sum_{v=1}^\alpha (\beta_{2v} \sqrt{2} \sin 2\pi vt + \beta_{2v+1} \sqrt{2} \cos 2\pi vt) \quad (6)$$

Let  $\hat{f}_\lambda(t) = (\hat{f}_\lambda(t_1), \dots, \hat{f}_\lambda(t_n))^T$ , and then rehearsing it in the matrix form.

$$\hat{f}_\lambda = X_\alpha \beta_\alpha = \frac{1}{n} X_\alpha D X_\alpha^T y = H(\lambda) y \quad (7)$$

Where  $y = (y_1, \dots, y_n)^T, \beta_\alpha = (\beta_1, \dots, \beta_{2\alpha+1})^T$

$$X_\alpha = \begin{bmatrix} \sqrt{2}\sin 2\pi t_1 & \sqrt{2}\cos 2\pi t_1 & \dots & \sqrt{2}\sin 2\pi t_1 & \sqrt{2}\cos 2\pi t_1 \\ \sqrt{2}\sin 2\pi t_2 & \sqrt{2}\cos 2\pi t_2 & \dots & \sqrt{2}\sin 2\pi t_2 & \sqrt{2}\cos 2\pi t_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sqrt{2}\sin 2\pi t_n & \sqrt{2}\cos 2\pi t_n & \dots & \sqrt{2}\sin 2\pi t_n & \sqrt{2}\cos 2\pi t_n \end{bmatrix}$$

$$D = \text{diag}(1, 1/(1 + \lambda(2\pi)^4), 1/(1 + \lambda(2\pi)^4) \dots, 1/(1 + \lambda(2\pi\alpha)^4), 1/(1 + \lambda(2\pi\alpha)^4)) \quad (8)$$

The generalized cross-validation (GCV) criterion proposed in [18] will be adopted to provide the optimal smoothing parameter  $\lambda$

$$\text{GCV}(\lambda) = \frac{\frac{1}{n} \sum_{i=1}^n \|y_i - \hat{f}_\lambda(t_i)\|^2}{\|1 - \text{tr} H(\lambda)/n\|^2} \quad (9)$$

Where n is the number of the wavelet coefficients. Then we can get the  $\hat{f}_\lambda(t)$  and  $\hat{\sigma}$

Algorithm for Smoothing Spline estimated with discrete wavelet transform

Input : The image with noise.

Processing :

Step 1: Decompose the noised image into sub bands with J levels using wavelet transform;

Step 2: Extract the coefficient vector  $y_i$  of the first level in the diagonal direction of 2D wavelet;

Step 3: Perform the smoothing spline estimate

Step 4: Utilize generalized cross validation criterion to get minimization of the GCV score and the noise variance

Output: The estimated noise variance

The distinct wavelet based smoothing spline estimation gives the optimum denoising parameter noise variance to ensure the superior denoising.

**2.2.3. Bilateral Filtering**

The bilateral filter is a nonlinear filter that does spatial averaging without smoothing edges; it has shown to be an effective image denoising technique. An important issue with the application of the bilateral filter is the selection of the filter parameters, which affect the results significantly. Adaptive non-Local algorithm was proposed in [15].

The bilateral filter takes a weighted sum of the pixels in a local neighborhood; the weights depend on both the spatial distance and the intensity distance. In this way, edges are preserved well while noise is averaged out. Mathematically, at a pixel location x, the output of the bilateral filter[22] is calculated as follows:

$$\hat{I}(X) = \frac{1}{c} \sum_{y \in N(x)} e^{-\frac{\|y-x\|^2}{2\sigma_d^2}} e^{-\frac{|I(y)-I(x)|^2}{2\sigma_r^2}} I(y) \quad (10)$$

Where  $\sigma_d$  and  $\sigma_r$  are parameters controlling the fall-off of the weights in spatial and intensity

domains, respectively.

$N(x)$  is the spatial neighborhood of  $I(x)$ , and  $C$  is the normalization constant

$$C = \sum_{y \in N(x)} e^{-\frac{\|y-x\|^2}{2\sigma_d^2}} e^{-\frac{|I(y)-I(x)|^2}{2\sigma_r^2}} \quad (11)$$

For  $\sigma_d = 3$  or  $5$ ,  $\sigma_r$  should be chosen around  $2 \times \sigma_n$ ; for  $\sigma_d = 1.5$ ; the optimal ratio for  $\sigma_r/\sigma_n$  is around  $3$  and  $\sigma_n$  is noise standard deviation. This method turns out to produce better results than the standard bilateral filter does as effective bilateral window size can be increased without losing details of the image.

