DESIGN OF ADAPTIVE BACKSTEPPING WITH
GRAVITATIONAL SEARCH ALGORITHM FOR NONLINEAR
SYSTEM

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ABSTRACT
Adaptive backstepping controller is designed for tracking purpose of nonlinear system with unknown parameter is injected to it. Gravitational search algorithm (GSA) is integrated with the designed controller in order to automatically tune its control parameters and adaptation gain since the tracking performance of the controller relies on these parameters. Performance evaluation is observed based on the tracking output and the tracking error between reference input and the system’s output. The effectiveness of the adaptive backstepping controller is verified by looking at the lowest amount value of Sum of Squared Error (SSE) attained from the simulation process. The results show that the system’s output follow the reference input given with remarkably small tracking error.

Keywords: Adaptive Backstepping, Tracking Error, Gravitational Search Algorithm, Sum of Squared Error, Disturbance

1. INTRODUCTION
Backstepping is designed by stepping back toward the control input starting with the scalar equation separated from it by the largest number of integrations. Thus, the design process of this controller starts at the known-stable system and “back out” new controllers that progressively stabilize each outer subsystem. When the final external control is reached, the process is terminated. Backstepping technique is based directly on the mathematical model of the examined system. It is being developed by introducing new variables into it in a form depending on the state variables, controlling parameters and stabilising functions, which compensate for nonlinearities, exists in the system. The fundamental concept of backstepping method is introduced in detail for regulating and tracking problem [1]. The backstepping design strategy also had been used to develop a Lyapunov-based nonlinear controller for a hydraulic servo system [11],[12],[30] and [31]. Besides, backstepping also had been used in position tracking of an electropneumatic system [13].

Adaptive control is a group of methods which provides a systematic approach for automatic adjustment of controllers in order to achieve or maintain a desired level of control performance when the parameters of the plant dynamic model are unknown or change in time. Adaptive law is provided in control system such that the required performance of the testing system is achieved. The value of adaptation gain is selected properly by various method in order to overcome the uncertainties of the chosen system. An adaptive control law is designed for various type of systems [14], [15],[16],[17], [18], [19], [20]. Combination of adaptive control law with other control techniques also can be seen in [21], [22], [23],[24] and [25]. Combination of backstepping and
An adaptive controller is presented using input/output measurements [26]. Switching laws are used to increase robustness to parametric uncertainties and disturbances and improve transient response of the system. Integration of adaptive and backstepping is also implied for a servo system in a flight simulator [27]. The controller is designed to overcome the parameter uncertainties and load disturbances in the system. Backstepping also being assimilated with adaptive controller in [28], to prevail over friction and external force. Backstepping also had been used as an adaptive method and combined with neural network for strict-feedback nonlinear systems [29]. The developed control scheme guarantees the uniform ultimate boundedness of the closed-loop adaptive systems.

This research work chooses an electro hydraulic actuator servo system as numerical example. An adaptive backstepping controller is designed for position tracking of the system. The control parameters and adaptation gain are then tuned by using Gravitational Search Algorithm (GSA) technique in order to acquire the suitable values for automatic and accurate tracking response. The performance of the designed controller with this optimization technique is compared based on its tracking error. The effectiveness of the backstepping controller is verified in a simulation environment with two different types of reference inputs.

2. PROBLEM FORMULATION

One of the characteristic of electro hydraulic actuator system is small size-to-power ratio and the ability to apply extremely large force and torque. Therefore, it has been used widely in industrial applications. On the other hand, the dynamics of hydraulic systems are highly nonlinear [32] due to input saturation, change of valve opening, valve overlap and friction. Besides, nonlinearities also come from unmodelled dynamic such as uncertainties, external disturbances and leakage. Therefore, nonlinear robust control techniques are needed in order to obtain successful performance of electro hydraulic system. This research work chose electro hydraulic actuator system as numerical example because its position tracking control is highly nonlinear. The system parameters are assumed to be known [31]. External force is given to the system actuator as an additional disturbance to the system. This force is assumed as unknown parameter. With the assumption that the system has a non-saturating load, state space model of EHA system is given in equation (1) [31].

