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THE STATIC SECTORIZATION OF AIRSPACE VIA GENETIC ALGORITHM APPROACH

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ABSTRACT

The sectorization of airspace has become one of the most important problems of operational research; saturation and non-optimality of this system are the main reasons behind a new sectorization. The objective of SAP is to minimize the workload coordination between adjacent sectors and the controller's workload which is balanced among the sectors. The main purpose of this work is to suggest a new method to solve the SAP problem. The proposed method is divided into two steps: the first one involves modeling the SAP problem in terms of 0-1 quadratic programming model subject to linear constraints. The second one concern solving this model via the Genetic Algorithm approach. Finally, some experimental results are introduced.

Key words: *Sectorization of Airspace Problem (SAP), Air Traffic Control (ATC), Operational Research (OP), Quadratic Programming Problem (QPP), Genetic Algorithm (GA)*

1. INTRODUCTION

According to the european commission, the european airports and air routes reach saturation [30]. that is why, a new sectorization of the airspace is become a necessary task. in this context, the airspace beacons are classified into a number of sectors; each of them is assigned to a team of controllers. these allocations take into a count the capacity of the controller to manage several aircrafts simultaneously [10],[11].

Delahaye and al. [11] have tried the (SAP) problem via the Genetic Algorithm approach. A sector is defined by a set of connected vertices of the network and the chromosome contains all information needed to define the sectors. However, sectors built by synthesis of connected vertices do not ensure the convexity constraint. Other investigations on this problem have been published in [28], but without the connectivity constraint.

Huy Trandac and Philippe Baptiste have proposed a constraint programming approach to optimize the sectorization that shall satisfy all specific constraints [3],[29], the Airspace is modeled by a valueted graph and a sector in a solution of (SAP) is defined as a set of vertices, without geometrical boundaries. A genetic algorithm to solve ASP has been proposed in [10] .

BICHOT Charles-Edmond have proposed a fusion-fission method [7], allows to find the best airspace partitioning for our objective function. To diversify its applications, the fusion-fission method has also been applied to image segmentation and documents clustering.s

Recently, airspace has been divided into small volume units and a sector is obtained by joining some of these elementary units [13]. Unfortunately, the most specific constraints cannot be taken into account and for instance, the sectors can be fragmented in the solution[10], [11]. More recently, we have used the continuous Hopfield network to solve the SAP [14]. In this context, a specific energy function is proposed and all the simulations terminated in a valid sectorization. The goal of this work is to solve the Sectorization of Airspace Problem SAP via a new tool. The proposed method is divided into two steps: the first one involves modeling the SAP problem in terms of 0-1 quadratic programming model subject to linear constraints[26]. The second one concern solving this model via the Genetic Algorithm approach [25], [9].

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Genetic Algorithm (GA) belongs to a class of probabilistic methods called «evolutionary algorithms » based on the principles selection, crossover and mutation. It was introduced by J. HOLLAND that it's based on natural evolution theory of Darwin [14], [6]. The GA method has been applied in a large number of optimization's problems in several domains telecommunication, routing, scheduling, and it proves it's efficiently to obtain good solutions [11].

The paper falls in six sections. The second introduces the Sectorization of Airspace Problem (SAP). In the third section, we model the problem in terms of 0-1 quadratic programming model subject to linear constraints. In the forth section, we use the Genetic Algorithm to solve the proposed model. Finally, some computational experiences are shown in the fifth section.

2. PRESENTATIONT SECTRORIZATION OF AIRSPACE PROBLEM

In this part, we give a succinct description of the components of the airspace; then, we will present the Sectorization of Airspace Problem (SAP).

- ***Presentation of the airspace and Air Traffic Controllers***

The airspace is composed of a set of beacons or network connecting these beacons (Fig.1); this area is under the responsibility of air traffic controllers.

Air traffic is currently controlled by human operators (Air Traffic Controllers) who monitor aircraft trajectories and give instructions to pilots so as to avoid mid-air collisions and dangerous situations. The airspace is partitioned into managerial units, Air Traffic Control Centers (ATCC), which are themselves partitioned into elementary airspace modules. These basic airspace modules may be combined together so as to form air traffic control (ATC) sectors each operated by a small team of 2–3 controllers.

