Journal of Theoretical and Applied Information Technology
28th February 2014. Vol. 60 No.3
© 2005 - 2014 JATIT & LLS. All rights reserved·
ISSN: **1992-8645** www.jatit.org E-ISSN: **1817-3195**

AUTOMATIC MOTIF DETECTION FOR ISLAMIC GEOMETRICAL PATTERNS

[1]ABDELBAR NASRI, [2]RACHID BENSLIMANE

[1,2] Laboratoire de Transmission et de Traitement de l'Information
Université Sidi Mohamed Ben Abdellah, USMBA, Ecole supérieure de Technologie
Route d'Imouzzer BP. 2427, Fès, Maroc
E-mail: [1] abdelbar.nasri@usmba.ac.ma, [2] rachid.benslimane@usmba.ac.ma

ABSTRACT

In this paper, we present a new method to detect the basic unit cell of a periodic Islamic Geometrical Pattern (IGP). This method is based on the autocorrelation function (ACF), a function known to be appropriate to analyze and extract a repetitive motif from a regular texture. The motif can be successfully extracted when the peaks detected in the autocorrelation function of an image are pertinent. To optimize the peaks detection, we propose a new method based on the stability of the motif surface which is defined by two short displacement vectors. Compared to classical extraction methods of periodicity, the proposed method hereby is tolerant to geometric distortion, noise and changes in intensity. Tests on 166 images with different visual quality demonstrate the capability of the proposed method to extract the periodic motif automatically without the need of human intervention.

Keywords: *Autocorrelation, Wallpaper Groups, Islamic Art, Pattern Extraction, Displacement Vector.*

1. INTRODUCTION

Texture periodicity analysis has become popular lately and has been used for texture tracking [14], synthesis [22, 11, 31], and retrieval [8, 9].

It is proven that every 2D periodic patterned texture can be classified in one of five groups based on the type of network or lattice (unit cell or motif) as illustrated in figure (1).

Previous works proposed for motif extraction can be grouped broadly into two categories: the local feature-based approach [6, 13, 14, 15, 16, 17, 18, 19, 25], and the global structure-based approach [1, 3, 7, 8, 9, 20, 21, 22, 23].

The local feature-based approach starts by identifying a number of texel candidates. Matching based on visual similarity between these potential texels and their neighbors is then performed. Successful matching leads to the connection of texels with a lattice structure. The approach is repeated until no more new texels are found.

The global structure-based approach [1, 3, 4, 7, 8, 9, 20, 21, 22, 23.] tries to extract periodicity using methods that emphasize the idea of periodic patterns as global processes.

T.Matsuyama [4] proposes an approach based on Fourier transformation. The author of [5, 23.]

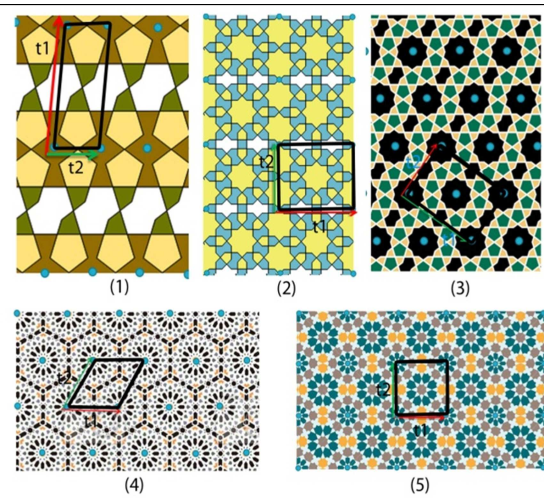


Image used in (1, 2, 3) are captured from [29]

(1): Parallelogram, $\lvert t_1\rvert \neq \lvert t_2\rvert, \varphi \neq 90^\circ$
(2): Rectangular, $\lvert t_1\rvert \neq \lvert t_2\rvert, \varphi = 90^\circ$
(3): Centered rectangular $\lvert t_1\rvert \neq \lvert t_2\rvert, \varphi = 90^\circ$
(4): Hexagonal, $\lvert t_1\rvert = \lvert t_2\rvert, \varphi = 60^\circ$
(5): square, $\lvert t_1\rvert = \lvert t_2\rvert, \varphi = 90^\circ$

Figure 01: The Five Fundamental Two-Dimensional Bravais Lattices.