\[ x_1 = x_2 \\
\frac{\dot{x}_2}{m} = -\frac{k}{m}x_1 - \frac{f}{m}x_2 + \frac{S}{m}x_3 - \theta \\
\frac{\dot{x}_3}{k_c} = -\frac{s}{k_c}x_2 - \frac{k_1}{k_c}x_3 + \frac{c}{k_c}\sqrt{\frac{pa-x_3}{2}}k_d\mu (1) \]

with \( c = c_dw\frac{2}{\rho} \)

\( x_1 \) = displacement of the load
\( x_2 \) = load velocity
\( x_3 \) = pressure difference \( p_1 - p_2 \) between the cylinder chambers caused by load
\( \theta \) = unknown parameter of the system

Adaptive backstepping controller design is started with defining error for each state \( x_1, x_2 \) and \( x_3 \) respectively as

\[ z_i = x_i - x_{id} \] (2)

with \( i = 1, 2, 3 \) is the error for each state

\( x_{id} \) = reference input
\( x_{2d} \) and \( x_{3d} \) = virtual control

The control objective is to have EHA track of a specified \( x_{id} \) position trajectory such that \( e_1 \rightarrow 0 \).

2.1 Proposition 1

Equation (1) is assumed with non-saturating load which means that \( x_3 < p_a \). Let \( c_1, c_2, \) and \( c_3 \), be positive tuning parameters and \( \gamma \) as the parameter of the adaptive law, the best and asymptotically stabilized position tracking of equation (1) with respect to the desired input can be achieved by the control \( u \) and virtual control \( x_{2d} \) and \( x_{3d} \) given by

\[ u = k_c \sqrt{\frac{2}{pa-x_3}}\left[ -\frac{S}{m}z_2 + \frac{s}{k_c}x_2 + \frac{k_1}{k_c}x_3 + x_{3d} - c_3z_3 \right] \] (3)

\[ x_{3d} = \frac{m}{S}k_1x_1 + \frac{f}{m}x_2 - z_1 + x_{2d} \]
Taking the derivative of (9) along the system (1)

Substituting

with \( \hat{\theta} \) is the estimate value of unknown parameter \( \theta \).

2.2 Proof of proposition 1

Let the Lyapunov function

The derivative of (6) along the system (1) is

Substituting equation (5) into (7), \( V_1 \) becomes

A new Lyapunov function \( V_2 \) is defined as

with \( \tilde{\theta} = \theta - \hat{\theta} \) and \( \gamma > 0 \) is an adaptation gain.

Taking the derivative of (9) along the system (1) gives

Substituting (4) into (10), \( V_2 \) becomes

Finally \( V_3 \) is defined as

Taking derivative of (12) along the system (1)

By selecting control signal \( u \) as given in (3) and substituting into (13),

with adaptive control law

In order to obtain satisfactory tracking performance, the control parameter of backstepping, \( c_1, c_2, c_3 \), and the gain of adaptive law, \( \gamma \) should be chosen with suitable method. In this work, gravitational search algorithm (GSA) is used to optimise these parameters so that the suitable value can be attained to reduce the tracking error between reference input and system’s output.

3. ALGORITHM

The control parameter of backstepping is significant in order to achieve performance target. This parameter can be obtained by several methods such as heuristic approach, artificial intelligent technique and optimization algorithm. Optimization is a mathematical discipline that concerns of the finding of minima and maxima of functions subject to so-called constraints. It comprises a wide variety of techniques from operation research, artificial intelligence and computer science. Examples of optimization techniques are ant colony optimization, particle swarm optimization, genetic algorithm and gravitational search algorithm. Optimization had been reported as a powerful modelling and problem solving methodology has a broad range of applications in management science, industry and engineering [33]. In the engineering field, these optimization techniques always used for signal processing, image detection and as tuning method for control parameters. A set of benchmarks of heuristic optimization methods is described in [34]. The description can be used for evaluation of the optimization process. Jun Ye used neural network to find the parameter of backstepping controller in order to improve the tracking performance of mobile robots [35] while fuzzy logic and least mean square are used for parameter tuning for backstepping controller to stabilize the attitude of quadrotor UAV [36]. For power system stability enhancement in [37], particle swarm optimization (PSO) technique is used to tune the backstepping controller. PSO also been used for backstepping parameter tuning for maglev transportation system [38]. An adaptive backstepping control tuned by particle swarm optimization (PSO) is designed for stability enhancement of multi-machine power system [39]. The proposed technique shows that the designed controller is effective in stabilizing the system under severe contingencies and perform better than conventional power system stabilizers. Other optimization method is ant colony optimization algorithm [40]. This research used a combination of fuzzy logic controller and neural network to acquire parameters of backstepping and ant colony optimization technique is used to attain the best value for parameters of fuzzy neural network. As an
approach to ship course control, backstepping is developed with genetic algorithm technique, to optimise its parameter [41]. The similar algorithm also has been used in designing backstepping for flight control system [42].

