Journal of Theoretical and Applied Information Technology
20th September 2013. Vol. 55 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

ON SOFT GENERALIZED CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we continue the study of soft generalized closed sets in a soft topological space introduced by Kannan [1]. Firstly, we give a representation of soft sets and soft topological spaces. Secondly, we investigate behavior relative to soft subspaces of soft generalized closed sets. We show that a soft generalized closed set in a soft compact (soft Lindelöf, soft countably compact) space is also soft compact. Then, we show that a soft compact set in a soft regular space is soft generalized closed and disjoint soft g-closed sets in a soft normal space generally cannot be separated by soft open sets. Finally, we investigate some properties of soft generalized open sets.

Keywords: *Soft Sets, Soft Topological Space, Soft Generalized Closed (Open) Sets.*

1. INTRODUCTION

Several theories, such as the theory of fuzzy sets [2] and theory of rough sets [3] can be considered as mathematical tools for dealing with uncertainties. These theories have their inherent difficulties as pointed out in [4]. The reason for these difficulties is, possibly, the inadequency of the parametrization tool of the theories. In 1999 Molodtsov [4] initiated the concept of soft set theory as a mathematical tool for dealing with uncertinties. Molodtsov [4] presented some applications of the soft set theory in several directions viz. study of smoothness of functions , game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement, etc. Maji et. al. [5,6] presented an application of soft set theory in a decision making problem and made a theoretical study of the soft set theory in more detail.

Recently, Shabir and Naz [7] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X and gave basic notions of soft topological spaces such as soft open and closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft T_i-spaces, for i=1,2,3,4. After then many authors [8, 9, 10, 11, 12] studied some of basic concepts and properties of soft topological spaces.

Levine [13] introduced generalized closed and open sets in topological spaces. In 2012, Kannan [1] defined soft generalized closed and open sets in soft topological spaces and studied their some properties. He introduced these concepts which are defined over an initial universe with a fixed set of parameters and investigated behavior relative to the union and intersection of soft generalized closed sets. He introduced new soft separation axiom and its basic properties.

In this work, we continue investigating the properties of soft generalized closed and open sets. We explore some basic properties of these concepts. We investigate behavior relative to soft subspaces of soft generalized closed sets. We show that a soft generalized closed set in a soft compact (soft Lindelöf, soft countably compact) space is also soft compact. Then, we show that a soft compact set in a soft regular space is soft generalized closed and disjoint soft g-closed sets in a soft normal space generally cannot be separated by soft open sets.

2. PRELIMINARIES

Let X be an initial universe set and E be the set of all possible parameters with respect to X. Parameters are often attributes, characteristics or properties of the objects in X. Let P(X) denote the power set of X. Then a soft set over X is defined as follows.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

DEFINITION 2.1

[4] A pair (F,A) is called a soft set over X where A⊆E and F:A→P(X) is a set valued mapping. In other words, a soft set over X is a parameterized family of subsets of the universe X. For ∀ε∈A, F(ε) may be considered as the set of ε-approximate elements of the soft set (F,A). It is worth noting that F(ε) may be arbitrary. Some of them may be empty, and some may have nonempty intersection.

DEFINITION 2.2

[6] A soft set (F,A) over X is said to be null soft set denoted by Φ if for all e∈A, F(e)=∅. A soft set (F,A) over X is said to be an absolute soft set denoted by Ã if for all e∈A, F(e)=X.

DEFINITION 2.3

[7] Let Y be a nonempty subset of X, then Ỹ denotes the soft set (Y,E) over X for which Y(e)=Y, for all e∈E. In particular, (X,E) will be denoted by X̃.

DEFINITION 2.4

[6] For two soft sets (F,A) and (G,B) over X, we say that (F,A) is a soft subset of (G,B) if A⊆B and for all e∈A, F(e) and G(e) are identical approximations. We write (F,A)⊑(G,B). (F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by (G,B)⊑(F,A). Then (F,A) and (G,B) are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

DEFINITION 2.5

[6] The union of two soft sets of (F,A) and (G,B) over X is the soft set (H,C), where C=A∪B and for all e∈C, H(e)=F(e) if e∈A-B, G(e) if e∈B-A, F(e)∪G(e) if e∈A∩B. We write (F,A)⊔(G,B)=(H,C). [14] The intersection (H,C) of (F,A) and (G,B) over X, denoted (F,A)⊓(G,B), is defined as C=A∩B, and H(e)=F(e)∩G(e) for all e∈C.

