



# THE ROLE OF KALMAN FILTER IN THE MODELLING OF GPS ERRORS

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## ABSTRACT

This paper describes about the modeling of errors (like ionospheric delays, atmospheric delays, Tropospheric delays, Multipath effects and dilution of precision etc.) affecting the GPS signals as they travel from satellite to user on Earth. These errors degrade the accuracy of GPS position. An attempt is made to improve the accuracy in locating the GPS receiver by filtering the range measurements with the datum conversion between Universal Transverse Mercator (UTM) and World Geodetic System (WGS - 84) using single frequency ML-250 hand held GPS receiver and smoothing of these coordinates using Kalman filter. A linear recursive filtering technique, Kalman filter is used for greater accuracy in estimating the position of user by considering the initial state of the system, statistics of system noise and measurement errors from sensor noise measurements. The results of proposed Kalman filter technique give better accuracy with more consistency and are found superior to the standard one

**Key words:** *GPS, Datum conversion, Kalman filter, Recursive algorithm, Error covariance, Measurement noise.*

## 1. INTRODUCTION OF GLOBAL POSITIONING SYSTEM

A Satellite-based system Global Positioning System uses a constellation of 24 satellites to give an accurate position of user. GPS receivers have been developed to observe signals transmitted by the satellites and achieve sub-meter accuracy in point positioning and a few centimeters in relative positioning. GPS can be operated in all weather, day and night with out any requirement of Inter visibility between points. GPS provides a global absolute positioning capability with respect to a consistent terrestrial reference frame and considered as an absolute global geodetic positioning system. The GPS satellites are positioned in such a way that at least five to eight satellites are accessible at any point on earth and at any time (Hoffmann-Wellonhaff et al, 1998). GPS is based on a system of coordinates called the World Geodetic System 1984 (WGS-84 whose coordinates are the latitude, longitude, and height) (Kaplan.E.D, 1996 and Parkinson, 1996). GPS data is observed in WGS 84 and Universal Transverse Mercator (UTM). The most common map projection and grid system used for land navigation is the Universal Transverse Mercator (UTM) system.

A key advantage of the UTM projection is its preservation of the shape of the small areas on a map and its grid coordinates permit easy calculations using plane trigonometry (Langely B Richard, 2000). In UTM, the ellipsoid is portioned in to 60 zones with a width of six degrees longitude each. A scale factor of 0.996 is applied to the central meridian (Leick, 1995). The scale factor is to avoid fairly large distortions in the outer areas of zone.

There are several sources of error that degrade the GPS position from few meters to tens of meters (Pratap Misra, 2001). These error sources are Ionospheric, Atmospheric delays, Satellite and Receiver Clock Errors, Multipath, Dilution of Precision, Selective Availability (S/A) and Anti Spoofing (A-S) as described by Hoffmann - Wellenhof et al (1998). The errors could be transmitted via VHF/UHF links and the users can make use of the corrections to fix their positions more accurately. These errors can be reduced to arrive at a more accurate estimate of coordinates of user by means of a recursive algorithm- KALMAN FILTER. The emphasis is given on the above errors to analyze the Kalman filter (Grewel.M. S et al, 2001).



Beran et al (2001) could try to develop Kalman filter based functional model for single frequency positioning which is suitable for low dynamic platforms. Bisnath and Langely (2002) with the use of dual frequency geodetic-quality receiver. Kevin Milton et al (2006) have also adapted a technique, which utilizes semi-codeless technique to with the use of dual frequency geodetic quality receiver. Kevin Milton et al (2006) have adopted a technique which utilizes a semi-codeless technique to provide high accuracy GPS measurements and L1-L2 carrier phase measurement for wide-lane applications. The delay of the P code is varied and the signals are cross correlated until a maximum value is reached. When the cross correlation of the L1 and L2 signals is at a peak value, the relative delay between the L1 and L2 signals is proportional to the ionospheric delay. The derived ionospheric delay may be accounted for in measurement analysis of the L1 signal to provide for a high precision position solution. (<http://www.google.com/patents?hl=en&lr=&vid=USPAT5903654&id=TkcXAAAEBAJ&oi=fnd&dq=ionospheric+delays;29<sup>TH</sup>DEC2006>).