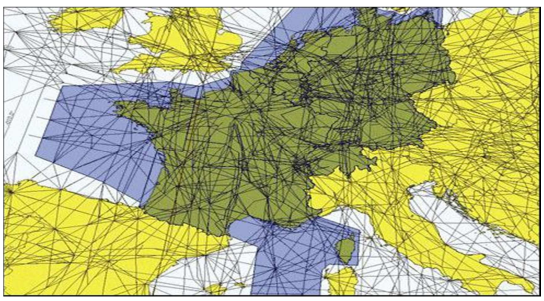

Fig.1: Air Route Network

Nowadays, air traffic increases throughout the world; thanks to safety, the sectorization of airspace has become one of the most important solutions of Air Traffic Control; in this context, the airspace is divided into a number of sectors; each of them is assigned to a team of controllers. The sector number is then determined by the capacity of a controller to manage N aircraft simultaneously: In practice, the average seems to be 10 to 15 aircrafts, when this limit is attaining we say that the sector is saturated [10], [11].

- ***Modelling the Controllers Workload***

Generally, there exist three kinds of controller's workload [11], [9], [7]:

- **Monitoring Workload M_w**:

This charge is proportional to crossing time of an aircraft in the sector. If this route passes through two sectors, the load is distributed across the sectors in proportion to the part of the edge in the sector.

- **Conflict Workload C_W**:

This charge is proportional to the total number of potential conflicts that can occur on a crossing point.

- **Coordination Workload O_w:**

This is proportional to the number of the aircraft which pass along a road section, each of which ends charge are located in different sectors.

The airspace is formed by together airways and $n \in IN^n$ beacons $b_1, \dots, b_n$ will be divided into $k \in IN^n$ sectors $V_1, \dots, V_k$.

We define the $n \times k$-matrix of coordination W and n-vector w of workload in beacons i as follows:

$$W_{ipjq} = \begin{cases} O_w & \text{if beacons } b_1 \text{and } b_2 \\ & \text{are directly connected} \\ \forall\, i,j \in \{1,\dots,n\} & p,q \in \{1,\dots,k\} \\ 0 & \text{else} \end{cases}$$

$$w_i = (C_w + \frac{M_w}{2})_i \quad \forall\, i \in \{1,\dots,n\}$$

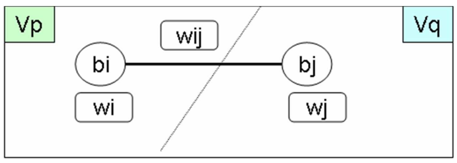

Fig.2: Modeling The Workload

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In the next part, we will model the SAP problem in terms of 0-1 quadratic programming problem subject to linear constraints [26], [15].

3. MODELING OF THE (SAP) PROBLEM

The goal of this part is modelling the problem (SAP) in terms of 0-1 quadratic programming problem subject to linear constraints.

Definition 1:
We define the total workload of controllers in the all airspace by following formula:

$$W_T = \sum_{i=1}^{n} w_i + \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}$$

Definition 2:
The number of sectors is defined by the capacity of controllers to manage N aircraft simultaneously; in practice, the means seems to be 10 to 15 aircrafts, so:

$$k = \frac{\sum_{i=1}^{n} w_i + \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}}{12} = {W_T}/{12}$$

- Binary variables

First, we define the following binary variables:

$$\forall\, i \in \{1, \dots, n\} \text{ and } \forall\, p \in \{1, \dots, k\}$$

$$x_{ip} = \begin{cases} 1 \text{ if the beacons } b_i \text{ is part of sector } V_p \\ 0 \text{ else} \end{cases}$$

We convert these suits of variables:
$$x = (x_{11}, \dots, x_{1k}, \dots, x_{i1}, \dots, x_{ik}, \dots, x_{n1}, \dots, x_{nk})^T$$

In the next, we use this vector to define the objective function and the constraints of the problem under study.

- *Objective function*

The objective function of the mathematical programming model consists to minimize the total workload coordination:

$$f(x) = \sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k} (1-\delta_{ij})(1-\delta_{pq})\, x_{ip} x_{jq}\, W_{ipjq}$$

The Boolean δ_{ij} is the Kroenecker delta.

$$\delta_{ij} = \begin{cases} 1 \text{ si } i = j \\ 0 \text{ si } i \neq j \end{cases}$$

- *The family of the allocation constraints:*

Each beacon must be allocated to only one sector; in this context, we obtain the following family constraints:

$$\sum_{p=1}^{k} x_{ip} = 1 \;\forall\, i \in \{1, \dots, n\}$$

We remark that we associate one constraint to each beacon.