DEFINITION 2.6

[7] The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by (F,E)\(G,E), is defined as H(e)= F(e)\G(e) for all e∈E.

DEFINITION 2.7

[7] The relative complement of a soft subset (F,E) is denoted by $(F,E)^c$ and is defined by $(F,E)^c=(F^c,E)$ where F^c:E→P(X) is a mapping given by $F^c(e)=X-F(e)$ for all e∈E.

PROPOSITION 2.8

[7] Let (F,E) and (G,E) be the soft sets over X. Then

(1) $((F,E)\sqcup(G,E))^c=(F,E)^c\sqcap(G,E)^c$

(2) $((F,E)\sqcap(G,E))^c=(F,E)^c\sqcup(G,E)^c$.

DEFINITION 2.9

[10] Let I be an arbitrary index set and $\{(F_i,E)\}_{i\in I}$ be a subfamily of soft sets over X. The union of these soft sets is the soft set (G,E), where $G(e)=\cup_{i\in I}F_i(e)$ for each e∈E. We write $\sqcup_{i\in I}(F_i,E)=(G,E)$. The intersection of these soft sets is the soft set (H,E), where $H(e)=\cap_{i\in I}F_i(e)$ for all e∈E. We write $\sqcap_{i\in I}(F_i,E)=(H,E)$.

PROPOSITION 2.10

[6, 10] Let (F,E), (G,E), (H,E), (K,E) be the soft sets over X. Then

(1) (F,E)⊓(F,E)=(F,E), (F,E)⊓Φ=Φ, (F,E)⊓Ẽ =(F,E).

(2) (F,E)⊔(F,E)=(F,E), (F,E)⊔Φ=(F,E), (F,E)⊔Ẽ=Ẽ.

(3) (F,E)⊓(G,E)=(G,E)⊓(F,E)
(F,E)⊔(G,E)=(G,E)⊔(F,E).

(4) (F,E)⊔((G,E)⊔(H,E))=((F,E)⊔(G,E))⊔(H,E),
(F,E)⊓((G,E)⊓(H,E))=((F,E)⊓(G,E))⊓(H,E).

(5)
(F,E)⊔((G,E)⊓(H,E))=((F,E)⊔(G,E))⊓((F,E)⊔(H,E)),

(F,E)⊓((G,E)⊔(H,E))=((F,E)⊓(G,E))⊔((F,E)⊓(H,E)).

(6) (F,E)⊑(G,E) if and only if (F,E)⊓(G,E)=(F,E).

(7) (F,E)⊑(G,E) if and only if (F,E)⊔(G,E)=(G,E).

(8) If (F,E)⊓(G,E)=Φ, then $(F,E)\sqsubseteq(G,E)^c$.

(9) If (F,E)⊑(G,E) and (G,E)⊑(H,E), then (F,E)⊑(H,E).

(10) If (F,E)⊑(G,E) and (H,E)⊑(K,E), then (F,E)⊓(H,E)⊑(G,E)⊓(K,E).

(11) $(F,E)\sqcup(F,E)^c=\tilde{X}$, $(F,E)\sqcap(F,E)^c=\Phi$.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

(12) $(F,E) \sqsubseteq (G,E)$ if and only if $(G,E)^c \sqsubseteq (F,E)^c$.

DEFINITION 2.11

[7] Let (F,E) be a soft set over X and $x \in X$. $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$. For any $x \in X$, $x \notin (F,E)$, if $x \notin F(e)$ for some $e \in E$.

DEFINITION 2.12

[7] Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is said to be a soft topology on X if
(1) $\Phi, \tilde{X} \in \tilde{\tau}$
(2) If $(F,E),(G,E) \in \tilde{\tau}$, then $(F,E) \sqcap (G,E) \in \tilde{\tau}$
(3)If $\{(F_i,E)\}_{i \in I} \in \tilde{\tau}$, $\forall i \in I$, then $\sqcup_{i \in I}(F_i,E) \in \tilde{\tau}$.

The pair $(X,\tilde{\tau})$ is called a soft topological space. Every member of $\tilde{\tau}$ is called a soft open set. A soft set (F,E) is called soft closed in X if $(F,E)^c \in \tilde{\tau}$.