Journal of Theoretical and Applied Information Technology
28th February 2014. Vol. 60 No.3
© 2005 - 2014 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

presents methods that use periodicity measures defined by co-occurrence matrices computed globally over the whole pattern.

A more traditional image processing approach to detect repetition uses autocorrelation function ACF [1, 3, 7, 20, 21.]. The main problem with using this approach is the difficulty of finding the pertinent peaks in the ACF.

Chetverikov [20] proposes a regularity measure based on finding the maximum overall direction on the ACF. Leu [21] uses the several highest peaks in the ACF, computed on the gradient field of the image to capture translation vectors.

H.C. Lin et al [1] present a threshold-free approach based on a filtered ACF and obtained by using an optimal Gaussian smoothing. Yanxi Liu et al [7] propose an efficient method based on the region of dominance. All these methods select arbitrarily the number of peaks in the ACF. To overcome this drawback, Jingrui He et al [3] propose an optimization scheme to automatically extract the pertinent peaks and the correct number of peaks. Despite its good performance, this method is sensible to geometrical distortion, noise and quality of the processed images.

In this paper we propose a new automatic method based on ACF. This method is rapid and no sensitive to geometric distortions, noises and intensity changes.

The rest of this paper is organized as follows: Section 2 describes an approach to extract motifs based on the autocorrelation function. Section 3 presents the proposed method. Experimental results of these latter are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. PERIODICITY EXTRACTION BASED ON ACF:

The process to extract periodicity with ACF consists of three major steps: (A) Calculation of the Autocorrelation function, (B) Finding of pertinent peaks, and (C) Displacement of vectors and extraction of the motif. Figure 2 gives an illustration of these steps.

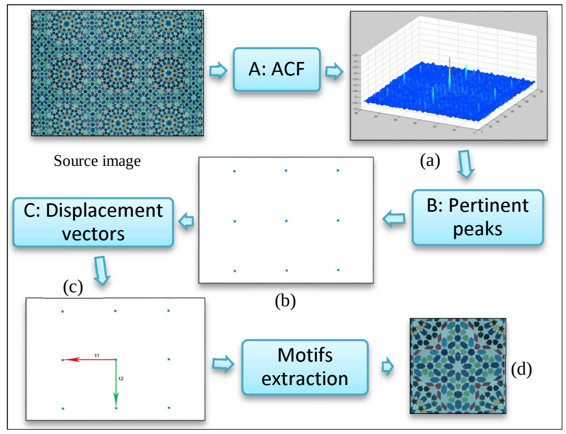


Figure 2: Process To Extract Periodic Motif Based On ACF

A. Autocorrelation function

Lin et al [1] prove that if a grey-scale image I(x, y) presents a periodic motif, the shape and arrangement of this motif can be found using the locations of the peaks in its autocorrelation function ac(x,y), computed as follows:

$$ac(x,y)=\sum_{i=1}^{A}\sum_{j=1}^{B} I^*(i,j)I(x+i,y+j) \quad (1)$$

Where " *" is the complex conjugate and A and B define the dimension of the original image. According to the correlation theorem Gonzalez and Woods [26], Eq. (1) can be expressed as:

$$ac(x,y)=F^{-1}\left[F^*(I(x,y))F(I(x,y))\right] \quad (2)$$

Where F and F^{-1} denote respectively the Fourier transform and inverse Fourier transform.

B. Finding pertinent peaks

Pertinent peaks detection follows generally two steps:

- A coarse peaks detection step: ac(x,y) is considered as a peak if it is the maximum of all the ac(x0, y0), where (x0,y0) is a neighbor of (x,y) in a window of fixed size VxV with the center (x,y). The number of peaks detected in this step is distorted by false peakes.

Journal of Theoretical and Applied Information Technology
28th February 2014. Vol. 60 No.3
© 2005 - 2014 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

- A fine peak detection step allowing to find only pertinent peaks. There exist manual and automatic methods.

