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STATE FEEDBACK CONTROLLER AND OBSERVER DESIGN FOR TCP NETWORK WITH CONSIDERATION OF UDP FLOW

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ABSTRACT

In this paper we consider the phenomenon of congestion of a router in a communication network, initially, we propose a model of TCP based on the fluid flux model and, secondly, we propose a synthesis of an observer and state feedback to stabilize the system. In this study we have taken into account both the time variant delay and the perturbations due to UDP flow. Such traffic control will be implemented by a mechanism of Active Queue Management (AQM), located at the router.

Keywords: *Delay-Dependent Stability, Linear Matrix Inequality (LMI), AQM, TCP network, Stabilization, Time-Delay Systems.*

1. INTRODUCTION

We were interested in this paper at the problem of sharing a communication link between several remote sources. This problem is represented schematically in Figure 1. Each slave has a different communication time according to the path taken by the connection and the delay associated, which leads to variable delays in the time. The denomination "slave" does not refer necessarily to a single machine, but possibly to a set of transmitters geographically close. Many studies have been conducted in order to ensure a certain level of service quality between network users. Indeed, the authors of [17] call the Nyquist criterion to adjust the parameters of RED and PI for an arbitrary number of sources and routers. However, this frequency method does not provide satisfactory results when delays become time-varying.

In [24], a predictive approach is used to compensate the delays. if this study provides a generic formulation of the synthesis of AQM, incorporating various mechanisms (RED, PI, PID with cost LQ), the control law is relatively "heavy" requiring multiple matrix calculations. Although interesting in theory, these methods are difficult to use in practice.

So many studies in the literature [21, 3, 5, 1, 18, 19, 4, 6] had considered the simplistic case where all sources have the same communication time, that is to say the same round-trip time (RTT). In this paper we considered the same case, but with a master-slave modeling taking into account the perturbations due to applications using the UDP protocol.

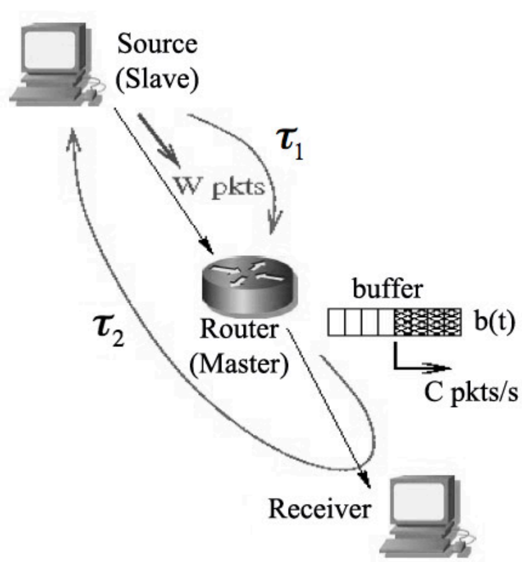

Figure 1: A single communication in the TCP network

This paper is organized as follows. In the second part we introduce a mathematical modeling of the behavior of TCP, section 3 is dedicated to the synthesis of an observer and a development of a control law ensuring the stabilization of the system and a certain level of performance. Section 4 presents an example extracted from the literature and simulation validate the proposed methodology. Finally, we present our conclusions in section 5.

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2. MODELLING OF TCP TRAFFIC: THE FLUID-FLOW MODEL

2.1 Preliminaries

The dynamic model of the congestion window of a TCP source, developed by Misra et al [2], and which has been simplified by Hollot et al [18] is as follows:

$$\begin{cases} \dot{W}(t) &= \dfrac{1}{R(t)} - \dfrac{W(t)W(t-R(t))}{2R(t-R(t))}p(t-R(t)) \\ \dot{q}(t) &= \dfrac{W(t)}{R(t)}N(t) - C, \end{cases} \quad (1)$$

where W, N, C, p and q are respectively the TCP window size, number of TCP sessions, link capacity, probability of dropping packets and the queue length. $R(t)$ represents the round-trip time(RTT), and it is defined by

$$R(t) = \tau_1(t) + \tau_2(t) \quad \text{(see Figure 1)} \quad (2)$$

where τ_1 and τ_2 represent respectively the time delay from the source to the router and from the router to the source via the receiver.

Remark 1 The delay τ_1 contains the quantity $q(t)/C$, and so it varies in time, the delay τ_2 represents only the propagation time from the router to the source, therefore it can be considered constant. However, the constancy of delay is an assumption rarely verified in reality (see e.g., [13, 14, 11]), this leads to consider its potential variations, describing generally the latter by a continuous function of time. Once again, the case of variable delays was the subject of much research (see e.g., [12, 8, 23]).

In this paper, we consider the delay as a differentiable function bounded, which satisfies the following conditions :

$$\forall i = 1,2;\ \forall t \geq 0; \quad 0 \leq \tau_i(t) \leq \tau_{max} \quad and \quad \dot{\tau}_i(t) < d_i < 1 \quad (3)$$

where d_i is a positive real, which represents the upper bound of the derivative of delay $\tau_i(t)$.