PROPOSITION 2.13

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X and $\tilde{\tau}'$ denotes the collection of all soft closed sets. Then
(1) $\Phi, \tilde{X} \in \tilde{\tau}'$
(2) If $(F,E),(G,E) \in \tilde{\tau}'$, then $(F,E) \sqcup (G,E) \in \tilde{\tau}'$
(3) If $\{(F_i,E)\}_{i \in I} \in \tilde{\tau}'$, $\forall i \in I$, then $\sqcap_{i \in I}(F_i,E) \in \tilde{\tau}'$.

DEFINITION 2.14

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X and (F,E) be a soft set over X. Then the soft closure of (F,E), denoted by $\overline{(F,E)}$, is the intersection of all soft closed super sets of (F,E).

Clearly, $\overline{(F,E)}$ is the smallest soft closed set over X which contains (F,E).

THEOREM 2.15

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X, (F,E) and (G,E) are soft sets over X. Then
(1) $\overline{\Phi} = \Phi$ and $\overline{\tilde{X}} = \tilde{X}$
(2) $(F,E) \sqsubseteq \overline{(F,E)}$
(3) (F,E) is a soft closed set if and only if $(F,E) = \overline{(F,E)}$
(4) $\overline{\overline{(F,E)}} = \overline{(F,E)}$
(5) $(F,E) \sqsubseteq (G,E)$ implies $\overline{(F,E)} \sqsubseteq \overline{(G,E)}$
(6) $\overline{(F,E) \sqcup (G,E)} = \overline{(F,E)} \sqcup \overline{(G,E)}$
(7) $\overline{(F,E) \sqcap (G,E)} \sqsubseteq \overline{(F,E)} \sqcap \overline{(G,E)}$.

DEFINITION 2.16

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X, (G,E) be a soft set over X and $x \in X$. Then x is said to be a soft interior point of (G,E) and (G,E) is said to be a soft neighbourhood of x if there exists a soft open set (F,E) such that $x \in (F,E) \sqsubseteq (G,E)$.

DEFINITION 2.17

[11] Let $(X,\tilde{\tau})$ be a soft topological space over X and (F,E) be a soft set over X. Then the soft interior of (F,E), denoted by $(F,E)°$, is the union of all soft open sets contained in (F,E).

Thus $(F,E)°$ is the largest soft open set contained in (F,E).

THEOREM 2.18

[11] Let $(X,\tilde{\tau})$ be a soft topological space over X and (F,E) and (G,E) are soft sets over X. Then
(1) $\Phi° = \Phi$ and $(\tilde{X})° = \tilde{X}$
(2) $(F,E)° \sqsubseteq (F,E)$
(3) (F,E) is a soft open set if and only if $(F,E) = (F,E)°$
(4) $((F,E)°)° = (F,E)°$
(5) $(F,E) \sqsubseteq (G,E)$ implies $(F,E)° \sqsubseteq (G,E)°$
(6) $((F,E) \sqcap (G,E))° = (F,E)° \sqcap (G,E)°$
(7) $((F,E) \sqcup (G,E))° \sqsupseteq (F,E)° \sqcup (G,E)°$.

THEOREM 2.19

[11] Let $(X,\tilde{\tau})$ be a soft topological space over X and (F,E) be a soft set over X. Then
(1) $((F,E)^c)° = (\overline{(F,E)})^c$
(2) $\overline{((F,E)^c)} = ((F,E)°)^c$.

DEFINITION 2.20

[7] Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the soft subset of (F,E) over Y denoted by $({}^YF,E)$, is defined as ${}^YF(e) = Y \cap F(e)$, for all $e \in E$. In other words $({}^YF,E) = \tilde{Y} \sqcap (F,E)$.

DEFINITION 2.21

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X and Y be a nonempty subset of X. Then $\tilde{\tau}_Y = \{({}^YF,E) : (F,E) \in \tilde{\tau}\}$ is said to be the soft relative topology on Y and $(Y,\tilde{\tau}_Y)$ is called a soft subspace of $(X,\tilde{\tau})$. We can easily verify that $\tilde{\tau}_Y$ is, in fact, a soft topology on Y.

THEOREM 2.22

[7] Let $(Y,\tilde{\tau}_Y)$ be a soft subspace of a soft topological space $(X,\tilde{\tau})$ and (F,E) be a soft set over X, then
(1) (F,E) is soft open in Y if and only if $(F,E) = \tilde{Y} \sqcap (G,E)$ for some $(G,E) \in \tilde{\tau}$.
(2) (F,E) is soft closed in Y if and only if $(F,E) = \tilde{Y} \sqcap (G,E)$ for some soft closed set (G,E) in X.

