

RULE-BASED REASONING ALGORITHM FOR INTUITIONISTIC FUZZY PETRI NETS

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ABSTRACT

Looney's fuzzy reasoning algorithm supported by Fuzzy Petri Net (FPN) is further extended for representing the fuzzy production rules in a knowledge based system which is provided with fuzzy reasoning ability of fuzzy systems. In intuitionistic fuzzy sets (IFS) there is an extension of Zadeh Fuzzy sets that possess a new additional attribute function i.e non membership and intuitionistic index take into account with stronger expression ability to deal with uncertain information. In this paper a rule based reasoning algorithm is presented for intuitionistic fuzzy petrinets. intuitionistic fuzzy members are used to represent the confidence degree and threshold of the transition along with token values of each place. A generalized fuzzy petri net to intuitionistic fuzzy petri net and vice versa is presented in an effective way. Some new features are added to take care of the generalization of the intuitionistic nature of the fuzzy petri nets.

Keywords - *Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Petri Nets, Fuzzy Reasoning, Fuzzy Relations, Intuitionistic Fuzzy Numbers*

1. INTRODUCTION

Petri Net (PN) is an excellent mathematical modeling language for the representation of Discrete Event Systems (DES). It is one of the quite effective tools for graphical modeling, mathematical modeling, simulation and real time control by the use of places and transitions. However there was an intuitive need for a system which would be able to address uncertainties and imprecision of the real world system because of increase complexity of industrial and communication systems.

Looney [1] first presented Fuzzy reasoning algorithm supported by Fuzzy Petri Nets (FPN). The model can be successfully used for exhibiting the Fuzzy production rules in a knowledge based system, comprising with not only the graphical and structural ability of Petri Nets (PN), but also provides with the Fuzzy reasoning ability of Fuzzy system. We come across two types of reasoning: forward and backward reasoning in graphical structure by

using PN's graphical nature, which helps in visualizing the structure of rule-based system and making the model simple and legible. The second reasoning is based on PN's mathematical foundation which allows one to express the dynamic behavior of a system in algebraic forms [2, 3].

In this paper, we have presented improved version of Intuitionistic Fuzzy Petri Nets (IFPN) model [4, 5] and [6], which is the combination of FPN and IFPN. The token value of each Proposition and the confidence degree and the threshold of the transition are represented by intuitionistic Fuzzy numbers. We first converted Fuzzy petrinets to intuitionistic Fuzzy petrinets and then used reasoning algorithm. The conversion is carried on by simple mapping from Fuzzy to intuitionistic nets. We also used negative literal for reasoning process. The paper is organized as

follows, in section 1, we present introduction and brief review of work done in FPN . In section 2,we present fuzzy production rule concept.In section 3we include preliminaries on Instuitionistic Fuzzy Sets.Section 4 represents definition and notation of IFPN along with the conversion of FPN toIFPN.In section 5 we

present the algorithm.Section 6 represents an illustrative example.Section 7 represents the conversion of IFPN to FPN.The last section contains the conclusion and future scope of the work.

2. FUZZY PRODUCTION RULES

Let $R = \{ R_1, R_2, \dots, R_m \}$ be a finite set of production rules such that i^{th} production rules

R_i is presented as R_i : if A then B, ($CF = \mu_i$) where, $A = \{ \alpha_1, \alpha_2, \dots, \alpha_n \}$ represents the antecedent part which comprises of one or more propositions connected by either “AND” or “OR” in the rule.

$B = \{ B_1, B_2, \dots, B_n \}$ represents the consequent part consist of one or more propositions connected by “AND” or “OR” and μ_i denotes the certainty factor (CF_i) of the rule R_i .

Following [7, 8] and [9], fuzzy production rules can be classified into four types as follows:

Type 1: $R_i(\mu_i) : P_1(\alpha_1) \wedge P_2(\alpha_2) \wedge \dots \wedge P_{k-1}(\alpha_{k-1}) \rightarrow P_k(\alpha_k) \alpha_k = \min(\alpha_1, \alpha_2, \dots, \alpha_{k-1}) * \mu_i$

Type 2 : $R_i(\mu_i) : P_1(\alpha_1) \rightarrow P_2(\alpha_2) \wedge P_3(\alpha_3) \wedge \dots \wedge P_{k-1}(\alpha_{k-1}) \wedge P_k(\alpha_k) \alpha_2 = \alpha_1 * \mu_i$

$\alpha_3 = \alpha_1 * \mu_i$
:
:
 $\alpha_k = \alpha_1 * \mu_i$
Type 3 : $R_i(\mu_i) : P_1(\alpha_1) \vee P_2(\alpha_2) \vee \dots \vee P_{k-1}(\alpha_{k-1}) \rightarrow P_k(\alpha_k) \alpha_k = \max(\alpha_1, \alpha_2, \dots, \alpha_{k-1}) * \mu_i$

Here α_i denotes the truth degree of antecedent consequence part of P_i in fuzzy rule R_i and μ_i denotes the confidence degree of applying rule R_i .

Rule Representation

In rule representation, proposition is represented by PN places. Tokens inside the places represent the truth degree of the proposition. The Transition of PN serves as rule ,the reasoning process can be initiated by firing the transition in FRNs.The mutual causality interconnects among the proposition and reasoning rules can be expressed by means of the analogy with thearcs connecting the Transitions and places.