The GPS receiver is giving both WGS-84 data observed in latitude, longitude & altitude and UTM data observed in Northing & Easting (Langely.R.B, 2000). In the conversion process of UTM to WGS 84, accuracy must be obtained without distortions. In the work attempted by Ravindhra .et al (Feb, 2002), only the datum conversion from WGS- 84 to UTM and inaccuracy were discussed. The role of the noise in GPS is only at satellite and receiver segments. The modeling of the noise in satellites and receiver segments are discussed by Langely (March 1997).

Y. Yuan and J. Ou (2001) developed one robust recurrence technique, based on the efficient combination of single-frequency GPS observations by users and the high-precision differential ionospheric delay corrections from WAAS. For the commonly used GPS wide-area augmentation systems (WAAS) with a grid ionospheric model, the efficient modeling of ionospheric delays in real time, for single-frequency GPS users, is still a crucial issue which needs further research. This is particularly necessary when differential ionospheric delay corrections cannot be broadcast, when users cannot receive them or when there are ionospheric anomalies. Ionospheric delays have a severe effect on navigation performance of single-frequency receivers. A new scheme is

extended their work further and recent precise point positioning results show decimeter accuracy

proposed which can efficiently address the above problems.

## 2. THE KALMAN FILTER

A significant mathematical toolbox used for stochastic estimation from noisy sensor measurements is *Kalman filter*. Kalman filtering is based on linear mean square error filtering (estimation) and it is essentially a set of mathematical equations that implement a Predictor-corrector type estimator which is *optimal*. It minimizes the estimated *error* covariance —when some presumed conditions are met. For the given spectral characteristics of an additive combination of signal and noise, the linear operation on this input combination yields the best (meaning minimum square error) separation of the signal from the noise is to be known.

The distinctive feature of Kalman filter is about the description of its mathematical formulation in terms of state space analysis as per (Bozic, 1999) and its solution is computed recursively. As each update estimate is computed from the previous estimate and the input data, only previous estimate requires storage. The filter is a computational algorithm that processes measurements to deduce a minimum error estimate of the system by utilizing knowledge of the system, measurement dynamics, and statistics of the system, noises measurement errors and initial condition information. It is to improve the quality of datum conversion using this smoothening technique. It also reduces the error while converting from UTM to WGS-84 GPS data. Here by employing this smoothening technique, Kalman filter puts up better UTM to WGS-84 conversion efficiency. The effects of ionospheric delays have already been discussed by Klobuchar (May 1987). Smoothening of WGS- 84 with the help of Kalman filter has been discussed by Malleswari et al (2005). But in this proposed technique, the Kalman filter is used to smoothen the UTM coordinates. Since the time of its introduction, the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. This is likely due in large part to advances in digital computing, relative simplicity and robust nature of the filter itself. Rarely do the conditions



necessary for optimality actually exist, and yet the filter apparently works well for many applications in spite of this situation as emphasized by (Peter S Maybeck, 2001). Langely (October, 2000) discussed about the GPS receivers tracking 8 or more satellites simultaneously and determining the positions using all of the available observations with a least – squared – error algorithm.

**3 MATHEMATICAL FORMULATION OF KALMAN FILTER**

The Kalman filter addresses the general problem of trying to estimate the state  $x \in \mathfrak{R}^n$  of a discrete-time controlled process that is governed by the linear stochastic difference equation as in equation 1.

$$X_k = A X_{k-1} + B U_k + W_{k-1} \text{----- (1)}$$

With a measurement  $x \in \mathfrak{R}^m$  that is (as stated in equation 2).

$$Z_k = H X_k + V_k \text{----- (2)}$$

The random variables  $W_k$  and  $V_k$  represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions

$$P(W) - N(0, Q) \text{----- (3)}$$

$$P(V) - N(0, R) \text{----- (4)}$$

The process noise covariance  $Q$  and measurement noise covariance  $R$  matrices as in equations 3 & 4 might change with each time step or measurement, however here we assume they are constant (Peter S Maybeck (2001).