2.2 The Linearized TCP Fluid Model

The operating point (W_0, q_0, p_0) is defined by :

$$\dot{W} = 0 \text{ and } \dot{q} = 0,$$

therefore, we find the two conditions :

$$W_0^2 p_0 = 2 \text{ and } W_0 = \frac{R_0 C}{N} \quad (4)$$

where R_0 verifies the following equalities :

$$R_0 = \frac{q_0}{C} + T_p \quad (5)$$

with T_p denotes the propagation delay.

From the point of equilibrium (W_0, q_0, p_0), we approach the model (1) by the linear model :

$$\begin{cases} \delta\dot{W}(t) &= -\dfrac{2N}{R_0^2 C}\delta W(t) - \dfrac{R_0 C^2}{2N^2}\delta p(t-\tau_1(t)) \\ \delta\dot{q}(t) &= \dfrac{N}{R_0}\delta W(t) - \dfrac{1}{R_0}\delta q(t) \end{cases} \quad (6)$$

For more details about the linearization, see [7, 9].

(2)

2.3 Augmented TCP Fluid Model

For controlling the TCP traffic, during the synthesis of the AQM, we will take into account the traffic unmodeled from applications using the UDP protocol. In order to reduce the effect of this perturbation, insensible to the loss of packets, we will add the term $d(t)$ to the dynamic of the queue.

$$\delta\dot{q}(t) = \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t) + d(t) \quad (7)$$

As a result, the linearized TCP Model (6) can be written as follows :

$$\begin{cases} \dot{x}(t) &= Ax(t) + B_1 u(t-\tau_1(t)) + B_2 d(t) \\ y(t) &= Cx(t) \end{cases} \quad (8)$$

where $x(t) = [\delta W(t)\ \delta q(t)]^T$ is the state vector, $u(t) = p(t)$ is the input , and

$$A = \begin{bmatrix} \dfrac{-2N}{R_0^2 C} & 0 \\ \dfrac{N}{R_0} & \dfrac{-1}{R_0} \end{bmatrix}, \quad B_1 = \begin{bmatrix} \dfrac{-R_0 C^2}{2N^2} \\ 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

In order to simplify representation of the system in state space, we increase the order of the system,

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let $\tilde{x}(t)$ the new state vector, such as : $\tilde{x}(t)=[x(t)\ \ d(t)]^T$.

Model (8) becomes

$$\begin{cases} \dot{\tilde{x}}(t) &= \bar{A}\,\tilde{x}(t)+\bar{B}u(t-\tau_1(t)) \\ \tilde{y}(t) &= \bar{C}\,\tilde{x}(t) \end{cases} \tag{9}$$

where

$$\bar{A}=\begin{bmatrix} A & B_2 \\ 0\ \ 0 & 0 \end{bmatrix},\bar{B}=\begin{bmatrix} B_1 \\ 0 \end{bmatrix},\bar{C}=\begin{bmatrix} C & 0 \end{bmatrix}$$

3. CONTROL SYSTEM WITH OBSERVER

This section will be devoted to the development of a state feedback control, so we will first reconstruct the states of the output measured using an observer. Note that the choice of the observer is very important, seeing that the measured output reached the system with a delay $\tau_2(t)$.

Given this fact, and the assumption of linearity of the system, our attention has focused on the consideration of a Luenberger observer whose dynamics can be described by :

$$\begin{cases} \dot{\hat{x}}(t) = \bar{A}\,\hat{x}(t)+\bar{B}u(t-\tau_1(t)) \\ \qquad\quad -L[\tilde{y}(t-\tau_2(t))-\hat{y}(t-\tau_2(t))] \\ \hat{y}(t) = \bar{C}\,\hat{x}(t) \end{cases} \tag{10}$$

where L is a gain whose value to define, must ensure a sufficiently fast convergence of the estimated state $\hat{x}(t)$ to the real state $\tilde{x}(t)$, in spite of variable delay $\tau_2(t)$ affecting the measured output $\tilde{y}(t)$.

3.1 Synthesis of State Feedback Control Law

Our problem lies in the synthesis of the gains of the state feedback control and the observer, to ensure the asymptotic stability of the closed loop system and rapid convergence of the estimated state to the actual state of the system.

In the following, we developed conditions allowing the stabilization of the delayed system, then we determine the gains of state feedback stabilizing the system.

The control law developed by the router (Master) from the estimated state $\hat{x}(t)$ has the following form :

$$u(t) = K\,\hat{x}(t) \tag{11}$$

where K is a linear gain allowing to ensure the asymptotic stability (closed-loop) of the linear system (9). However, we will be focusing first to studying the ideal controller $u(t)=K\tilde{x}(t)$, by considering a perfect observer (such as $\hat{x}(t)=\tilde{x}(t)$). Thus our system (9) becomes

$$\begin{cases} \dot{\tilde{x}}(t) &= \bar{A}\,\tilde{x}(t)+\bar{B}K\,\tilde{x}(t-\tau_1(t)) \\ \tilde{y}(t) &= \bar{C}\,\tilde{x}(t) \\ \tilde{x}(t) &= \varphi(\theta)\quad,\quad \theta\in[-\tau_{1max},0] \end{cases} \tag{12}$$

where τ_{1max} is the upper bound of the delay $\tau_1(t)$, and the function $\varphi(\theta)$ is assumed continuous and differentiable.