DEFINITION 2.23

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X, (G,E) be a soft closed set in X and $x \in X$ such that $x \notin (G,E)$. If there exist soft open sets (F_1,E)

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

and (F_2,E) such that $x\in(F_1,E)$, $(G,E)\sqsubseteq(F_2,E)$ and $(F_1,E)\sqcap(F_2,E)=\Phi$, then $(X,\tilde{\tau})$ is called a soft regular space.

DEFINITION 2.24

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X, (F,E) and (G,E) soft closed sets in X such that $(F,E)\sqcap(G,E)=\Phi$. If there exist soft open sets (F_1,E) and (F_2,E) such that $(F,E)\sqsubseteq(F_1,E)$, $(G,E)\sqsubseteq(F_2,E)$ and $(F_1,E)\sqcap(F_2,E)=\Phi$, then $(X,\tilde{\tau})$ is called a soft normal space.

DEFINITION 2.25

[8] Let $(X,\tilde{\tau})$ be a soft topological space over X,

(1) A family $C=\{(F_i,E):i\in I\}$ of soft open sets in $(X,\tilde{\tau})$ is called a soft open cover of X, if satisfies $\sqcup_{i\in I}(F_i,E)=\tilde{X}$. A finite subfamily of soft open cover C of X is called a finite subcover of C, if it is also a soft open cover of X.

(2) X is called soft compact if every soft open cover of X has a finite subcover.

DEFINITION 2.26

[15] A soft topological space $(X,\tilde{\tau})$ is called soft countably compact if every countable soft open cover of X has a finite subcover.

DEFINITION 2.27

[16] A soft topological space $(X,\tilde{\tau})$ is called soft Lindelöf if every soft open cover of X has a countable subcover.

DEFINITION 2.28

[17] Let $(X,\tilde{\tau})$ be a soft topological space over X, (F,E) and (G,E) soft sets in X. Two soft sets (F,E) and (G,E) are said to be soft disconnected sets if $\overline{(F,E)}\sqcap(G,E)=\Phi$ and $\overline{(G,E)}\sqcap(F,E)=\Phi$.

3. SOFT GENERALIZED CLOSED SETS

Now we recall the definition of soft generalized closed sets in a soft topological space introduced by Kannan [1].

DEFINITION 3.1

[1] Let $(X,\tilde{\tau})$ be a soft topological space. A soft set (F,E) is called a soft generalized closed (briefly soft g-closed) in X if $\overline{(F,E)}\sqsubseteq(G,E)$ whenever $(F,E)\sqsubseteq(G,E)$ and (G,E) is soft open in X.

EXAMPLE 3.2

Let $X=\{x_1, x_2, x_3\}$, $E=\{e_1, e_2\}$ and $\tilde{\tau}=\{\Phi, \tilde{X}, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E)\}$ where

$F_1(e_1)=\{x_1\}$, $F_1(e_2)=\{x_1\}$, $F_2(e_1)=\{x_2\}$, $F_2(e_2)=\emptyset$, $F_3(e_1)=\{x_1, x_2\}$, $F_3(e_2)=\{x_1\}$, $F_4(e_1)=\{x_2, x_3\}$, $F_4(e_2)=\{x_2, x_3\}$, $F_5(e_1)=\emptyset$, $F_5(e_2)=\{x_1\}$, $F_6(e_1)=\{x_2\}$, $F_6(e_2)=\{x_1\}$, $F_7(e_1)=\{x_2, x_3\}$, $F_7(e_2)=X$. Then $(X,\tilde{\tau})$ is a soft topological space over X. Let (F_5,E) be a soft set over X such that $F_5(e_1)=\{x_3\}$, $F_5(e_2)=\{x_2\}$.Clearly, (F_5,E) is soft g-closed in $(X,\tilde{\tau})$.

THEOREM 3.3

[1] If (F,E) is soft g-closed in X and $(F,E)\sqsubseteq(G,E)\sqsubseteq\overline{(F,E)}$, then (G,E) is soft g-closed.

THEOREM 3.4

[1] If (F,E) and (G,E) are soft g-closed sets then so is $(F,E)\sqcup(G,E)$.