- **Manual peak number detection**

Lin et al [1] present a threshold-free approach based on finding an optimal Gaussian smoothing of the autocorrelation function. To obtain a smoothed autocorrelation function, they convolve the 2D ACF with a 2D Gaussian filter (Eq (3)).

$$g(x,y,\sigma)=\frac{1}{2\pi\sigma^2}\exp\left[\frac{-x^2-y^2}{2\sigma^2}\right] \quad \text{Eq (3)}$$

As it is proposed by Yianx et al [7] "the absolute height of a peak is not as important as the size of its region of dominance (RD), defined as the largest circle centered on the candidate peak such that no higher peaks are contained in the circle. A peak with a low height, but located far from any larger neighbors, is much more perceptually important than a high peak that is close to an even higher one. The first 32 most dominant peaks found using this method are well distributed over the whole image, with very few spurious peaks".

This method is good at removing false peaks, especially in an IGP that contain a large number of repetitive patterns (cf. Fig A.2). However, with this method, the number of pertinent peaks has to be selected manually. Moreover, it does not detect pertinent peaks in the IGP in the case of a repetitive pattern taking a large proportion of the processed image. (cf. Fig A.2) illustrates an example of our reimplementation of this method.

- **Automatic peak number detection**

Jingrui He et al [3] propose an optimization-based scheme for automatic peak number detection. They first formulate it as an optimization problem by a straightforward yet effective criterion function, and then resort to Simulated Annealing to optimize it. Despite its good performance, this method needs an adaptive parameterization based on some prior knowledge of the database [3]. Figure A.3 illustrates an example of automatic peak detection using our reimplementation of this method.

C. Displacement vectors

Once peak number is determined, the Generalized Hough Transform (GHT) [1] can be utilized to find the two displacement vectors [1]. These two non-collinear vectors are used as two translation vectors that generate the lattice, from which the repetitive motif is extracted (cf. Fig 3).

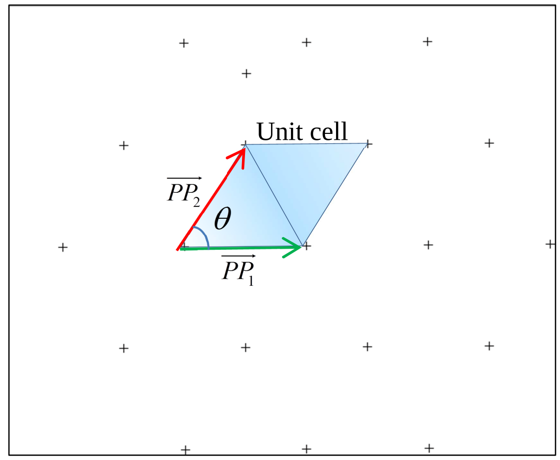


Figure 3: Unit Cell Or Lattice Surrounded By 2 Calculated Displacement Vectors $\overrightarrow{PP_1}$ *And* $\overrightarrow{PP_2}$

3. PROPOSED METHOD

We propose an iterative method to automatically extract the repetitive motif that identifies Islamic Geometry patterns: The optimal number of peaks and the corresponding two displacement vectors are determined from the autocorrelation function.

By varying the number N of peaks, which are the N highest peaks in the ACF, we calculate the most frequent surface 'S' of the periodic motif, defined by two displacement vectors.

This iterative procedure stops when the surface 'S' of the periodic motif remains unchanged between two successive iterations n and n+1, S(n)=S(n+1). This iterative process results in the determination of the optimal number of peaks and two linearly independent translation vectors that generate the lattice, from which the periodic motif is extracted.

The main procedure is to determine for a chosen number of peaks N the most frequent two displacement vectors. To do so, we proceed as follows:

- Compute the autocorrelation function as mentioned in section 2-A.
- Sort peaks in descending order of magnitude.

Journal of Theoretical and Applied Information Technology
28th February 2014. Vol. 60 No.3
© 2005 - 2014 JATIT & LLS. All rights reserved.
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

- Repeat while S(n)==S(n+1).
- Select the N highest peaks
- For each highest peak P(x,y) do:
  - Find the two nearest neighbor peaks $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ with the constraint that the angle between the resulting vectors $\overrightarrow{PP_1}$ and $\overrightarrow{PP_2}$ must be between 60° and 90° [28] (cf. Fig 3).
  - Compute the surface of the unit cell (motif) surrounded by the two calculated displacement vectors $\overrightarrow{PP_1}$ and $\overrightarrow{PP_2}$ (cf. Fig 4).
  - Compute the most frequent displacement vectors using their corresponding surface motif S(n).