To determine the LMI conditions characterizing the asymptotic stability of system (12), we propose the Following theorem :

Theorem 1 Given a state feedback gain K, the system (12) is asymptotically stable, if there exist $n\times n$ symmetric positive definite matrices P_1,S_1,R_1, and $n\times n$ matrices P_2,P_3,Y_{1i},Z_{1k}, such that the following LMIs are feasible for $i=1,2;k=1,2,3$:

$$\Gamma = \begin{bmatrix} \psi_1 & P^T\begin{bmatrix} 0 \\ BK \end{bmatrix}-Y_1^T \\ * & -(1-d_1)S_1 \end{bmatrix}<0 \tag{13}$$

and

$$\begin{bmatrix} R_1 & Y_1 \\ * & Z_1 \end{bmatrix}\geq 0 \tag{14}$$

where

$$Y_1=\begin{bmatrix} Y_{11} & Y_{12} \end{bmatrix},Z_1=\begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{13} \end{bmatrix},P=\begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

and the matrix ψ_1 is defined as follows :

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$$\psi_1 = P^T \begin{bmatrix} 0 & I_n \\ A & -I_n \end{bmatrix} + \begin{bmatrix} 0 & I_n \\ A & -I_n \end{bmatrix}^T P + \tau_{1max} Z_1 + \begin{bmatrix} S_1 & 0 \\ 0 & \tau_{1max} R_1 \end{bmatrix} + \begin{bmatrix} Y_1 \\ 0 \end{bmatrix} + \begin{bmatrix} Y_1 \\ 0 \end{bmatrix}^T \quad (15)$$

Proof : By writing the system (12) in the form of a model descriptor, we obtain :

$$\begin{aligned} \dot{\tilde{x}}(t) &= z(t) \\ 0 &= -z(t) + A\tilde{x}(t) + BK\tilde{x}(t-\tau_1(t)). \end{aligned}$$

Then, by posing $\bar{x}(t) = col(\tilde{x}(t), z(t))$ and $E = diag(I_n, 0)$, the system (12) can be rewritten as follows:

$$E\dot{\bar{x}} = \begin{bmatrix} \dot{\tilde{x}}(t) \\ 0 \end{bmatrix} = \begin{bmatrix} z(t) \\ -z(t) + (A+BK)\tilde{x}(t) \end{bmatrix} - \begin{bmatrix} 0 \\ BK \end{bmatrix} \int_{t-\tau_1(t)}^{t} z(s)ds \quad (16)$$

Consider then, following the example of [15], a Lyapunov Krasovskii functional defined by:

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

where

$$V_1(t) = \bar{x}(t)^T EP\bar{x}(t)$$

$$V_2(t) = \int_{-\tau_{1max}}^{0} \int_{t+\theta}^{t} z(\beta)^T R_1 z(\beta) d\beta d\theta$$

$$V_3(t) = \int_{t-\tau_1(t)}^{t} \tilde{x}(s)^T S_1 \tilde{x}(s) ds$$

with

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1, R_1, S_1 > 0$$

$$\text{and} \quad EP = P^T E.$$

This functional is positive definite because we note that $\bar{x}(t)^T EP\bar{x}(t) = \tilde{x}(t)^T P_1 \tilde{x}(t)$ and P_1 is assumed to be positive definite.

We will now determine the derivatives of the three terms of V(t) :

$$\dot{V}_1(t) = 2\tilde{x}(t)^T P_1 \dot{\tilde{x}}(t) = 2\bar{x}(t)^T P^T \begin{bmatrix} \dot{\tilde{x}}(t) \\ 0 \end{bmatrix}$$

using the expression given by (16), we find

$$\dot{V}_1(t) = 2\bar{x}(t)^T P^T \begin{bmatrix} z(t) \\ -z(t) + (A+BK)\tilde{x}(t) \end{bmatrix} - 2\bar{x}(t)^T P^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \int_{t-\tau_1(t)}^{t} z(s)ds$$

otherwise expressed,

$$\dot{V}_1(t) = \bar{x}(t)^T \psi_0 \bar{x}(t) + \delta(t) \quad (17)$$

where ψ_0 and $\delta(t)$ are respectively defined by :

$$\psi_0 = P^T \begin{bmatrix} 0 & I_n \\ (A+BK) & -I_n \end{bmatrix} + \begin{bmatrix} 0 & I_n \\ (A+BK) & -I_n \end{bmatrix}^T P$$

$$\delta(t) = -2\bar{x}(t)^T P^T \begin{bmatrix} 0 \\ BK \end{bmatrix} \int_{t-\tau_1(t)}^{t} z(s)ds$$