THEOREM 3.5

[1] A soft set (F,E) is soft g-closed in X if and only if $\overline{(F,E)}\setminus(F,E)$ contains only null soft closed set.

COROLLARY 3.6

[1] A soft g-closed (F,E) is soft closed if and only if $\overline{(F,E)}\setminus(F,E)$ is soft closed.

THEOREM 3.7

[1] Let (F,E) be a soft g-closed set and suppose that (G,E) is a soft closed set. Then $(F,E)\sqcap(G,E)$ is a soft g-closed set.

If $(X,\tilde{\tau})$ be a soft topological space over X and $Y\subset Z\subset X$ be non-empty subsets of X, then we shall denote the soft closure of $\tilde{Y}$ with respect to relative soft topology on $(Z,\tilde{\tau}_z)$ by $\overline{(\tilde{Y})}_z=\overline{(\tilde{Y})}\sqcap\tilde{Z}$.

THEOREM 3.8

Let $(X,\tilde{\tau})$ be a soft topological space over X and $Y\subset Z\subset X$ be non-empty subsets of X. If $\tilde{Y}$ is a soft g-closed set relative to $(Z,\tilde{\tau}_z)$ and $\tilde{Z}$ is a soft g-closed set relative to $(X,\tilde{\tau})$, then $\tilde{Y}$ is soft g-closed relative to $(X,\tilde{\tau})$.

PROOF

Let $\tilde{Y}\sqsubseteq(F,E)$ and $(F,E)\tilde{\in}\tilde{\tau}$. Since $Y\subset Z$, we have $\tilde{Y}\sqsubseteq\tilde{Z}$. Then $\tilde{Y}\sqsubseteq\tilde{Z}\sqcap(F,E)$. Since $\tilde{Y}$ is a soft g-closed set relative to $(Z,\tilde{\tau}_z)$, and $\tilde{Z}\sqcap(F,E)$ is a soft open set in $(Z,\tilde{\tau}_z)$, we obtain $\overline{(\tilde{Y})}_z\sqsubseteq\tilde{Z}\sqcap(F,E)$. It follows than that $\overline{(\tilde{Y})}\sqcap\tilde{Z}\sqsubseteq\tilde{Z}\sqcap(F,E)$ and $\overline{(\tilde{Y})}\sqcap\tilde{Z}\sqsubseteq(F,E)$. Hence $\tilde{Z}\sqcap[\overline{(\tilde{Y})}\sqcup(\overline{(\tilde{Y})})^c]\sqsubseteq(F,E)\sqcup(\overline{(\tilde{Y})})^c$. That is, $\tilde{Z}\sqcap\tilde{X}\sqsubseteq(F,E)\sqcup(\overline{(\tilde{Y})})^c$. Since $Z\subset X$, we have $\tilde{Z}\sqsubseteq\tilde{X}$. So $\tilde{Z}\sqsubseteq(F,E)\sqcup(\overline{(\tilde{Y})})^c$ and $(F,E)\sqcup(\overline{(\tilde{Y})})^c$

Journal of Theoretical and Applied Information Technology
20th September 2013. Vol. 55 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

soft open in $(X,\tilde{\tau})$. Since $\tilde{Z}$ is a soft g-closed set relative to $(X,\tilde{\tau})$ and $\overline{(\tilde{Y})} \sqsubseteq \overline{(\tilde{Z})}$, we have $\overline{(\tilde{Y})} \sqsubseteq (F,E)\sqcup(\overline{(\tilde{Y})})^c$. Therefore, since $\overline{(\tilde{Y})}\sqcap(\overline{(\tilde{Y})})^c=\Phi$, we obtain $\overline{(\tilde{Y})}\sqsubseteq(F,E)$.

THEOREM 3.9

Let $(X,\tilde{\tau})$ be a soft topological space over X, $Y\subseteq X$, (F,E) be a soft set in Y and suppose that (F,E) is soft g-closed in X. Then (F,E) is soft g-closed relative to $(Y,\tilde{\tau}_Y)$.

PROOF

Let $(F,E)\sqsubseteq\tilde{Y}\sqcap(G,E)$ and suppose that (G,E) is soft open in X. Then $(F,E)\sqsubseteq(G,E)$ and hence $\overline{(F,E)}\sqsubseteq(G,E)$. It follows then that $\tilde{Y}\sqcap\overline{(F,E)}\sqsubseteq\tilde{Y}\sqcap(G,E)$.