- *The balance constraints*

For each vector x, the controller of the sector p has to manage the next workload:

$$\sum_{i=1}^{n} x_{ip} w_{ip}$$

To make an equitable allocation between the airspace controllers, we impose the next family of constraints:

$$\sum_{i=1}^{n} x_{ip} w_i = C^k \quad \forall\, p \in \{1, \dots, k\}$$

Where:

$$C^k = {\left(\sum_{p=1}^{k} w_p\right)}/{k}$$

From now on, we call this quantity the desired workload. Since it is difficult to realize the later family of constraints, we prefer, only, to control the total error produced by an allocation of the beacons to the various sectors; thus, we replace the objective function by the following one:

$$f(x) = \sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k} (1-\delta_{ij})(1-\delta_{pq})\, x_{ip} x_{jq}\, W_{ipjq} + \lambda \sum_{p=1}^{k}\left(\sum_{i=1}^{n} x_{ip} w_i - C^k\right)^2$$

The real number λ is a control variable.

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Based on the binary variables, on the objective function f and the family of on the allocation constraints, the mathematical form of the Sectorization of Airspace Problem is given by:

$$SAP:\begin{cases} Min\left(\begin{array}{c}\sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k}(1-\delta_{ij})(1-\delta_{pq})\,x_{ip}x_{jq}\,W_{ipjq} \\ +\lambda\sum_{p=1}^{k}\left(\sum_{i=1}^{n}x_{ip}W_{ip}-C^{k}\right)^{2}\end{array}\right) \\ Subject\ to: \\ \sum_{p=1}^{k}x_{ip}=1 \\ \quad with\ \forall\, i\in\{1,\dots,n\},\forall\, p\in\{1,\dots,k\} \\ and\ x_{ip}\in\{0,1\}\end{cases}$$

The matrix form of the SAP problem is the following:

$$\text{SAP:}\begin{cases} Min\ f(x)=x^{T}Qx+C^{T}x \\ \quad Subject\ to: \\ \quad Ax=b \\ \quad x\in\{0,1\}^{n\times k}\end{cases}$$

Where:
$m = n \times k$
$Q = R_{ipjq}$ $is\ a\ m\times m$ Symmetric matrix, for $i,j \in \{1,\dots,n\}$ and $p,q \in \{1,\dots,k\}$, C is a n real vector, $b \in IR^{n}$ is a n real vector and A is a $n \times m$ matrix. Without loss of generality, we assume that all diagonal terms of Q are equal to 0.
Quadratic 0-1Programming (QP) with linear constraints is a general model that allows formulating numerous important problems in combinatorial optimization including, for example: quadratic assignment, graph partitioning, task allocation, capital budgeting and densest k-subgraph.

The difficulty of quadratic problems is mainly due to the non-linearity of the function and the integrity of the variables. That's why most of the techniques proposed to solve it consist first to break the non-linearity of the functions and to relax the integrity constraints.

Various heuristics and exact methods have been proposed to solve QP. Due to the non-convexity of the objective function, QP is often reformulated before searching for its optimal solution. So, several methods have been developed to solve it exactly through 0-1 linear reformulations [1], [16] or 0-1 convex quadratic reformulations [19], [27]. Although 0-1 linear reformulations of QP are the most common approaches, other methods have been proposed. Let us cite, for example, algebraic and dynamic programming methods [8], reformulation to a continuous concave minimization problem [23] and enumerative methods based on different types of relaxations as lagrangian relaxation, semidenite relaxation or convex quadratic relaxation [4],[5], [17], [21].

In this paper we reformulate SAP by an equivalent zero-one quadratic program with a convex objective function. Consequently, we can solve the transformed problem using general purpose optimization software which implements the Genetic Algorithm (GA).

4. GENETIC ALGORITHM: APPLICATION TO SOLVE THE (SAP) PROBLEM

In this part, we use the Genetic Algorithm approach to solve the problem SAP. Genetic Algorithm belongs to a class of probabilistic methods called « Evolutionary Algorithms » based on the principles selection, Crossover and mutation. GA was introduced by J. HOLLAND [22] that it's founded on natural evolution theory of Darwin [7].

Each solution represents an individual who is coded in one or several chromosomes. These chromosomes represent the problem's variables. First, an initial population composed by a fix number of individuals is generated, then operators of reproduction are applied to a number of individuals selected switch their fitness. This procedure is repeated until the maximums number of iterations is attained. GA has been applied in a large number of optimization's problems in several domains [30], [14], [24], [18], telecommunication, routing, scheduling, and it proves it's efficiently to obtain good solutions [10], [25]. We have formulated the problem as a non linear program with mixed variables.