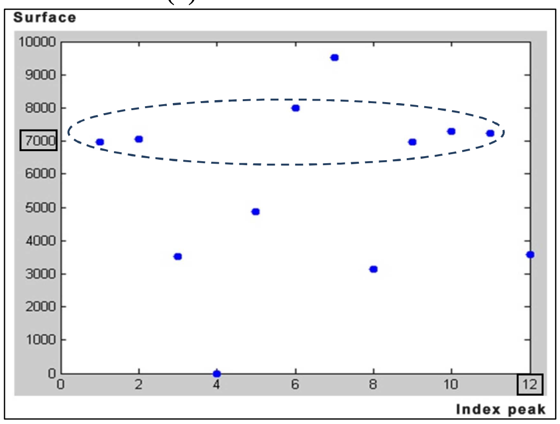


Figure 4: This Figure Shows An Example For N=12. This Figure Plot The Surface Of Unit Cell Calculated In Each Peak And The Most Occurred Surface.

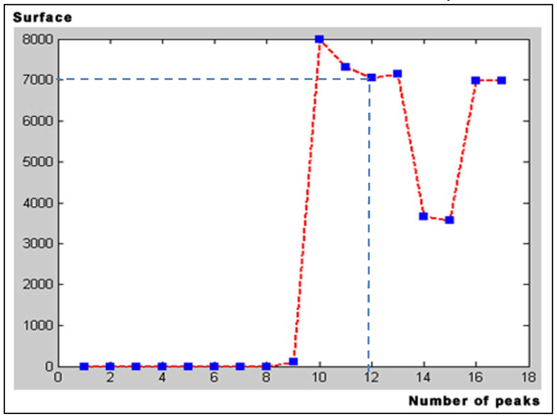


Figure 5: This Figure Shows That The Optimal Peak Number Is N=16, Because The Surface Remains Unchanged From N=16 To N=17.

4. EXPERIMENTAL RESULTS

The performance of the proposed method is evaluated by comparing its correct rate and time complexity with those proposed by Jingrui He et al [3] and by Lin [1]. 166 images were used to test this performance. Some representative unit cells are shown in Figure (A.8). The tested images have different photometric and geometric quality. They also have different sizes varying from 128x128 pixels to 1700x1300 pixels.

Figure (A.5, A.7) shows some examples, for which our method leads to good detection in comparison with Jingrui's method which fails to detect the right motifs (cf. Fig A.4, A.6).

Table 1 shows that the proposed method presents the best rate of the correct detection of the periodic motifs compared to Jingrui He et al method.

Table 2 shows for the first 5 images of figure 8, that our method is more rapid that Jingrui et al method. In fact, our method calculates surface N times in each iteration $o(n)$, and method [3] calculates SA, NxN times in each iteration of algorithm $o(n^2)$, the difference is more important when the number of peaks N increases.

Table 1: Comparison Between [3] And Our Method As A Correct Rate

Method	***Correct detection***	***False detection***	***Correct detection Rate***
Jingrui He et al [3]	143	23	86.14%
Our processes to extract periodicity	155	11	93.37%

Table 2: Comparison Between [3] And Our Method As A Time Complexity

Source images	Jingrui He et al [3]	Our processes to extract periodicity
Image 1 587x384	1.5130 s	1.3246 s
Image 2 430x323	4.2081 s	0.7243 s
Image 3 600x480	4.0677 s	1.9096 s
Image 4 645x419	5.2447 s	1.7013 s
Image 5 610x467	2.5080 s	2.0945 s

5. CONCLUSION

© 2005 - 2014 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

In this paper, a motif detection method for periodic Islamic patterns based on the autocorrelation function was proposed. The test results show the robustness of the proposed method to extract periodicity from images with different photometric and geometric quality, and with different size.