THEOREM 3.10

In a soft topological space $(X,\tilde{\tau})$, $\tilde{\tau}=\tilde{\tau}'$ if and only if every soft set over X is a soft g-closed set.

PROOF

Suppose that $\tilde{\tau}=\tilde{\tau}'$ and that $(F,E)\sqsubseteq(G,E)$, $(G,E)\in\tilde{\tau}$. Then $\overline{(F,E)}\sqsubseteq\overline{(G,E)}=(G,E)$ and (F,E) is soft g-closed. Conversely, suppose that every soft set over X is soft g-closed. Let $(G,E)\in\tilde{\tau}$. Then since $(G,E)\sqsubseteq(G,E)$ and (G,E) is soft g-closed, we have $\overline{(G,E)}\sqsubseteq(G,E)$ and $(G,E)\in\tilde{\tau}'$. Thus $\tilde{\tau}\sqsubseteq\tilde{\tau}'$. If $(H,E)\in\tilde{\tau}'$, then $(H,E)^c\in\tilde{\tau}\sqsubseteq\tilde{\tau}'$ and hence $(H,E)\in\tilde{\tau}$. Finally, $\tilde{\tau}=\tilde{\tau}'$.

THEOREM 3.11

[15] Let $(X,\tilde{\tau})$ be a soft compact space. If (F,E) is a soft closed set in X, then (F,E) is soft compact.

THEOREM 3.12

Let $(X,\tilde{\tau})$ be a soft compact space. If (F,E) is soft g-closed in X, then (F,E) is soft compact.

PROOF

Let $C=\{(G_i,E):i\in I\}$ be a soft open cover of (F,E). Since (F,E) is soft g-closed, $\overline{(F,E)}\sqsubseteq\sqcup_{i\in I}(G_i,E)$. From Theorem 3.11, $\overline{(F,E)}$ is soft compact and it follows that $(F,E)\sqsubseteq\overline{(F,E)}\sqsubseteq(G_1,E)\sqcup(G_2,E)\sqcup...\sqcup(G_n,E))$ for some $(G_i,E)\in C$ $(i=1,2,...,n)$.

THEOREM 3.13

[15] Let $(X,\tilde{\tau})$ be a soft Lindelöf (or soft countably compact) space. If (F,E) is a soft closed set in X, then (F,E) is soft Lindelöf (or soft countably compact.

THEOREM 3.14

Let $(X,\tilde{\tau})$ be soft Lindelöf (or soft countably compact) and suppose that (F,E) is soft g-closed in X. Then (F,E) is soft Lindelöf (or soft countably compact).

PROOF

The proof is similar with Theorem 3.12.

THEOREM 3.15

[15] A soft topological space $(X,\tilde{\tau})$ is soft regular if and only if for every $x\in X$ and every soft open set (F,E) of x, there is a soft open set (G,E) of x such that $x\in(G,E)\sqsubseteq\overline{(G,E)}\sqsubseteq(F,E)$.

THEOREM 3.16

If $(X,\tilde{\tau})$ is soft regular and (F,E) is soft compact set in X, then (F,E) is soft g-closed.

PROOF

Let $(X,\tilde{\tau})$ be soft regular, (F,E) be soft compact and $(F,E)\sqsubseteq(H,E)$, $(H,E)\in\tilde{\tau}$. For every point $x\in(F,E)$ there exists a soft open set $(G,E)_x$ of x such that $x\in(G,E)_x\sqsubseteq\overline{(G,E)}_x\sqsubseteq(H,E)$ by Theorem 3.15. From soft compactness of (F,E), there exists a finite open cover $(G,E)_{x1}$, $(G,E)_{x2}$, …, $(G,E)_{xk}$ of (F,E) such that $\overline{((G,E)_{xi})}\sqsubseteq(H,E)$ for each i. Then the define (G,E) as their finite union of $(G,E)_{x1}$, $(G,E)_{x2}$, …, $(G,E)_{xk}$. Then $(F,E)\sqsubseteq(G,E)$ and $\overline{(G,E)}=\{\overline{((G,E)_{xi})}: i=1,2,...,k\}\sqsubseteq(H,E)$. Then we obtain $(F,E)\sqsubseteq(G,E)\sqsubseteq\overline{(G,E)}\sqsubseteq(H,E)$ and it follows that $\overline{(F,E)}\sqsubseteq(H,E)$.

