SPATIAL GENERAL EPIDEMIC MODEL

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ABSTRACT

The general consideration in Spatial General Epidemic Model is controlling the propagation of the disease starting from an infected individual. By a suitable control process, this epidemic can be rooted out. Here, we suggest the control process using the percolation probability to be attached with the formation of edges having the suitable probabilities, giving the random graphs which will explain the extinction probability of the epidemic.

Keywords: *Spatial General Epidemic Model, Extinction of the Epidemic, Random Graphs, Threshold Behaviour of the Epidemic*

1. INTRODUCTION

Currently in the period of dynamic indeterminism in Science, there is hardly a serious piece of research, which, if treated realistically, does not involve operations on Stochastic Processes. Stochastic Processes concern , themselves with the sequences of events governed by the probabilistic laws.

Bailey[1] has observed that the total load of human misery and suffering from a communicable disease in the world today is incalculable, and presents a formidable challenge to the public health authorities, epidemiologists, parasitologists, entomologists, biomathematicians and any other experts whose skills may have some bearing on the problems involved. He has also observed that the fearful tool of human life and happiness exacted through ages by the prevalence of widespread diseases and pestilence affords a spectacle that is both fascinating and repellent.

The theory of random graphs can be viewed as a modest beginning from which we can learn a variety of techniques and can find out what kind of results we should try to prove with regard to more complicated random structures.

Kuulasmaa[4] has explained the use of random graphs in epidemic modelling. A study on epidemic models through random graphs is focused in this paper.

2. BASIC DEFINITIONS AND RESULTS

DEFINITION 2.1 SPATIAL GENERAL EPIDEMIC GE (Z^d, α, μ, F):

Let the set of sites be Z^d, the d – dimensional integer lattice, and let S be a finite subset of Z^d. We assume that α is a strictly positive real number, μ is a probability density defined on Z^d such that $\mu(0) = 0$ and F is a probability distribution function concentrated on $(0, \infty)$. At time zero there is an infectious individual at each site of S, and the other sites are occupied by healthy individuals. The infectives emit germs independently in Poisson processes with rates α until they are removed, each independently after having been infectious for a random length of time with distribution F. After an individual has been removed, his or her site remains empty for ever. Each emitted germ goes independently to a site whose location with respect to the location of the parent is chosen, according to the contact distribution. If a healthy individual gets a germ, he or she becomes infected and starts emitting germs until he or she is removed after an infectious time with distribution F. If an infected individual or an empty site receives a germ nothing happens.

The most important question about the general epidemic is whether it is possible that the infection never dies out. It happens almost surely if and only if infinitely many individuals are ultimately infected.

DEFINITION 2.2 THE PROBLEM OF EXTINCTION OF THE EPIDEMIC AS RANDOM GRAPHS:

Let $G=(V,E)$ be the simple graph, where $V=Z^d$ and for each, $v\in V$, E contains an edge from v to w, if and only if $\mu(w-v)>0$. Corresponding to the general epidemic, $GE(V,\alpha,\mu,F)$, we define a locally dependent random graph (G,P) such that, there is an infective at every vertex of V, the edge from v to w is black if and only if the infective at v sends a germ to w before he or she is removed. The random graph (G,P) determines the ultimate spread of the epidemic: the individual at $v\in V, v\notin S$ will sooner or later be infected if and only if in the random graph there is a black path from S to V.

DEFINITION 2.3 The percolation probability $P(\zeta^{\Sigma\infty})$ indicates the probability that infection never becomes extinct.

THEOREM 2.1 Let (G,P) and (G,Q) be two locally dependent random graphs, defined on the same directed graph G, with avoidance functions $\{p_v\}$ and $\{q_v\}$ respectively. If $p_v\geq q_v$ for every $v\in V$, we have $P(\xi^{\Sigma})\leq Q(\xi^{\Sigma})$ for any set Σ of paths and $P(\Im^{\Sigma})\leq Q(\Im^{\Sigma})$, if Σ is a countable set of paths.

This theorem is very useful for comparison of percolation probabilities on random graphs with different probability measures.

As an application of the above theorem, we get immediately the result of Hammersley[3] that the percolation probability in a bond percolation process is higher than in the corresponding site process.

3. LEMMA: A THRESHOLD THEOREM FOR THE GENERAL EPIDEMIC PROCESS

"If in the general epidemic $GE(Z^d,\alpha,\mu,F)$, d$\geq$2 and μ is properly at least two dimensional, then there exists a critical infection rate α_c such that $\alpha<\alpha_c$ the infection becomes extinct almost surely, whereas for $\alpha>\alpha_c$, the probability of extinction is less than one. It is interesting to note that if μ is one dimensional and has finite mean and if also F has finite mean, then the probability of extinction is always one".

4. INVESTIGATION OF THE THRESHOLD BEHAVIOUR OF THE EPIDEMIC MODEL IN AN EFFICIENT MANNER

PROOF: Let $GE(Z^d,\alpha,\mu,F)$ be an arbitrary epidemic with (G,P) as the corresponding random graph. We can define two constant lifetime epidemics such that in the random graph, (G,P^*) say, of one of them, the marginal probability for any edge to be black is the same as in (G,P), and in the other , which has constant distribution μ, the probability that an infective emits no germs is the same as in $GE(Z^d,\alpha,\mu',F)$. Let (G,P^0) be the random graph of the latter constant lifetime process. We can use Theorem 2.1 to find out these constant lifetime epidemics provide both an upper bound and a lower bound for the probability of no extinction of

$GE(Z^d,\alpha,\mu,F)$:

$$P^0(\zeta^{\Sigma\infty})\leq P(\zeta^{\Sigma\infty})\leq P^*(\zeta^{\Sigma\infty}).$$

:

Also we note that not all but many random graphs corresponding to general epidemics have product representations. Let $\psi:[0,\infty)\to R$ be the function defined by

$$\psi(x)=-\log\int_0^\infty e^{-xt}dF(t).$$

The random graph (G,P) of $GE(Z^d,\alpha,\mu,F)$ has a product representation if

$(-1)^{i-1}\psi^{(i)}(\alpha) \geq 0$ in the interval $0 \leq x \leq \alpha$ for every i = 1,2,3,…m if $m = \left|\left\{v \in Z^d : \mu(v) > 0\right\}\right| < \infty$ or for every i = 1,2,3,… if m= ∞. In particular, the condition is true if F is infinitely divisible.

5. CONCLUSION

We are interested in the incidence, propagation and control of many infectious diseases. We could identify the incidence and local propagation from the observations we could undertake on any particular disease. From the data, we identify the probability density function which could be fitted for propagation. This leads to the application of the above result and we conclude that the application of random graph theoretical results are more suitable to discuss this real time situation.

Earlier models are based on the assumptions pertaining to the Birth and Death process. The parameters involved in Birth and Death rates may be viewed as functions of both number of individuals present at any level and at any time. There are also models involving assumptions on isolation of infected individuals. In these models, finding the probability generating function using the difference differential equation of the model and thereby getting the extinction probability is found to be a difficult. In our approach, we do not confront with any such difficult situation.

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