The  $n \times n$  matrix  $A$  in the difference equation (1) relates the state at the previous time step  $K-1$  to the state at the current step  $K$ , in the absence of either a driving function or process noise. Note that in practice  $A$  might change with each time step, but here we assume it is constant. The  $n \times 1$  matrix  $B$  relates the optional control input  $u \in \mathfrak{R}^1$  to the state  $x$ . The  $m \times n$  matrix  $H$  in the measurement equation (2) relates the state to the measurement  $Z_k$ . In practice  $H$  might change with each time step or measurement, but here we assume it is constant. The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update

equations as shown in figure 1. Discrete Kalman filter time update equations (5 & 6) are given as

$$\hat{x}_k^- = A \hat{x}_{k-1} + B u_k \text{----- 5 \& 6}$$

$$P_k^- = A P_{k-1} A^T + Q$$

Time update equations project the state and covariance estimates forward from time step  $k-1$  to step  $k$ .  $A$  and  $B$  are from equation (1), while  $Q$  is from is from equation (.3). Initial conditions for the filter are discussed in the earlier references. Discrete Kalman filter measurement update equations (7, 8 & 9) are given below.

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

$$\hat{x}_k = x_k^- + K_k (Z_k - H \hat{x}_k^-) \text{-----7, 8 \& 9}$$

$$P_k = (I - K_k H) P_k^-$$

The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback--i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problems.

The first task during the measurement update is to compute the Kalman gain,  $K_k$ . The next step is to actually measure the process to obtain  $Z_k$ , and then to generate an *a posteriori* state estimate by incorporating the measurement as in equation (8). The final step is to obtain an *a posteriori* error covariance estimate via equation (9). After each time and measurement update pair, the process is repeated with the previous *a posteriori* estimates used to project or predict the new *a priori* estimates. This recursive nature is one of the very appealing features of the Kalman filter—it makes practical implementations much more feasible than (for example) an implementation of a Wiener filter (Brown and Hwang 1992) which is designed to operate on all of the data directly for each estimate. The Kalman filter instead recursively conditions the current estimate on all of the past measurements. Figure 1 below offers a complete picture of the operation of the filter, combining the high-level equations 5 & 6.

#### 4. FILTER PARAMETERS AND TUNING

In the actual implementation of the filter, the measurement noise covariance  $R$  is usually measured prior to operation of the filter. Measuring the measurement error covariance  $R$  is generally practical (possible) because we need to be able to measure the process anyway (while operating the filter) so we should generally be able to take some off-line sample measurements in order to determine the covariance of the measurement noise. The determination of the process noise covariance  $Q$  is generally more difficult as we typically do not have the ability to directly observe the process we are estimating. Sometimes a relatively simple (poor)

process model can produce acceptable results if one "injects" enough uncertainty into the process via the selection of  $Q$ .

Certainly in this case one would hope that the process measurements are reliable. In either case, whether or not we have a rational basis for choosing the parameters, often times superior filter performance (statistically speaking) can be obtained by tuning the filter parameters  $Q$  and  $R$ . The tuning is usually performed off-line, frequently with the help of another (distinct) Kalman filter in a process generally referred to as system identification, which is clearly stated in (Bozic, 1999).