Thus, by simple calculations, we find

$$\dot{V}_2(t) = \tau_{1max} z(t)^T R_1 z(t) - \int_{t-\tau_{1max}}^{t} z(s)^T R_1 z(s) ds \quad (18)$$

Similarly, the derivative of the third term of $V(t)$ (16) is given by :

$$\dot{V}_3(t) = \tilde{x}(t)^T S_1 \tilde{x}(t) - (1-\dot{\tau}_1(t))\tilde{x}(t-\tau_1(t))^T S_1 \tilde{x}(t-\tau_1(t))$$

Considering the hypothesis on the upper bound of the derivative $\dot{\tau}_1(t)$ of the delay $\tau_1(t)$, we deduce

$$\dot{V}_3(t) \le \tilde{x}(t)^T S_1 \tilde{x}(t) - (1-d_1)\tilde{x}(t-\tau_1(t))^T S_1 \tilde{x}(t-\tau_1(t)) \quad (19)$$

Substituting (17), (18) and (19) in the expression for the derivative of the Lyapunov functional Krasovskii, we obtain the following inequality:

$$\begin{aligned} \dot{V}(t) \le\; & \bar{x}(t)^T \psi_0 \bar{x}(t) + \delta(t) + \tau_{1max} z(t)^T R_1 z(t) \\ & - \int_{t-\tau_{1max}}^{t} z(s)^T R_1 z(s) ds + \tilde{x}(t)^T S_1 \tilde{x}(t) \\ & - (1-d_1)\tilde{x}(t-\tau_1(t))^T S_1 \tilde{x}(t-\tau_1(t)) \end{aligned}$$

We now substitute the term $\delta(t)$, for it, we apply the technique proposed in [22], which establishes for all vectors $a \in R^n, b \in R^n$ and for all matrices $R \in R^{n \times n}, Y \in R^{n \times 2n}, Z \in R^{2n \times 2n}$ and $N \in R^{2n \times n}$, the following inequalities:

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$$-2b^T Na \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} R & Y - N^T \\ Y^T - N & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

with $\begin{bmatrix} R & Y \\ Y^T & Z \end{bmatrix} \ge 0$

taking

$$N = P^T \begin{bmatrix} 0 \\ BK \end{bmatrix}, a = z(s), b = \bar{x}(t),$$

$$R = R_1, Z = Z_1, Y = Y_1$$

such as $\begin{bmatrix} R_1 & Y_1 \\ * & Z_1 \end{bmatrix} \ge 0$

we find :

$$\delta(t) \le \int_{t-\tau_1(t)}^{t} \begin{bmatrix} z(s)^T & \bar{x}(t)^T \end{bmatrix} \begin{bmatrix} R_1 & Y_1 - \begin{bmatrix} 0 & (BK)^T \end{bmatrix} P \\ * & Z_1 \end{bmatrix} \begin{bmatrix} z(s) \\ \bar{x}(t) \end{bmatrix} ds$$

which implies

$$\delta(t) \le \int_{t-\tau_{1max}}^{t} z(s)^T R_1 z(s) ds + \tau_{1max} \bar{x}(t)^T Z_1 \bar{x}(t) + 2(\tilde{x}(t)^T - \tilde{x}(t-\tau_1(t))^T)(Y_1 - \begin{bmatrix} 0 & (BK)^T \end{bmatrix} P) \bar{x}(t) \tag{20}$$

Substituting (20) in the expression of $\dot{V}(t)$, after simplification of terms with an integral, we obtain

$$\dot{V}(t) \le \bar{x}(t)^T \psi_1 \bar{x}(t) - (1-d_1)\tilde{x}(t-\tau_1(t))^T S_1 \tilde{x}(t-\tau_1(t)) - 2\tilde{x}(t-\tau_1(t))^T (Y_1 - \begin{bmatrix} 0 & (BK)^T \end{bmatrix} P) \bar{x}(t)$$

Then, applying the Schur complement, we find

$$\dot{V}(t) \le \zeta(t)^T \Gamma \zeta(t)$$

where $\zeta(t) = col\{\bar{x}(t), \tilde{x}(t-\tau_1(t))\}$ and Γ is a matrix given by (13).

Therefore, if the LMIs defined by (13) - (14) are satisfied, then $\dot{V}(t)$ is negative definite.

As well, we have $V(t)$ is positive definite, then the system (12) is asymptotically stable for a state feedback gain K.

Remark 2 The LMIs conditions of Theorem 1 can be used to determine the values of allowable delays for which the LMIs (13) - (14) are satisfied for a given state feedback gain K.

Remark 3 As our objective is to synthesize a state feedback control law $u(t) = K\tilde{x}(t)$ ensuring the asymptotic stabilization of system (12). The next step is to determine the state feedback gain K, thus we propose the following theorem.