Gravitational Search Algorithm (GSA) is among the latest algorithms for the optimization process. This technique is developed based on the law of gravity and mass interactions [43]. A collection of masses which interact with each other based on the Newtonian gravity and the laws of motion is considered as the searcher agents/objects in this algorithm. The performance of agents is measured by their masses. They are attracted to each other by the gravity force which causes a global movement of all objects toward the objects with heavier masses. Each agent has four specifications; position, inertial mass, active gravitational mass and passive gravitational mass. Solution of the problem is represented by the position of the mass.

GSA had been applied to determine the parameters of PID controller for speed and position control of DC motor [44] by choosing mean squared error (MSE) performance index as the objective function in this work. Adaptive Gravitational Search Algorithm is developed for the optimal tuning of fuzzy controlled servo system, characterized by second-order models with an integral component and variable parameters [45], which produces a new generation of Takagi-Sugeno proportional-integral fuzzy controllers with a reduced time constant sensitivity. Besides, GSA also had been chosen to optimised the parameters of sensor monitoring selection for each round in a point coverage network [46]. Simulation results show that the method is superior to former algorithms in the aspects of parameters optimization, lifetime increase and energy consumptions. In [47], Fuzzy GSA miner is used to develop a novel data mining technique, where the fuzzy controller behaves as an adaptive control for the gravitational coefficient.

Since no published reports on the combination of GSA algorithm with backstepping techniques, this paper integrates both techniques for electro hydraulic systems. GSA is implied to tune the control parameter and the adaptation gain. GSA is also memory-less, and only the current position of the agent plays a role in the updating procedure [43].

The tracking error, \( e(t) \), is very important in the process of designing controller for tracking performance. It represents the different between reference input and real output. This signal is used in the Lyapunov functions, to develop the controller for EHA system. The same signal also being used for the optimization process. Sum of Squared Error (\( SSE \)) is used as an objective function for this study in order to optimize of control and adaptation gain parameters. The formula of \( SSE \) is given by equation (16).

\[
SSE = \sum_{i=1}^{n} (x_i - y_{ref})^2 
\]  

where \( SSE = \text{sum of squared error} \)

\( i = \text{number of iteration} \)

\( x_i = \text{system output at } i \text{ iteration} \)

\( y_{ref} = \text{reference input} \)

A good tracking response will produce minimum \( SSE \).

4. SIMULATION RESULTS

Electro hydraulic actuator system is chosen as numerical example in this work. Thus, the aim of control of this system is to have a good tracking of the specified desired position of reference input. The tracking error should be smaller. The parameter of the testing system is shown in Table 1 [31].

<table>
<thead>
<tr>
<th>Table 1: Parameter of EHA system</th>
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<tbody>
<tr>
<td>Load at the EHS rod, ( m )</td>
</tr>
<tr>
<td>Piston area</td>
</tr>
<tr>
<td>Coefficient of viscous friction, ( f )</td>
</tr>
<tr>
<td>Coefficient of aerodynamic elastic force, ( k )</td>
</tr>
<tr>
<td>Valve port width, ( w )</td>
</tr>
<tr>
<td>Supply pressure, ( P_1 )</td>
</tr>
<tr>
<td>Coefficient of volumetric flow of the valve port, ( c_d )</td>
</tr>
<tr>
<td>Coefficient of internal leakage between the cylinder chambers, ( k_1 )</td>
</tr>
<tr>
<td>Coefficient of servo valve, ( k_p )</td>
</tr>
<tr>
<td>Coefficient involving bulk modulus and EHA volume, ( k_c )</td>
</tr>
<tr>
<td>Oil density, ( \rho )</td>
</tr>
</tbody>
</table>

Four parameters are yielded in the process of designing adaptive backstepping controller for the system chosen. The first three parameters are
control parameters, $c_1$, $c_2$, $c_3$ while the last one, $\gamma$ is adaptation gain for adaptive law. In order to assure the designed controller produce the best performance, suitable value should be given to each of these parameters. These parameters are optimized by using GSA algorithm. The performance of the designed controller is finally evaluated in terms of tracking error. Reference input given to the system is demonstrated by Figure 1 and 2. The objective is to ensure system with the designed controller tracked these input signals.