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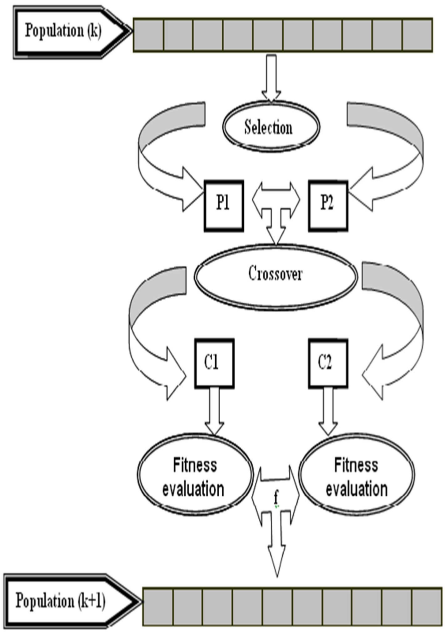

Fig.3: Genetic Algorithm

The different step of the GA is given by [6]:

1. Choose the initial population of individuals
2. Evaluate the fitness of each individual in that population
3. Repeat on this generation
4. Select the best-fit individuals for reproduction
   a. Crossover and Mutation operations
   b. Evaluate the individual fitness of new Individuals
   c. Replace least-fit population with new Individuals

Until termination (time limit, fitness achieved, etc.).

The criterion of convergence can be of diverse nature, for example:

- Minimum rate that wants to reach to adapt the population to problem.
- Limit the time of calculation in the algorithm.
- Or a combination of the two points.

- ***Coding***

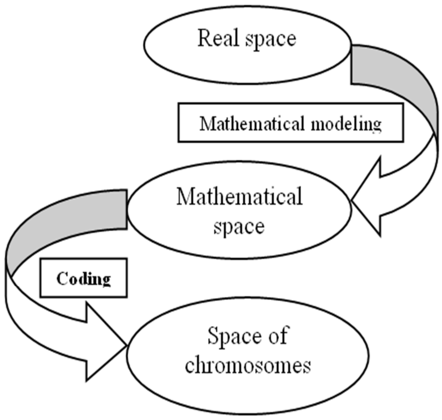

Fig.4: Coding Principle

In our application, we have encoded an individual by 1 chromosome represents the vector $X = (x_{ip})$ of binary variable, which the variable x_{ip} who takes 1 if the beacon b_i belongs to a sector V_p and 0 other.

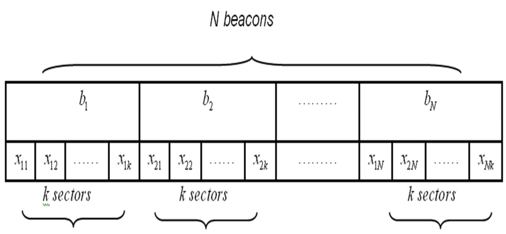

Fig.5: Chromosome

- ***Initial Population***

The first step in the functioning of a GA is, then, the generation of an initial population. Each member of this population encodes a possible solution to a problem. The individual of the initial population are randomly generated, and $X = (x_{ip})$ take the value 0 or 1. The w_i is a matrix takes random values in space $\mathbb{R}^n$ and the W_{ij} symmetric matrix with diagonally null, because $W_{ipjq} = W_{jqip}$ takes random values in $\mathbb{R}^{n \times n}$.

Where: $i, j \in \{1, \ldots, n\}$

- ***Evaluating individuals***

In this step, each individual is assigned a numerical value called fitness which corresponds to its performance; it depends essentially on the value of objective function in this individual. An individual who has a great fitness is the one who is

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the most adapted to the problem. The fitness suggested in our work is the following function:

$$Fitness(i) = \frac{1}{1 + Objective(i)}$$

Minimize the value of the objective function « Objective » is equivalent to maximize the value of the fitness function.