THEOREM 3.17

Let $(X,\tilde{\tau})$ be a soft normal space, Y be a nonempty subset of X and suppose that $\tilde{Y}$ be a soft g-closed set in X. Then $(Y,\tilde{\tau}_Y)$ is soft normal.

PROOF

Let (F_1,E) and (F_2,E) be soft closed in X and suppose that $(\tilde{Y}\sqcap(F_1,E))\sqcap(\tilde{Y}\sqcap(F_2,E))=\Phi$. Then $\tilde{Y}\sqsubseteq[(F_1,E)\sqcap(F_2,E)]^c\in\tilde{\tau}$ and hence $\overline{(\tilde{Y})}\sqsubseteq[(F_1,E)\sqcap(F_2,E)]^c$. Thus $[\overline{(\tilde{Y})}\sqcap(F_1,E)]\sqcap[\overline{(\tilde{Y})}\sqcap(F_2,E)]=\Phi$. Since $(X,\tilde{\tau})$ is soft normal, there exist disjoint soft open sets (G_1,E) and (G_2,E) such that $\overline{(\tilde{Y})}\sqcap(F_1,E)\sqsubseteq(G_1,E)$ and $\overline{(\tilde{Y})}\sqcap(F_2,E)\sqsubseteq(G_2,E)$. It follows then that $\tilde{Y}\sqcap(F_1,E)\sqsubseteq\tilde{Y}\sqcap(G_1,E)$ and $\tilde{Y}\sqcap(F_2,E)\sqsubseteq\tilde{Y}\sqcap(G_2,E)$.

THEOREM 3.18

If $(X,\tilde{\tau})$ is soft normal and $(F,E)\sqcap(H,E)=\Phi$ where (F,E) is soft closed and (H,E) is soft g-closed, then there exist disjoint soft open sets (G_1,E) and (G_2,E) such that $(F,E)\sqsubseteq(G_1,E)$ and $(H,E)\sqsubseteq(G_2,E)$.

PROOF

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$(H,E)\sqsubseteq(F,E)^c\tilde{\in}\tilde{\tau}$ and hence $\overline{(H,E)}\sqsubseteq(F,E)^c$. Thus $\overline{(H,E)}\sqcap(F,E)=\Phi$. Since $(X,\tilde{\tau})$ is soft normal, then there exit disjoint soft open sets (G_1,E) and (G_2,E) such that $(F,E)\sqsubseteq(G_1,E)$ and $\overline{(H,E)}\sqsubseteq(G_2,E)$. Since $(H,E)\sqsubseteq\overline{(H,E)}$, we obtain $(F,E)\sqsubseteq(G_1,E)$ and $(H,E)\sqsubseteq(G_2,E)$.

REMARK 3.19

Disjoint soft g-closed sets in a soft normal space generally cannot be separated by soft open sets.

EXAMPLE 3.20

In Example 3.2, also $(X,\tilde{\tau})$ is a soft normal space. (F_1,E) and (F_2,E) are soft g-closed in $(X,\tilde{\tau})$. Clearly, (F_1,E) and (F_2,E) are disjoint soft g-closed sets which cannot be separated by soft open sets. .

4. SOFT GENERALIZED OPEN SETS

DEFINITION 4.1

[1] Let $(X,\tilde{\tau})$ be a soft topological space. A soft set (F,E) is called a soft generalized open (briefly soft g-open) in X if the complement $(F,E)^c$ is soft g-closed in X.

THEOREM 4.2

[1] A soft set (F,E) is called a soft g-open in a soft topological space $(X,\tilde{\tau})$ if and only if $(G,E)\sqsubseteq(F,E)^\circ$ whenever $(G,E)\sqsubseteq(F,E)$ and (G,E) is soft closed in X.

THEOREM 4.3

[1] If (F,E) is soft g-open in X and $(F,E)^\circ\sqsubseteq(G,E)\sqsubseteq(F,E)$, then (G,E) is soft g-open.

THEOREM 4.4

[1] If (F,E) and (G,E) are soft g-open sets then so is $(F,E)\sqcap(G,E)$.

THEOREM 4.5

If (F,E) and (G,E) are soft disconnected and soft g-open sets, then $(F,E)\sqcup(G,E)$ is soft g-open.