### 2.2.4. Thresholding

BayesShrink was proposed by Chang, Yu and Vetterli. The goal of this method is to minimize the Bayesian risk, and hence its name, BayesShrink. It is a subband – dependent which means that threshold level is selected at each band of resolution in the wavelet decomposition.. The Bayes threshold,  $t_b$ , is defined as

$$t_b = \frac{\sigma^2}{\sigma_s} \quad (12)$$

where  $\sigma^2$  is the noise variance and  $\sigma_s^2$  is the signal variance

$$w(x, y) = s(x, y) + n(x, y) \quad (13)$$

Since the signal and noise are independent of each other it can be stated that

$$\sigma_w^2 = \sigma_s^2 + \sigma^2 \quad (14)$$

$\sigma_w^2$  can be calculated as shown below,

$$\sigma_w^2 = \frac{1}{n^2} \sum_{x,y=1}^n w^2(x, y) \quad (15)$$

The variance of the signal  $\sigma_s^2$  is computed as shown below

$$\sigma_s = \sqrt{\max(\sigma_w^2 - \sigma^2, 0)} \quad (16)$$

With these signal and noise variance, the Bayes threshold is computed from the below equation

$$t_b = \frac{\sigma^2}{\sigma_s} \quad (17)$$

The wavelet coefficients are thresholded at each band.

## 3. EXPERIMENT RESULTS

To analyze the performance of the proposed framework quantitatively and visually, we simulated noisy images by adding Gaussian random noise with various variance. The original input images were then restored using proposed

Multiresolution denoising Framework and PSNR and MSE results were compared. For visual comparisons. Figure 3 Lena is given. Table 1 shows the PSNR and MSE values for the three images. Those noise variances are estimated using both Donoho and Smoothing spline method. From table 1 we can see that our proposed method of smoothing Spline for the noise variance estimation is more precise and less error than the Donoho's method.



Figure 3 A) Input Image B) Noisy Image C) Denoised Image

## 4. CONCLUSION

The Multiresolution bilateral filtering along with BayesShrink Thresholding removes noise. The performance of denoising algorithm is measured using quantitative measures like PSNR and MSE as well as in terms of visual quality of images and noise variance is estimated using Smoothing spline which is more close to actual noise variance when compared to Donoho method. So the usage of Smoothing spline algorithm yields better performance metrics. The results shows that noise is removed effectively as high PSNR and low MSE is achieved.

The future work includes reducing the MSE for denoised image and increasing PSNR values and also improves the image resolution of image. Noise variance being estimated by Smoothing spline algorithm can be found by other optimization algorithms like Genetic and neural network algorithms which give a more accurate noise variance. The performance metrics can also be increased by using other Transforms which are more effective.

Table 1: Psnr And Mse Value

Input Image	Input Noise Variance	Estimated Noise Variance		Peak Signal to Noise Ratio (PSNR) in db		Mean square Error(MSE) in pixels	
		Donoho	Smoothing Spline	Donoho	Smoothing Spline	Donoho	Smoothing Spline
House	10	4.04	7.84	32.77	34.21	34.32	24.43
	15	11.04	13.21	29.59	32.09	71.81	40.17
	20	15.94	18.25	27.94	30.38	104.4	89.50
	25	20.85	23.7	24.98	29.18	130.33	78.52
	30	25.71	29	24.45	28.27	147.07	94.83
Peppers	10	4.07	8.12	31.91	33.7	42.50	28.47
	15	11.29	13.14	28.85	31.3	93.43	49.42
	20	15.80	18.44	28.67	29.50	180.1	74.44
	25	20.74	23.7	25.75	28.1	190.4	102.34
Lena	10	7.04	8.94	29.9	32.19	45.17	39.24
	15	11.70	14.2	27.42	30.07	112.42	43.91
	20	14.29	19.2	25.80	28.78	145.57	85.84
	25	21.04	24.5	25.37	27.78	170.48	108.3
	30	24.17	29.5	24.50	24.93	188	131.82

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