Theorem 2 Suppose that, for a positive real ε, there exist symmetric positive definite matrices $\bar{P}_1, \bar{S}_1, \bar{R}_1$, matrices $\bar{P}, \bar{Y}_{11}, \bar{Y}_{12}, \bar{Z}_{11}, \bar{Z}_{12}, \bar{Z}_{13}$, of dimensions $n \times n$ and a matrix W of dimension $m \times n$, satisfying the following LMIs:

$$\Gamma = \begin{bmatrix} \psi_2 & \begin{bmatrix} BW \\ \varepsilon BW \end{bmatrix} - \bar{Y}_1^T \\ * & -(1-d_1)\bar{S}_1 \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} \bar{R}_1 & \bar{Y}_1 \\ * & \bar{Z}_1 \end{bmatrix} \ge 0 \tag{22}$$

where

$$\bar{Y}_1 = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \end{bmatrix}, \bar{Z}_1 = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} \\ * & \bar{Z}_{13} \end{bmatrix}$$

the matrix ψ_2 is given by :

$$\psi_{21} = A\bar{P} + \bar{P}^T A^T + \bar{S}_1 + \tau_{1max}\bar{Z}_{11} + \bar{Y}_{11} + \bar{Y}_{11}^T$$

$$\psi_{22} = \varepsilon \bar{P}^T A^T + \bar{P}_1 - \bar{P} + \tau_{1max}\bar{Z}_{12} + \bar{Y}_{12}$$

$$\psi_{23} = -\varepsilon(\bar{P} + \bar{P}^T) + \tau_{1max}(\bar{Z}_{13} + \bar{R}_1)$$

Then, for any delay $\tau_1(t) \le \tau_{1max}$ of the form (3), the asymptotic stability of system (12) is ensured by the state feedback gain :

$$K = W\bar{P}^{-1} \tag{23}$$

Proof : As we saw in the previous section that the conditions LMIs (13) - (14) are satisfied for a given K. In this case K is unknown, the condition LMI (13) becomes a BMI. We must therefore transform it to a LMI. For this reason, we apply the method proposed in [16]. The first step of this technique is to link the variables P_3 and P_2 as follows

$$P_3 = \varepsilon P_2 \tag{24}$$

where $\varepsilon \in R$ is a parameter for adjusting. Then, taking into account the fact that the matrix P is invertible, we can define the matrix $\bar{P}$:

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$$\bar{P} = P_2^{-1}$$

and, for any matrix:

$$\Theta \in \{P_1, S_1, Y_{1i}, Z_{1k}, R_1\}.$$

For $i = 1,2; k = 1,2,3$ we can define a new matrix

$$\bar{\Theta} = \bar{P}^T \Theta \bar{P}$$

Then, we multiply LMIs (13) - (14), to the right by the matrix $\Delta_3 = Diag\{\bar{P}, \bar{P}, \bar{P}\}$ respectively left by Δ_3^T, and we take

$$W = K\bar{P}$$

Finally, we obtain the conditions of stabilization provided by the LMIs (21), (22) of this theorem.

3.2 Observer Synthesis

As stated in the introduction to this chapter, we will construct an estimate of the full state of the slave from the delayed output $\tilde{y}(t-\tau_2(t))$ of the system. By assuming that the pair (A,C) is observable and that the dynamic of the estimation error $e(t) = \tilde{x}(t) - \hat{x}(t)$ is governed by:

$$\dot{e}(t) = Ae(t) + LCe(t-\tau_2(t)) \qquad (25)$$

We therefore propose the following theorem, which ensures that the observed state converges quickly enough to the real state, despite the variable delay affecting the measurement.

Theorem 3 Suppose that, for a positive real ε, there exist symmetric positive definite matrices P_1, S_2, R_2, matrices $P, Y_{21}, Y_{22}, Z_{21}, Z_{22}, Z_{23}$ of dimensions $n \times n$ and a matrix X of dimension $n \times p$, satisfying the following LMIs:

$$\Gamma = \begin{bmatrix} \psi_1 & \begin{bmatrix} XC \\ \varepsilon XC \end{bmatrix} - Y_2^T \\ * & -(1-d_2)S_2 \end{bmatrix} < 0 \qquad (26)$$

$$\begin{bmatrix} R_2 & Y_2 \\ * & Z_2 \end{bmatrix} \geq 0 \qquad (27)$$

the matrix ψ_1 is given by :

$$\psi_{11} = P^T A + A^T P + S_2 + \tau_{2max} Z_{21} + Y_{21} + Y_{21}^T$$
$$\psi_{12} = \varepsilon A^T P + P_1^T - P^T + \tau_{2max} Z_{22} + Y_{22}$$
$$\psi_{13} = -\varepsilon(P + P^T) + \tau_{2max}(Z_{23} + R_2).$$