The following conclusions can be drawn from the proposed method:

- The method is simple in implementation and flexible in synthesizing the regular texture.
- There is no need for pre-processing or post processing such us filtering and thresholding of input image.
- The proposed method can be used to automatically detect periodicity in textures belonging to 17 wallpaper groups.

REFRENCES:

[1] H.-C. Lin, L.-L. Wang, and S.-N. Yang, “Extracting Periodicity of a Regular Texture Based on Autocorrelation Functions”, ELSEVER, Pattern Recognition Letters,vol. 18, pp. 433-443, 1997.

[2] V. Asha, P. Nagabhushan, N.U. Bhajantri, Automatic extraction of texture-periodicity using superposition of distance matching functions and their forward differences, ELSEVIER, Pattern Recognition Letters 33 (2012) 629–640.

[3] Jingrui He and al, Automatic Peak Number Detection in image Symmetry analysis, PCM 2004, LNCS 3333, pp. 111-118, 2004. Springer-Verlag Berlin Heidelberg 2004.

[4] T.Matsuyama, S.Miura, and M. Nagao. A structural analysis of natural textures by fourier transformation. CVGIP, 24(3):347-362, December 1983.

[5] R.W. Conners and C.A. Harlow, Toward a structural textural analyzer based on statistical methods. CGIP, 12(3):224-256, March 1980.

[6] Manuel Agusti-Melchor et al, lattice extraction based on symmetry analysis, VISAPP 2008- international conference on computer vision theory and applications 2008.

[7] Yanxi Liu, Robert T. Collins and Yanghai Tsin, A Computational Model for Periodic Pattern Perception Based on Frieze and Wallpaper Groups, IEEE Transactions on Pattern Analysis and Machine Intelligence, VOL. 26, NO. 3, MARCH 2004

[8] Junwei Han, Stephen J. McKenna, Classifying And Comparing Regular Textures For Retrieval Using Texel Geometry, Proceeding Of Visapp, Portugal, 5-8 February, 2009.

[9] Junwei Han, Stephen J. McKenna, and Ruixuan Wang, Regular Texture Analysis as Statistical Model Selection, Springer-Verlag Berlin Heidelberg 2008,ECCV 2008, Part IV, LNCS 5305, pp. 242–255, 2008.

[10] Leung, T., Malik, J.: Recognizing surfaces using three-dimensional textons. In: IEEE International Conference on Computer Vision, Corfu, Greece, pp. 1010–1017 (1999)

[11] Liu, Y., Tsing, Y., Lin, W.: The promise and perils of near-regular texture. Inter-national Journal of Computer Vision 62, 145–159 (2005)

[12] Malik, J., Belongie, S., Shi, J., Leung, T.: Textons, contours and regions: cue integration in image segmentation. In: IEEE International Conference of Computer Vision, Corfu, Greece, pp. 918–925 (1999).

[13] Hays, J., Leordeanu, M., Efros, A., Liu, Y.: Discovering texture regularity as a higher-order correspondence problem. In: European Conference on Computer Vi-sion, Graz, Austria, pp. 533–535 (2006)

[14] Lin, W., Liu, Y.: A lattice-based MRF model for dynamic near-regular texture tracking.IEEE Transactions on Pattern Analysis and Machine Intelligence 29, 777–792 (2007).

[15] Leung, T., Malik, J.: Detecting, localizing and grouping repeated scene elements from an image. In: European Conference on Computer Vision, Cambridge, UK, pp. 546–555 (1996).

[16] Tuytelaars, T., Turina, A., Gool, L.: Noncombinational detection of regular repeti-tions under perspective skew.IEEE Transactions on Pattern Analysis and Machine Intelligence 25, 418–432 (2003).

[17] Schaffalitzky, F., Zisserman, A.: Geometric grouping of repeated elements within images. In: Shape, Contour and Grouping in Computer Vision. Lecture Notes in Computer Science, pp. 165–181.Springer, Heidelberg (1999).

[18] Forsyth, D.A.: Shape from texture without boundries. In: European Conference in Computer Vision, Copenhagen, Denmark, pp. 225–239 (2002).

[19] Lobay, A., Forsyth, D.A.: Recovering shape and irradiance maps from rich dense texton fields.