- ***Selection***

The selection method used in this paper is the Roulette Wheel Selection (RWS) which is a proportional method of selection. In this method, individuals are selected according to their fitness. The principle of the RWS method can be summarized in the following schema:

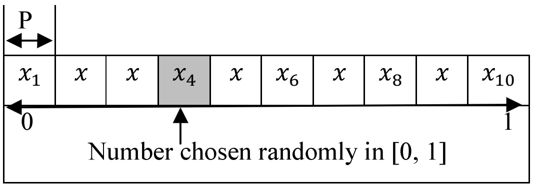

Fig.6: Selection By WRS

Where:

$$P_i = {f_i}\Big/{\textstyle\sum_{i=1}^{n} f_i}$$

- ***Crossover***

The crossover is very important in the algorithm, in this stage, new individuals called children are created by individuals selected from the population called parents. Children are constructed as follows: We fix a point of crossover, the parents are cuted switch this point, the first part of parent 1 and the second of parent 2 go to child 1 and the rest go to child 2 in the crossover that we adopted, we choose one crossover point, for the matrix $X = \left(x_{ip}\right)$ of binary variable.

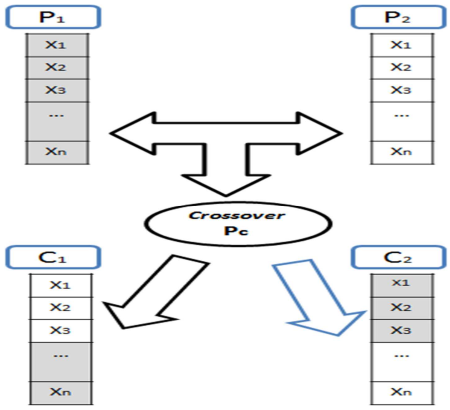

Fig.7: Slicing Crossover

This operation is effected with a probability of crossover P_c.

- ***Mutation***

The role of mutation is to keep the diversity of solutions in order to avoid local optimums. It corresponds on changing the values of one (or several) value.

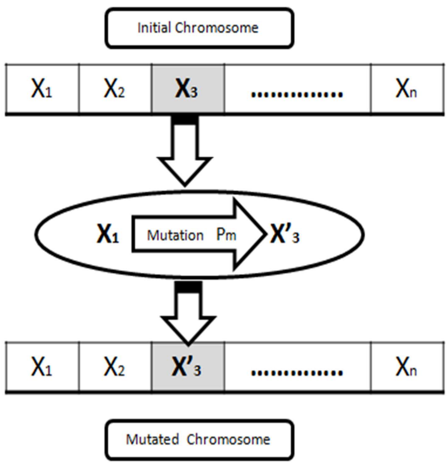

Fig.8: Principle Of The Mutation Operator

This operation is effected with probability of mutation P_m.

5. COMPUTATIONAL EXPERIMENTS

To compare the proposed sectorization with present one, some numerical results are introduced. The programs were run in a compatible IBM, Pentium(R) Dual− Core 2.5 GHz, and 1 Go of RAM through Borland C + + Builder 3.

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All of these simulations terminated in a valid sectorization. The quality of obtained solutions has been measured by means of the performance ratio:

$$E = \sqrt{\sum_{p=1}^{k} \left(C^k - CW_p\right)^2}$$

Where:

- CW_p , the calculate workload in the sector p:

$$CW_p = \sum_{i=1}^{n} w_i x_{ip} \; \forall \, p \in \{1, \dots, k\}$$

- CW_T, the total workload calculated:

$$CW_T = \sum_{i,j=1}^{n} \sum_{p,q=1}^{k} x_{ip} x_{jq} W_{ipjq}$$

From this fig.9, we remark the beacons are uniformly allocated to the different sectors.

- C^k , the average workload desired in the sector k.

The database of airspace supposed confidential; so, we generate randomly the matrix W and the vector w as follows:

$$\forall \, i,j \in \{1, \dots, n\} \quad W_{ij}, w_i \in [10, 100]$$

The sector number is estimated from the total workload of controllers, according to expert studies, in the domain of navigation, a controller has the capacity to manage in average 10 to 15 aircraft simultaneously.

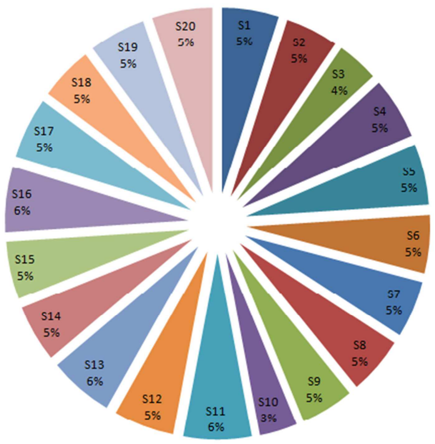

Fig.9: Classification Of The Beacons On The Sectors.