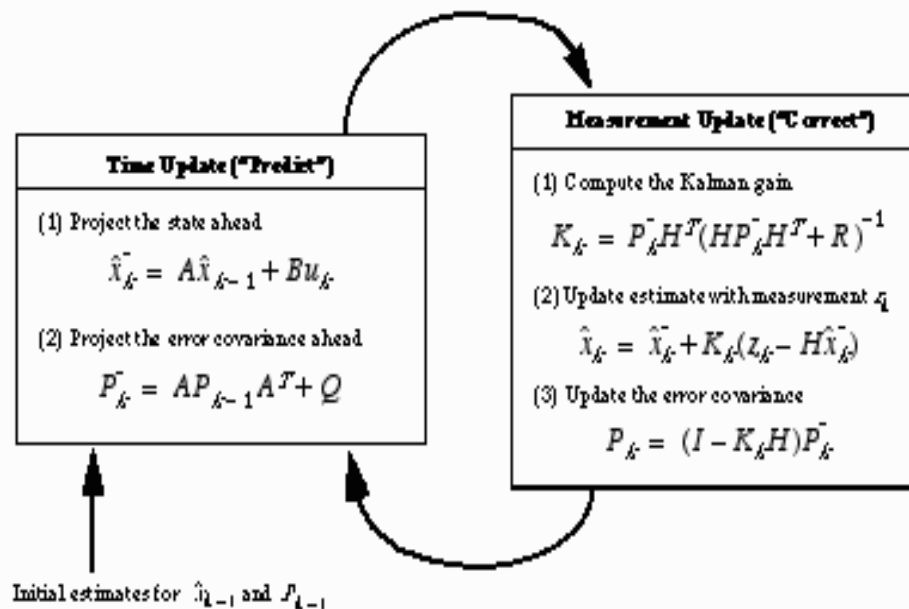


Figure1. A complete picture of the operation of the Kalman filter

In closing we note that under conditions where  $Q$  and  $R$  are in fact constant, both the estimation error covariance  $P_k$  and the Kalman gain  $K_k$  will stabilize quickly and then remain constant. If this is the case, these parameters can be pre-computed by either running the filter off-line, or for example by determining the steady-state value of  $P_k$ .

It is frequently the case however that the measurement error (in particular) does not remain constant. For example, when sighting beacons in our optoelectronic tracker ceiling panels, the noise in measurements of nearby beacons will be smaller than that in far-away beacons. Also, the process noise  $Q$  is sometimes

changed dynamically during filter operation – becoming  $Q_k$  – in order to adjust to different dynamics. For example, in the case of tracking the head of a user of a 3D virtual environment we might reduce the magnitude of  $Q_k$  if the user seems to be moving slowly, and increase the magnitude if the dynamics start changing rapidly. In such cases  $Q_k$  might be chosen to account for both uncertainties about the user's intentions and uncertainty in the model.

#### 5. RESULTS AND DISCUSSIONS

Using single frequency ML 250 GPS hand held receiver, the field data is collected at different



locations in around Hussain Sagar and Gandipet, Hyderabad. The receiver is giving WGS – 84 data ( $\Phi_{rx}$  and  $\lambda_{rx}$ ) and UTM data ( $N_{rx}$  &  $E_{rx}$ ). The variance for WGS 84 of Gandipet is:  $\lambda_{rx}$ -Longitude of Receiver is 27.0337204 in degrees and  $\Phi_{rx}$ - Latitude of Receiver is 6.89938593 in degrees. The variance for WGS 84 of Hussain Sagar is:  $\lambda_{rx}$ -Longitude of Receiver is 13.39910013 in degrees and  $\Phi_{rx}$ - Latitude of Receiver is 0.020659696 in degrees. Now the UTM coordinates are converted into WGS- 84 coordinates. It is quite evident that the converted values of latitude and longitude ( $\Phi_{prg}$  and  $\lambda_{prg}$ ) with out applying Kalman filter are giving poor resolution as shown in tables 1 and 2. And they are more inconsistent as the variance is more for the converted data, i.e.,  $\Phi_{prg}$  - Latitude in degrees before applying kalman filter: 7.3488 (Gandipet) and 0.001264126 (Hussain Sagar) Hence the UTM data to WGS 84 converted data now smoothed by Kalman filtering algorithm to test the accuracy and consistency ( $\Phi_{kf}$  and  $\lambda_{kf}$ ) which is giving a very small variance. It is shown from the tables 1 and 2 that,  $\Phi_{kf}$  – the Latitude in degrees after applying kalman filter is 0.004221766 for Gandipet and 0.00003667424 for Hussain Sagar.. Similarly,  $\lambda_{kf}$  - Longitude in degrees after applying kalman

**Table1a: Comparison of variance for different longitudes of Gandipet**

Longitude	variance
$\lambda_{rx}$ - Longitude in degrees (Receiver)	27.0337204
$\lambda_{prg}$ - Longitude in degrees before applying kalman filter	27.0337194
$\lambda_{kf}$ - Longitude in degrees after applying Kalman filter	0.03084715
$\lambda_{s/w}$ - Longitude in degrees after applying web soft	6.904284042
<b>Table1b: Comparison of variance for different latitudes of Gandipet</b>	
Latitude	variance
$\Phi_{rx}$ - Latitude in degrees (Receiver)	6.89938593
$\Phi_{prg}$ - Latitude in degrees before applying kalman filter	7.3488
$\Phi_{kf}$ - Latitude in degrees after applying Kalman filter	0.004221766
$\Phi_{s/w}$ - Latitude in degrees after applying web soft	0.011725697