Then the asymptotic convergence to 0 of the error $e(t) = \tilde{x}(t) - \hat{x}(t)$ is ensured by the gain

$$L = (P^T)^{-1} X \qquad (28)$$

Proof : By writing the system (25) as a model descriptor, we obtain the following system

$$E\dot{\bar{e}}(t) = \begin{bmatrix} z(t) \\ -z(t) + (A+LC)e(t) \end{bmatrix} - \begin{bmatrix} 0 \\ LC \end{bmatrix} \int_{t-\tau_2(t)}^{t} z(s)ds$$

with

$$\bar{e}(t) = col\{e(t), z(t)\}, \; E = diag\{I_n, 0\}$$

Considering now the same form of Lyapunov-Krasovskii functional as used in part 3.1, taking into account the variable delay $\tau_2(t)$. We thus obtain the following formulation:

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

with

$$V_1(t) = \bar{e}(t)^T EP\bar{e}(t)$$
$$V_2(t) = \int_{-\tau_{2max}}^{0} \int_{t+\theta}^{t} z(\beta)^T R_2 z(\beta) d\beta d\theta$$
$$V_3(t) = \int_{t-\tau_2(t)}^{t} e(s)^T S_2 e(s) ds$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1, R_2, S_2 > 0.$$

Thus, by applying the same techniques used to prove Theorem 2, we find the LMI (26) and (27). Then by taking $X = P^T L$, we find the proof of this theorem.

4. ILLUSTRATIVE EXAMPLE

This section aims to illustrate the theoretical results obtained in the previous sections. As indicated in the first part, the system that we have studied is the TCP network whose dynamics is illustrated by (12). In a first time, we are going to define the conditions to validate for synthesizing an observer able to guarantee a quickly enough convergence of the observation error, despite the presence of delay in the measurement loop. In a

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second time, assuming that $\hat{x}(t)=\tilde{x}(t)$, we determine the gain K ensuring stabilization of closed loop system, in spite of the delay in the control loop, and the perturbation due to UDP flow.

We consider the system presented in [20], where :

$$q_0 = 175 \ packets$$

with $T_p = 0.2s$, $C = 3750 \ packets/s$ and for $N = 60 \ TCP \ sessions$, we have $W_0 = 15 \ packets$, $p_0 = 0.008$, $R_0 = 0.246s$.

By substituting in (9), we obtain :

$$\begin{cases} \dot{\tilde{x}}(t) = \begin{bmatrix} -0.5287 & 0 & 0 \\ 243.9024 & -4.0650 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tilde{x}(t) \\ \qquad + \begin{bmatrix} -480.4687 \\ 0 \\ 0 \end{bmatrix} u(t-\tau_1(t)) \\ \tilde{y}(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tilde{x}(t) \end{cases} \tag{29}$$

Moreover, we consider that the lines of communication between the master (router) and slave (receiver), induce varying delays $\tau_1(t)$ and $\tau_2(t)$, characterized by the following values:

- Maximum values of delays :

$$\tau_{1max} = 0.4s, \ \tau_{2max} = 0.35s$$

- Maximum values of derivatives of delays :

$$d_1 = d_2 = 0.1$$

Thus by applying theorem 2 to the system (29) with adjustment parameter $\varepsilon = 2$, and by using the toolbox of Matlab YALMIP, we can conclude that the LMI (21) and (22) are satisfied for symmetric positive definite matrices follows:

$$P1 = \begin{bmatrix} 0.1916 & -2.8419 & -1.9192 \\ -2.8419 & 387.9813 & -2.7765 \\ -1.9192 & -2.7765 & 34.7129 \end{bmatrix},$$

$$R1 = \begin{bmatrix} 0.6431 & -0.0542 & -0.8383 \\ -0.0542 & 231.6539 & 30.7437 \\ -0.8383 & 30.7437 & 96.3691 \end{bmatrix}$$

$$S1 = \begin{bmatrix} 0.7101 & 3.4088 & 0.0000 \\ 3.4088 & 62.5035 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

and

$$\bar{P} = \begin{bmatrix} 0.0870 & 6.2587 & 0.9313 \\ 6.2587 & 417.2914 & 81.8614 \\ 0.9313 & 81.8614 & 77.1983 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0946 & 0.0891 & 0.0922 \end{bmatrix}$$

Then, by replacing the matrices $\bar{P}$ and W in (23), we find that the gain K is :

$$K = \begin{bmatrix} 1.6117 & 0.0127 & 0.0928 \end{bmatrix} \tag{30}$$

Moreover, applying Theorem 3 to the system (12) with $\varepsilon = 4$, we find that the condition LMI (26) is satisfied for the following symmetric positive definite matrices

$$P1 = \begin{bmatrix} 45.7206 & -0.8237 & -0.0436 \\ -0.8237 & 0.0170 & -0.0670 \\ -0.0436 & -0.0670 & 2.5922 \end{bmatrix},$$

$$S2 = \begin{bmatrix} 4.2638 & 0.0550 & 0.0303 \\ 0.0550 & 0.0035 & 0.0006 \\ 0.0303 & 0.0006 & 0.0004 \end{bmatrix}$$

$$R2 = \begin{bmatrix} 9.4282 & -0.1762 & 1.0866 \\ -0.1762 & 0.0135 & -0.0443 \\ 1.0866 & -0.0443 & 5.7235 \end{bmatrix}$$

with

$$P = \begin{bmatrix} 14.5156 & -0.0027 & 1.7516 \\ -0.0027 & 0.0007 & -0.0015 \\ 1.7516 & -0.0015 & 4.2733 \end{bmatrix},$$

$$X = \begin{bmatrix} -1.4813 \\ -1.5387 \\ -1.5149 \end{bmatrix}$$

by replacing the matrices P and X in (28), we obtain the observation gain L :