PROOF

Let (H,E) be a soft closed subset of $(F,E)\sqcup(G,E)$. Then $(H,E)\sqcap\overline{(F,E)}\sqsubseteq(F,E)$ and hence by Theorem 4.2, $(H,E)\sqcap\overline{(F,E)}\sqsubseteq(F,E)^\circ$. Similarly, $(H,E)\sqcap\overline{(G,E)}\sqsubseteq(G,E)^\circ$. Now $(H,E)=(H,E)\sqcap((F,E)\sqcup(G,E))\sqsubseteq((H,E)\sqcap\overline{(F,E)})\sqcup((H,E)\sqcap\overline{(G,E)})\sqsubseteq(F,E)^\circ\sqcup(G,E)^\circ\sqsubseteq((F,E)\sqcup(G,E))^\circ$. Hence $(H,E)\sqsubseteq((F,E)\sqcup(G,E))^\circ$ and by Theorem 4.2, $(F,E)\sqcup(G,E)$ is soft g-open.

THEOREM 4.6

A soft set (F,E) is soft g-open in $(X,\tilde{\tau})$ if and only if $(G,E)=\tilde{X}$ whenever (G,E) is soft open and $(F,E)^\circ\sqcup(F,E)^c\sqsubseteq(G,E)$.

PROOF

$\Rightarrow$ Suppose that (G,E) is soft open and $(F,E)^\circ\sqcup(F,E)^c\sqsubseteq(G,E)$.Now $(G,E)^c\sqsubseteq(\overline{(F,E)^c})\setminus(F,E)^c$. Since $(G,E)^c$ is soft closed and $(F,E)^c$ is soft g-closed, by Theorem 3.5 it follows that $(G,E)^c=\Phi$ or $\tilde{X}=(G,E)$.

$\Leftarrow$ Suppose that (H,E) is a soft closed set and $(H,E)\sqsubseteq(F,E)$. By Theorem 4.2, it sufficies to show that $(H,E)\sqsubseteq(F,E)^\circ$. Now $(F,E)^\circ\sqcup(F,E)^c\sqsubseteq(F,E)^\circ\sqcup(H,E)^c$ and hence we obtain $(F,E)^\circ\sqcup(H,E)^c=\tilde{X}$. Thus $(H,E)\sqsubseteq(F,E)^\circ$.

THEOREM 4.7

A soft set (F,E) is soft g-closed if and only if $\overline{(F,E)}\setminus(F,E)$ is soft g-open.

PROOF

$\Rightarrow$ Suppose that (F,E) is soft g-closed and $(H,E)\sqsubseteq\overline{(F,E)}\setminus(F,E)$, (H,E) being soft closed. By Theorem 3.5, $(H,E)=\Phi$ and hence $(H,E)\sqsubseteq(\overline{(F,E)}\setminus(F,E))^\circ$. By Theorem 4.2, $\overline{(F,E)}\setminus(F,E)$ is soft g-open.

$\Leftarrow$ Suppose $(F,E)\sqsubseteq(G,E)$ where (G,E) is a soft open set. Now $\overline{(F,E)}\sqcap(G,E)^c\sqsubseteq\overline{(F,E)}\sqcap(F,E)^c=\overline{(F,E)}\setminus(F,E)$ and since $\overline{(F,E)}\sqcap(G,E)^c$ is soft closed and $\overline{(F,E)}\setminus(F,E)$ is soft g-open, it follows that $\overline{(F,E)}\sqcap(G,E)^c\sqsubseteq(\overline{(F,E)}\setminus(F,E))^\circ=\Phi$. Therefore $\overline{(F,E)}\sqcap(G,E)^c=\Phi$ or $\overline{(F,E)}\sqsubseteq(G,E)$. Thus we get (F,E) is soft g-closed.

5. CONCLUSION

The concepts of soft generalized closed sets and soft generalized open sets in a soft topological space were first introduced by Kannan [1]. He studied their some properties. In the present work, we have investigated behavior relative to soft subspaces of soft generalized closed sets. We have showed that a soft generalized closed set in a soft compact (soft Lindelöf, soft countably compact) space is also soft compact. Then, we have showed that a soft compact set in a soft regular space is soft generalized closed and disjoint soft g-closed sets in a soft normal space generally cannot be separated by soft open sets. Finally, we have investigated some properties of soft generalized open sets. In future more general types of soft generalized closed sets may be defined and using of them characterizations related with soft separation axioms may be studied. Hence we expect that some

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

research teams will be actively working on soft generalized closed (open) sets and new types of soft generalized closed (open) sets in a soft topological space.

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