filter is 0.03084715 for Gandipet and 0.0006331302 for Hussain Sagar. So, lesser the variance more will be the consistency. Again, the same GPS receiver's UTM data ( $N_{rx}$  &  $E_{rx}$ ) is fed to Web Software "Coordinate. Transform" to validate WGS 84 data, i.e., ( $\Phi_{s/w}$  &  $\lambda_{s/w}$ ). This converted data is also again analyzed as  $\Phi_{s/w}$ - Latitude in degrees after applying websoft is 0.011725697 for Gandipet and 0.000198005 for Hussain Sagar. After comparing all the variances, that is ( $\Phi_{s/w}$ ,  $\Phi_{rx}$ ,  $\Phi_{kf}$  &  $\Phi_{prg}$  and  $\lambda_{s/w}$ ,  $\lambda_{prg}$ ,  $\lambda_{kf}$  &  $\lambda_{rx}$ ), it is found that the smoothed converted data, which is developed from Kalman filtering technique ( $\lambda_{kf}$  &  $\Phi_{kf}$ ), is having better consistency. A comparative analysis is emphasized in the figures 2, 3, 4 and 5. The accuracy of results obtained from program conversions has been validated. Table 1a and 1b indicate the comparison of all latitudes and longitudes of Gandipet with respect to source data (receiver), before and after applying Kalman filter and after applying web soft. Table 2a and 2b indicate the comparison of all latitudes and longitudes of Hussain Sagar with respect to source data (receiver), before and after applying Kalman filter and after applying web soft.

**Table2a: Comparison of variance for different longitudes of Hussain Sagar**

Longitude	variance
$\lambda_{rx}$ - Longitude in degrees (Receiver)	13.39910013
$\lambda_{prg}$ - Longitude in degrees before applying kalman filter	0.001264126
$\lambda_{kf}$ - Longitude in degrees after applying Kalman filter	0.0006331302
$\lambda_{s/w}$ - Longitude in degrees after applying web soft	0.00355667
<b>Table2b: Comparison of variance for different Latitudes of Hussain Sagar</b>	
Latitude	variance
$\Phi_{rx}$ - Latitude in degrees (Receiver)	0.020659696
$\Phi_{prg}$ - Latitude in degrees before applying kalman filter	0.000292464
$\Phi_{kf}$ - Latitude in degrees after applying Kalman filter	0.00003667424
$\Phi_{s/w}$ - Latitude in degrees after applying web soft	0.000198005