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$$L = \begin{bmatrix} -0.0004 \\ -2.0726 \\ -0.0010 \end{bmatrix}$$

(31)

To illustrate the effectiveness of our approach, we use the network simulator NS-2 [10] , knowing that in our example the buffer size is 400, and that the average size of a packet is 500 bytes. Thus, considering the values (30) - (31), relating to gains of state feedback and of the observer, we obtain the Figure 2 illustrating the evolution of the size of the queue.

We can observe that our AQM allows efficient control and maintains the queue length close to its equilibrium value, in spite of the communication delays τ_1 and τ_2.

We will now test the impact of the introduction of an additional traffic, insensible to packet loss, based on UDP. It is generated firstly with a flow of $0.5 Mbps$, secondly with $3 Mbps$ between 70 and 110 seconds. The simulation results given in Figures (3)-(4) shows that when the UDP traffic is intense, we see a slight increase in the queue length in relation to the equilibrium position $q_0 = 175\ packets$. However, once UDP communications have been completed, the operating point is quickly restored.

5. CONCLUSION

In this paper, we have discussed the phenomenon of congestion of router in the TCP network, by considering the case of slowly varying delays, bounded and asymmetric in the loops of control and measurement. Based on Lyapunov-Krasovskii functional and stability criteria defined in the forms of LMI, we have designed a state feedback controller with observer, ensuring asymptotic stabilization system. Finally, we have illustrated the viability of this approach through a numerical example. However, our hypothesis of a variable delay with a zero lower bound is unrealistic (equivalent to assuming that the data transfer through the TCP network can be done instantly) which can lead to a certain conservatism of the results achieved.

REFRENCES:

[1] Y. Fan, F Ren, and C. Lin. Design a pid controller for active queue management. IEEE International Symposium on Computers and Communication (ISCC), 2:985–990, 2003.

[2] V. Misra, W.-B. Gong, and D. Towsley. Fluid-based analysis of a nerwork of aqm routers supporting tcp flows with an application to red. Proc. ACM SIGCOMM, pages 151–160, 2000.

[3] Y. Ariba, Y. Labit, and F. Gouaisbaut. Congestion control of a single router with an active queue management. International Journal on Advances in Internet Technology, 2(1), 2009.

[4] P. F. Quet and H. Özbay. On the design of aqm supporting tcp flows using robust control theory. IEEE Trans. on Automat. Control, 49:1031–1036, June 2004.

[5] C.K. Chen, Y.C. Hung, T.L. Liao, and J.J. Yan. Design of robust active queue management controllers for a class of tcp communication networks. Information Sciences, 177(19):4059–4071, 2007.

[6] S. Tarbouriech, C. T. Abdallah, and J. Chiasson. Advances in communication control networks. Springer, 2005.

[7] H. S. Low, F. Paganini, and J. C. Doyle. Internet congestion control. IEEE Control Systems Magazine, 22:28–43, Feb 2002.

[8] J. Newbury and K. Morris. Power line carrier systems for industrial control applications. IEEE Transactions on Power Delivery, 14(4):1191–1196, 1999.

[9] K. Fall and K. Varadhan. The ns manual, notes and documentation on the software ns2-simulator. URL: www.isi.edu/nsnam/ns/, 2002.

[10] K. Fall, and K. Varadhan. ”the ns manual”, notes and documentation on the software ns2-simulator. URL: www.isi.edu/nsnam/ns/, November 2011.

[11] E. Witrant, C. Canudas de Wit, and M. Alamir. Remote output stabilization under two channels time-varying delays. Proc. of the 4th IFAC Workshop on Time Delay Systems, 2004.

[12] C. E. De Souza, R. E. Palhares, and P. L. D. Peres. Robust h-infinity filtering for uncertain linear systems with multiple time-varying state : An lmi approach. Conference on Decision and Control, Proceeding of the 38th Conference on Decision and Control, 46(5):2023–2028, 1999.

[13] I. Lopez, J. L. Piovesan, C. T. Abdallah, D. Lee, O. M. Palafox, M. Spong, and R. Sandoval. Pratical issues in networked control systems. ACC’06, pages 4202 – 4207, 2006.

[14] A. Seuret, F. Michaut, J. P. Richard, and T.Divoux. Networked control using gps

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

synchronization. American Control Conference, 49:American Control Conference, 2006.

[15] E. Fridman, and U. Shaked. An improved stabilization method for linear time-delay systems. IEEE Transactions on Automatic, 47:1931–1937, 2002.