© 2005 - 2014 JATIT & LLS. All rights reserved.
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

In: Computer Vision and Pattern Recognition, Washington, USA, pp. 400–406 (2004).

[20] Chetverikov, D.: Pattern regularity as a visual key. Image and Vision Comput-ing 18, 975–985 (2000)

[21] Leu, J.: On indexing the periodicity of image textures. Image and Vision Comput-ing 19, 987–1000 (2001)

[22] Charalampidis, D.: Texture synthesis: Textons revisited. IEEE Transactions on Image Processing 15, 777–787 (2006)

[23] Starovoitov, V., Jeong, S.Y., Park, R.: Texture periodicity detection: features, properties, and comparisons. IEEE Transactions on Systems, Man, and Cybernetics-A 28, 839–849 (1998)

[24] Gruenbaum, B. et al, Tilings and patterns. W .H. free-man and company, New Yourk (1987).

[25] Yan Gui and al, Texel based regular and Near-regular Texture Characterization, International conference of multimedia and signal processing 2011.

[26] Gonzalez, R.C. and R.E. Woods (1993). Digital Image Processing. Addison-Wesley, MA, 3rded.

[27] Lin, H.-C., L.-L. Wang and S.-N. Yang (1996). Automatic deter- mination of the spread parameter in Gaussian smoothing. Pattern Recognition Letters 17 (12), 1247-1252.

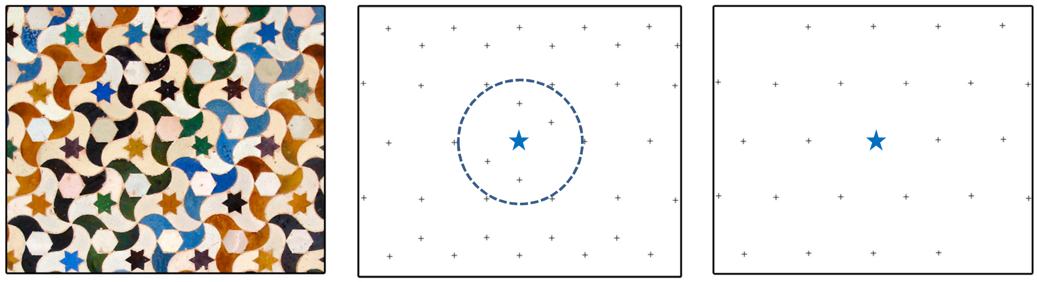
[28] D. Schattschneider.The plane symmetry groups:their recognition and notation.American Mathematical Monthly, 85:439, 1978.

[29] J H Conway, H Burgiel and C Goodman-Strauss. The symmetry of things. A K Peters Ltd. 2008. ISBN 978 1 56881 220 5.

[30] G Loy and J. Eklundh, “Detecting symmetry and symmetric constellations of features." inProc. European Conf. Computer Vision, 2006, pp. 508–521.

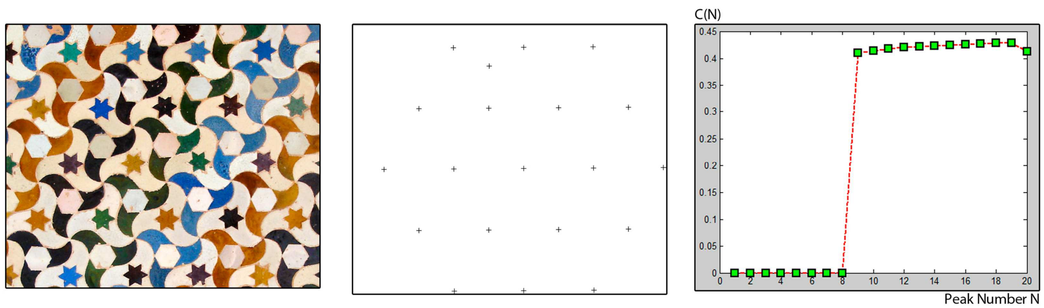
[31] T.Gangopadhyay XLRI C.H.Area(E), Jamshedpur, Further Tiling Patterns Involving Islamic Rosettes with an Odd Number of Vertices, International Journal of Computer Applications (0975 – 8887) Volume 71– No.6, May 2013

Journal of Theoretical and Applied Information Technology
28th February 2014. Vol. 60 No.3
© 2005 - 2014 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195



Source image without RD with RD

Figure A. 2: Dominant Peaks Found Using Region Of Dominance RD.