![Square Wave As Reference Input To The System](image1)

*Figure 1: Square Wave As Reference Input To The System*

100 agents within 100 iterations are chosen for GSA algorithm. Figure 3 illustrates the position output, $x_1$ of the chosen system obtained from the assimilation of GSA with adaptive backstepping when square wave signal is given as reference input to the system.

![Position Output, $x_1$ Obtained From The Designed Controller](image3)

*Figure 3: Position Output, $x_1$ Obtained From The Designed Controller*

Referring to Figure 3, the output $x_1$ of electro hydraulic actuator servo system followed the reference input given smoothly with almost zero tracking error. This situation also has been proved by Figure 4 which demonstrates the tracking error produced by the designed controller.

![The tracking error produced by the integration of adaptive backstepping with GSA](image4)

*Figure 4: The tracking error produced by the integration of adaptive backstepping with GSA*

Based on Figure 4, it can be seen that large error is produced in the early stage of the simulation process. However, this error is shrinking to almost zero as the time increased. These two outputs are generated based on the objective function of GSA algorithm. SSE as shown in equation (16) is chosen as the objective function for GSA in this application. SSE plot for the system with adaptive backstepping with GSA with square wave signal is injected to the system is shown in Figure 5.

![SSE With Respect To Number Of Iteration When Square Wave Signal Is Injected To The System](image5)

*Figure 5: SSE With Respect To Number Of Iteration When Square Wave Signal Is Injected To The System*

By observing figure 5, GSA yields higher SSE in the early iteration. This value is decreased with the increment in the number of iteration and stayed at that value until final iteration. GSA produced SSE at 0.1447 for this situation. Figure 6 illustrates the position output of the system when sine signal is given as reference input while its tracking error is shown in Figure 7.
Similar as before, Figure 6 and Figure 7 proved that electro hydraulic actuator system with the designed controller tracked the reference signal smoothly and beautifully with almost zero tracking error. GSA algorithm applied to the controller fulfilled the requirement of its objective function and produced remarkably small SSE as can be seen in Figure 8.

Higher SSE value is produced at the early number of iteration, and this value is decreased with the increment number of iterations. For this case, GSA yields SSE value at 0.0028. Table 2 presents the value of control parameters and adaptation gain of the designed controller generated by GSA for each reference input given to the system.

These parameters are obtained through GSA by giving the system tracking error to the algorithm. This algorithm operates based on its objective function, SSE. GSA find the optimum solution for each parameter such that SSE will be minimized.

5. CONCLUSION

Electro hydraulic actuator system with unknown parameter as uncertainties is chosen as a numerical example in this study since its position tracking is highly nonlinear. Adaptive backstepping controller is designed for the system. The designed controller consists of several control and adaptation gain parameters. The number of the defined state of the system determines the number of control parameters and the number of virtual control input. In order to achieve desired response, suitable value of these parameters needs to be chosen carefully so that tracking error obtained will be smaller. Instead of using trial and error approach to find appropriate values of these parameters, Gravitational Search Algorithm (GSA) technique is used in this research work, so that system output tracked reference input given properly with smaller or zero tracking error. These can be seen in each graph obtained. These parameters value are varying depends on the requirement of operation of the system. Robustness of the designed controller with GSA technique also had been proved by varying the reference input injected to the system. Although different type of input is injected to the system, the system output tracked reference input smoothly with small tracking error. GSA technique used in this work also improves the tracking performance of the designed controller by finding correct value for all control parameters. Small value of Sum of Squared Error (SSE) yielded by this optimization technique proves outstanding performance of the combined adaptive backstepping controller with GSA technique.

REFERENCES


System Theory University of Texas. March 7-9, 2010. Tyler, USA. IEEE. 2010. 81-84