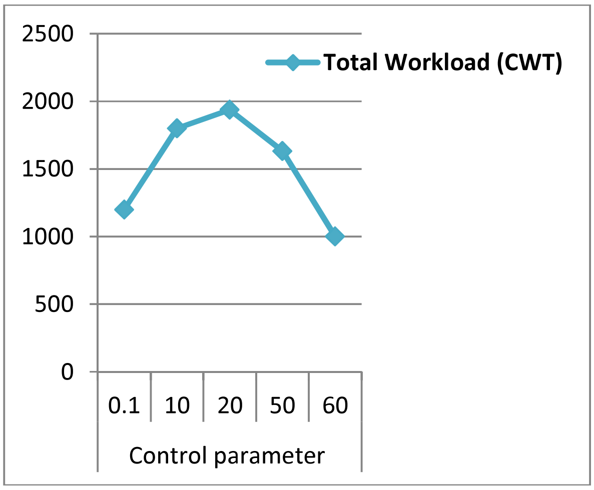

Fig.10: The Total Workload Versus Control Parameter λ

n	K	λ=0.1		λ=10		λ=20		λ=50		λ=60	
		E	CW_T	E	CW_T	E	CW_T	E	CW_T	E	CW_T
20	3	10.1	570.0	4.3	740	10.4	587	33.2	634.2	37.50	630.33
50	4	15.3	600.4	5.8	830	18.4	645	37.1	598	40.23	600.03
80	5	35.1	750.1	13.2	904.3	22.2	893.2	43.1	700	58.01	810.43
100	6	38.7	980.1	15.1	1035	30.4	950	44.2	807.2	53.04	882.35
150	7	42.2	1003.5	17.8	1102.5	38.7	983.4	58.2	897.3	60.20	890.50
200	8	48.5	1032.8	20.2	1345.2	40.1	1123.7	60.2	984.8	65.50	950.73
250	10	49.8	1150.1	28.2	1641.2	34.01	1508	65.2	1300	68.04	989.70
300	20	50.02	1202.3	30.34	1800.75	35.04	1938.13	62.03	1632.27	71.04	1031.16

TABLE I: The Total Coordination Workload And Error Versus Control Parameter λ.

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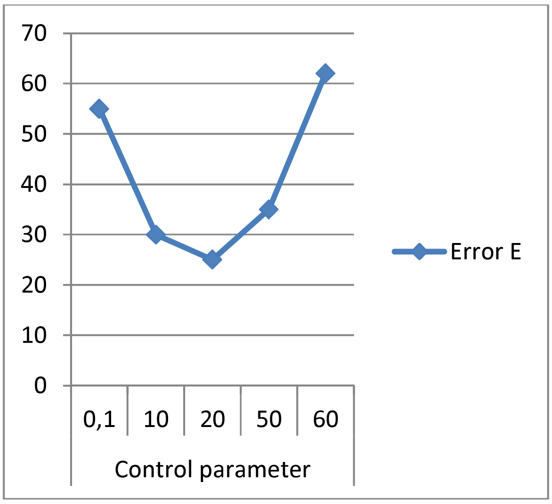

Fig .11:Error Functions According To Control Parameter $\boldsymbol{\lambda}$.

From the TABLE I, Fig.10 and Fig.11, the curve of $E(\lambda) and\ CW_T(\lambda)$ is a parabolic function of λ. In addition, the curve of a function E is concave, against the function CW_T which is convex.

Concerning the selection of the parameter λ, such select must make a balance between the Error E and the calculated of a total workload. In our case, when λ is closed in the interval [10, 50] we obtain a classification with acceptable Error E and admissible total coordination workload CW_T.

The values $||W||$ and $||w||$ are respectively the norm of the matrix W and the vector w .In practice, when W and w decrease some sectors are regrouped with the other ones. In the future, we will propose a dynamic model, which is determining automatically the sectors number.

6. ONCLUSION

In this work, the problem of sectorization is formulated as 0-1 quadratic programming subject to linear constraints. We have used the Genetic Algorithm approach to solve the proposed model. In this regard, all of these simulations terminated in a valid sectorization. To compare the proposed sectorization with present one, some numerical results are introduced. In this context, the proposed methods make balance, in terms of workload, between different sectors in comparison with the previews sectorization.

In future direction of this research we will propose a dynamic sectorization by determining automatically the number of sectors.

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