(a) (b) (c)

Figure A. 3: Example Of Automatic Peak Number Detection (B) Via Optimizing The Criterion Function (C), In This Example Nopt=19.

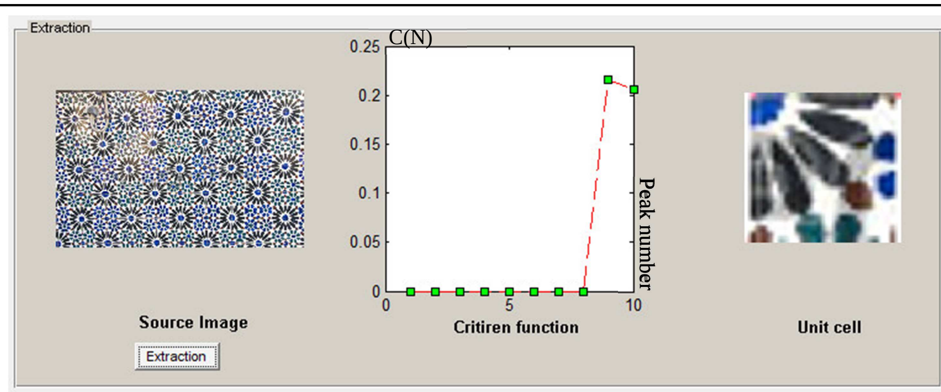


Figure A. 4: Example Of Automatic Peak Number Detection Via Optimizing The Criterion Function C(N) [3].

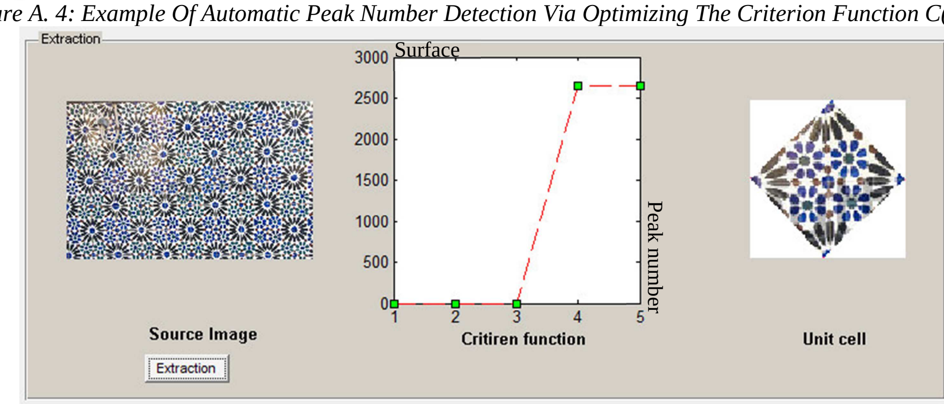


Figure A. 5: Example Of Images For Which The Proposed Method Extracts The Correct Unit Cell, While The Method Of Jingrui Et Al. Falls To Extract Them.

Journal of Theoretical and Applied Information Technology
28th February 2014. Vol. 60 No.3
© 2005 - 2014 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

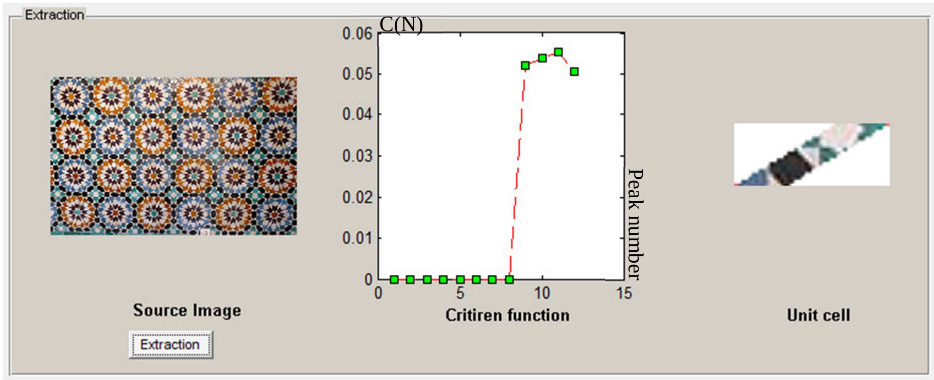


Figure A. 6: Example Of Falls Unit Cell Detection Using Method Of [3].