[16] V. Suplin, E. Fridman and U. Shaked. Hinfty control of linear uncertain time-delay systems a pojection approach. Proc. of the CDC, 2004.

[17] H. Han, C. V. Hollot, Y. Chait and V. Misra. Tcp networks stabilized by buffer-based aqms. IEEE INFOCOM, pages 964–974, March 2004.

[18] C. V. Hollot, V. Misra, D Towsley, and W. Gong. A control theoretic analysis of red. IEEE INFOCOM, pages 1510–1519, April 2001.

[19] C. V. Hollot, V. Misra, D Towsley, and W. Gong. On designing improved controllers for aqm routers supporting tcp flows. IEEE INFOCOM, 3:1726–1734, April 2001.

[20] C. V. Hollot, V. Misra, D Towsley, and W. Gong. Analysis and design of controllers for aqm routers supporting tcp flows. IEEE Trans. on Automat. Control, 47:945–959, June 2002.

[21] Y. Ariba, F. Gouaisbaut, and Y. Labit. Controle de congestion d'un routeur par retour d'état. Conference International Francophone d'Automatique (CIFA'08), July 2008.

[22] Y. Moon, P. Park, W.Kwon, and Y. Lee. Delay-dependent robust stabilization of uncertain state delayed systems. International Journal of Control, 74:1447–1455, 2001.

[23] Bugong. Xu. On delay-independent stability of large scale systems with time-delays. IEEE Trans. Automat. Contr, 40(5):930–933, 1995.

[24] K. B. Kim. Design of feedback controls supporting tcp based on the state space approach. IEEE Trans. on Automat. Control, 51 (7), July 2006.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

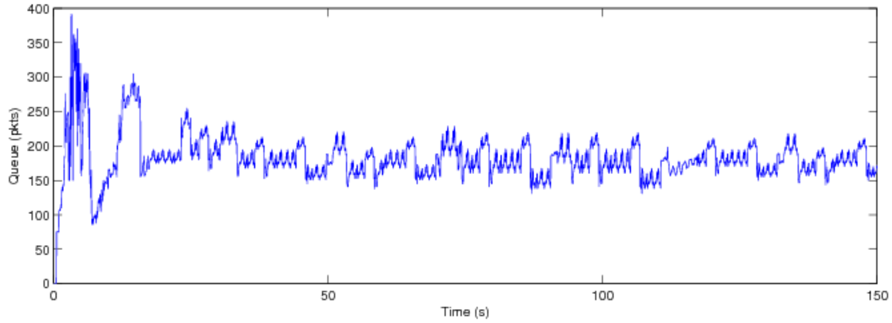

Figure 2: Evolution Of The Queue Length : Desired Length $q_0 = 175$ packets

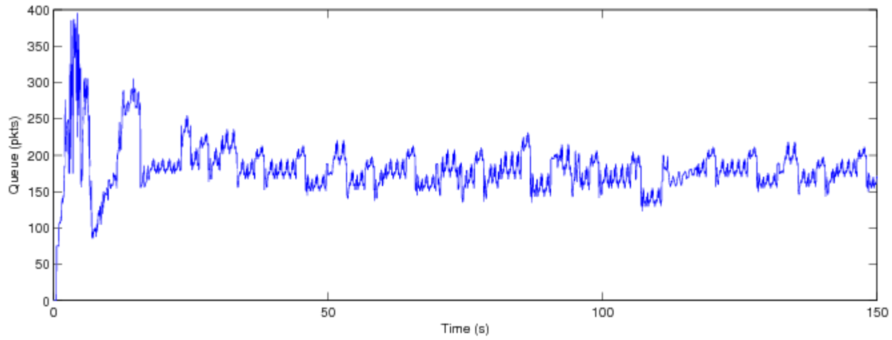

Figure 3: Observer-Based State-Feedback : The Introduction Of UDP Flows ($0.5 Mbps$) Between 70 And 110 Seconds

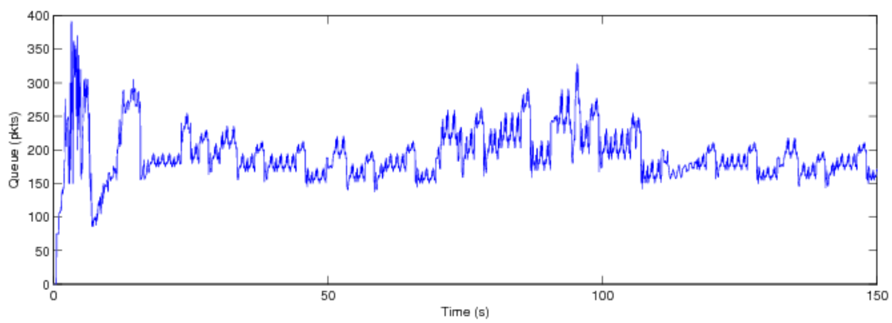

Figure 4: Observer-Based State-Feedback : The Introduction Of UDP Flows ($3 Mbps$) Between 70 And 110 Seconds