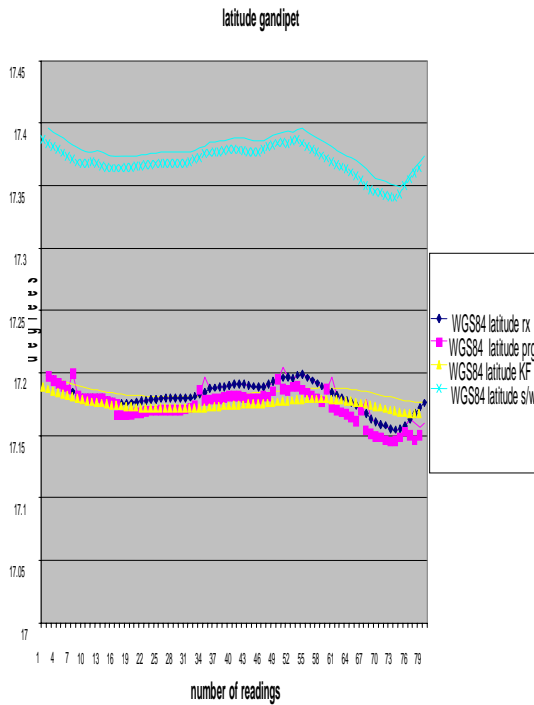


Figure2. Plot of Latitudes for the field data of Gandipet

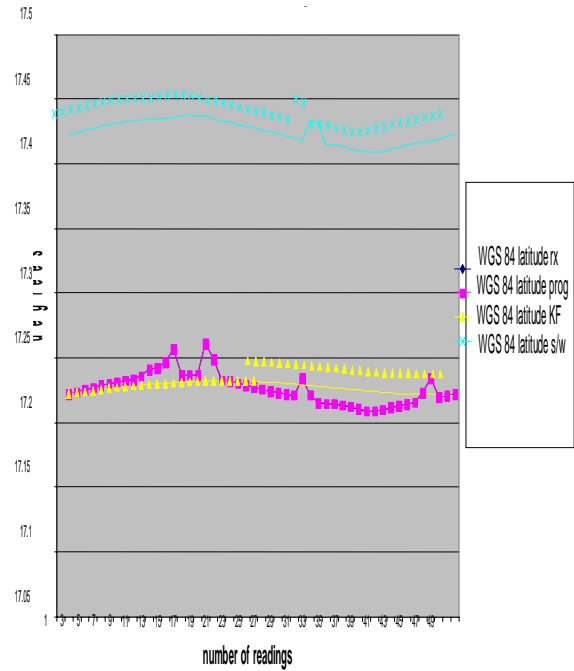


Figure 4. Plot of Latitudes for the field data of Hussain Sagar

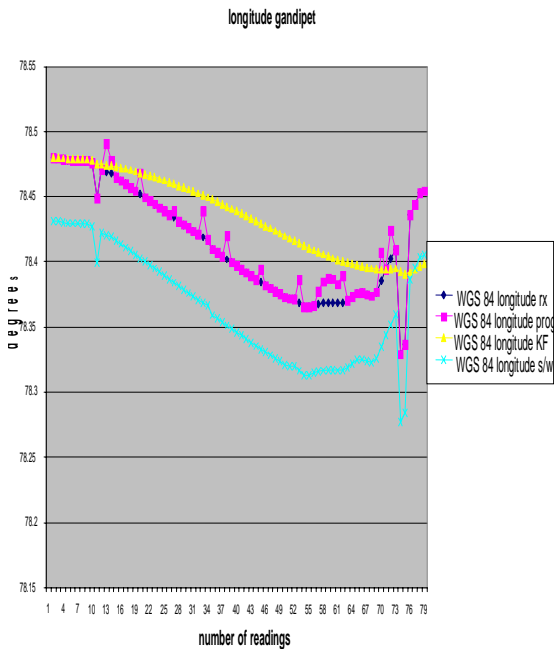


Figure 3. Plot of Longitudes for the field data of Gandipet

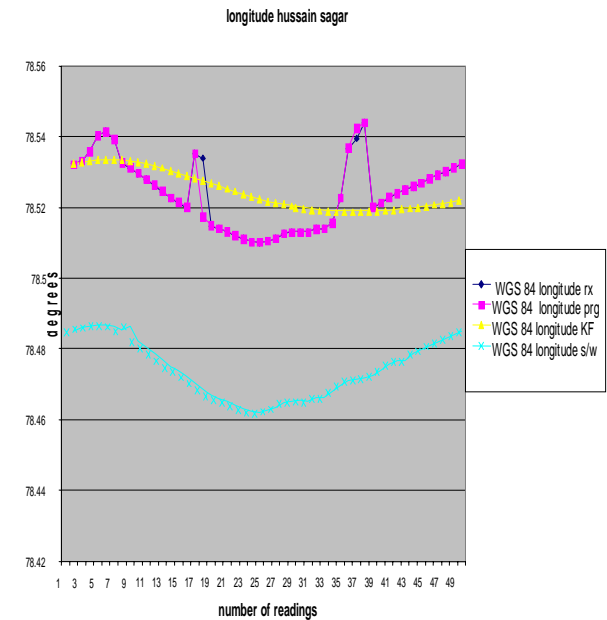


Figure 5. Plot of Longitudes for the field data of Hussain Sagar



## 6. CONCLUSIONS

A comparison of accuracy is suggesting, accuracy through Kalman filter application is certainly yielding better results. The present study & data analysis methodology showed that the variations in the signal related to WGS- 84 data can be smoothened using Kalman filter with in the range of studies made and the analysis is found to yield better accuracies as, it is shown from the tables 1 and 2 that,  $\Phi_{kf}$  – the Latitude in degrees after applying kalman filter is 0.004221766 for Gandipet and 0.00003667424 for Hussain Sagar. Similarly,  $\lambda_{kf}$  - Longitude in degrees after applying kalman filter is 0.03084715 for Gandipet and 0.0006331302 for Hussain Sagar. So, lesser the variance more will be the consistency. However the extensive application of the methodology for the data in bringing out the limitations of the smoothening of the signal, a statistical evaluation atleast in robust domain could throw some light on actual information content & loss of the information through Kalman filtering.

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