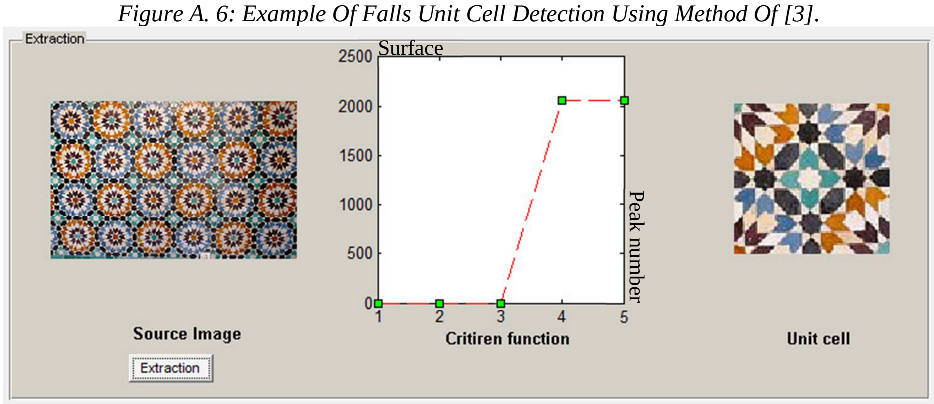


Figure A. 7: Correct Unit Cell Using Our Method

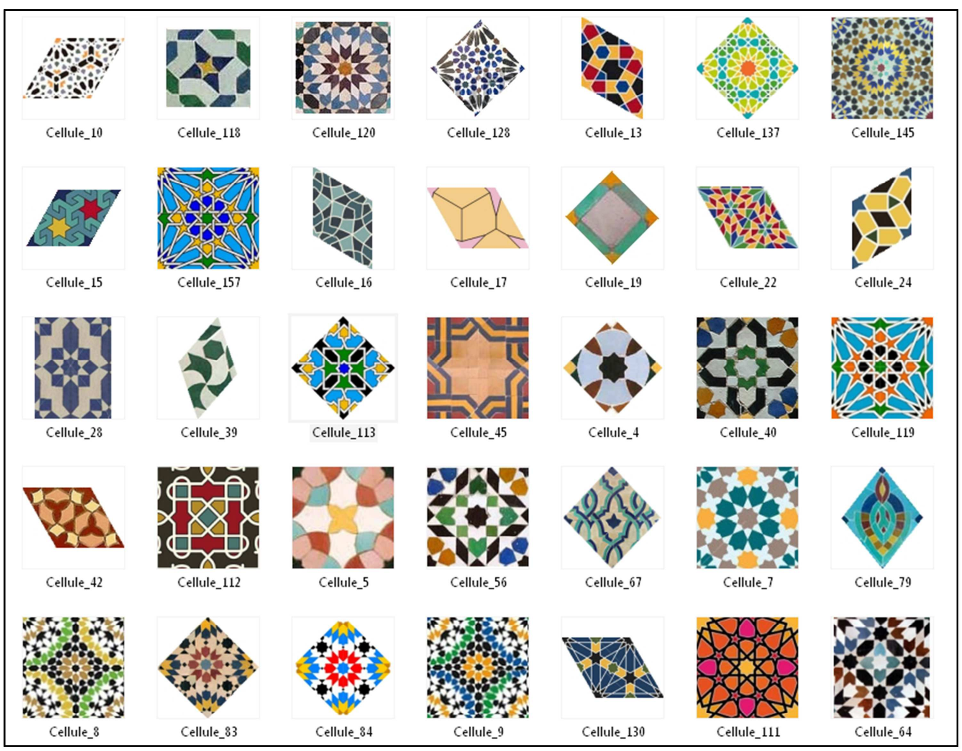


Figure A. 8: Results Of Automatically Detected Lattices Using Our Method.