The pictorial representations of the first three type of FRPN are shown inFig 1, Fig 2 and Fig 3

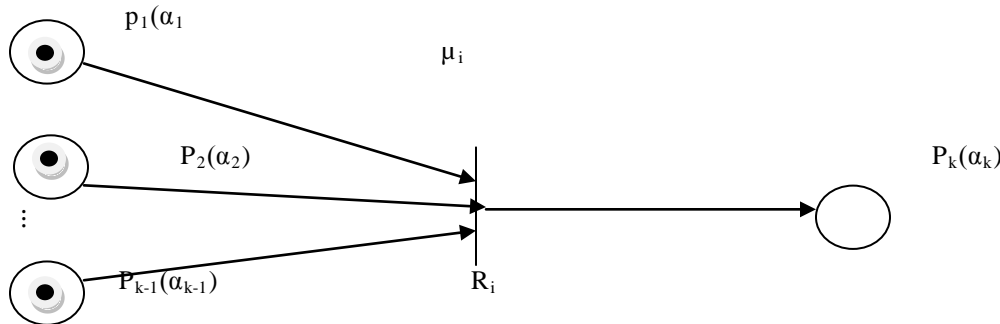


Figure 1: (Type-1 FRPN)

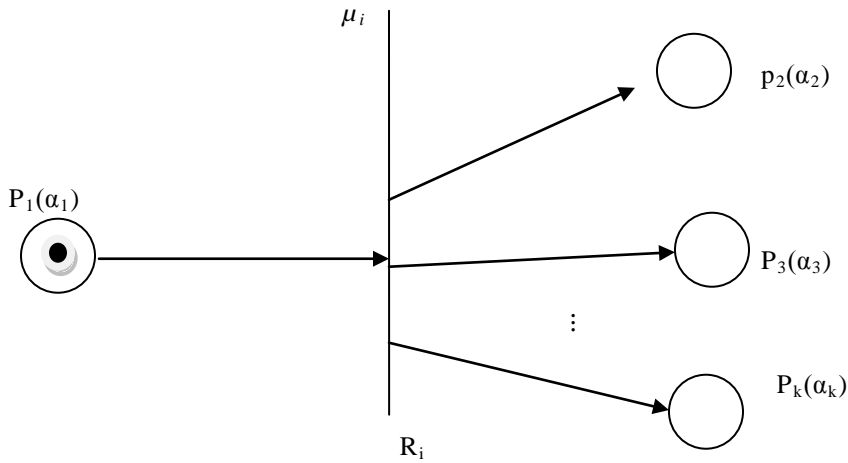


Figure 2: (Type-2 FRPN)

Type – 3 is normalized as

$$R_{i1}(\mu_i) : P_1(\alpha_1) \rightarrow P_k(\alpha_k)$$

$$R_{i2}(\mu_i) : P_2(\alpha_2) \rightarrow P_k(\alpha_k)$$

⋮

⋮

$$R_{ik-1}(\mu_i) : P_{k-1}(\alpha_{k-1}) \rightarrow P_k(\alpha_k)$$

and the PN structure is given as :

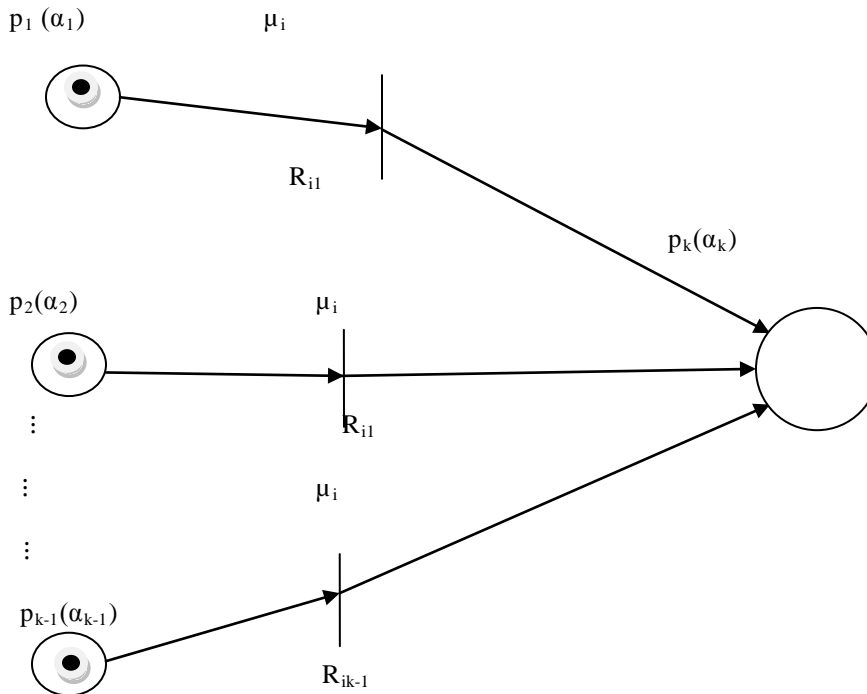


Figure 3: (Type-3 normalized FRPN)

2.2 Rule based issues

Consider the rule which is shown in the following figure.

$R_1(0.80) : \text{IF } P_1 \text{ AND } P_2 \text{ THEN } P_6$

$R_2(0.90) : \text{IF } P_2 \text{ AND } P_3 \text{ AND } P_4$
 THEN $P_7 \text{ AND } P_8$

$R_3(0.80) : \text{IF } P_4 \text{ AND } P_5 \text{ THEN } P_9$

$R_4(1.0) : \text{IF } P_6 \text{ THEN } P_7$

$R_5(0.80) : \text{IF } P_8 \text{ AND NOT } P_9 \text{ THEN } P_{10}$

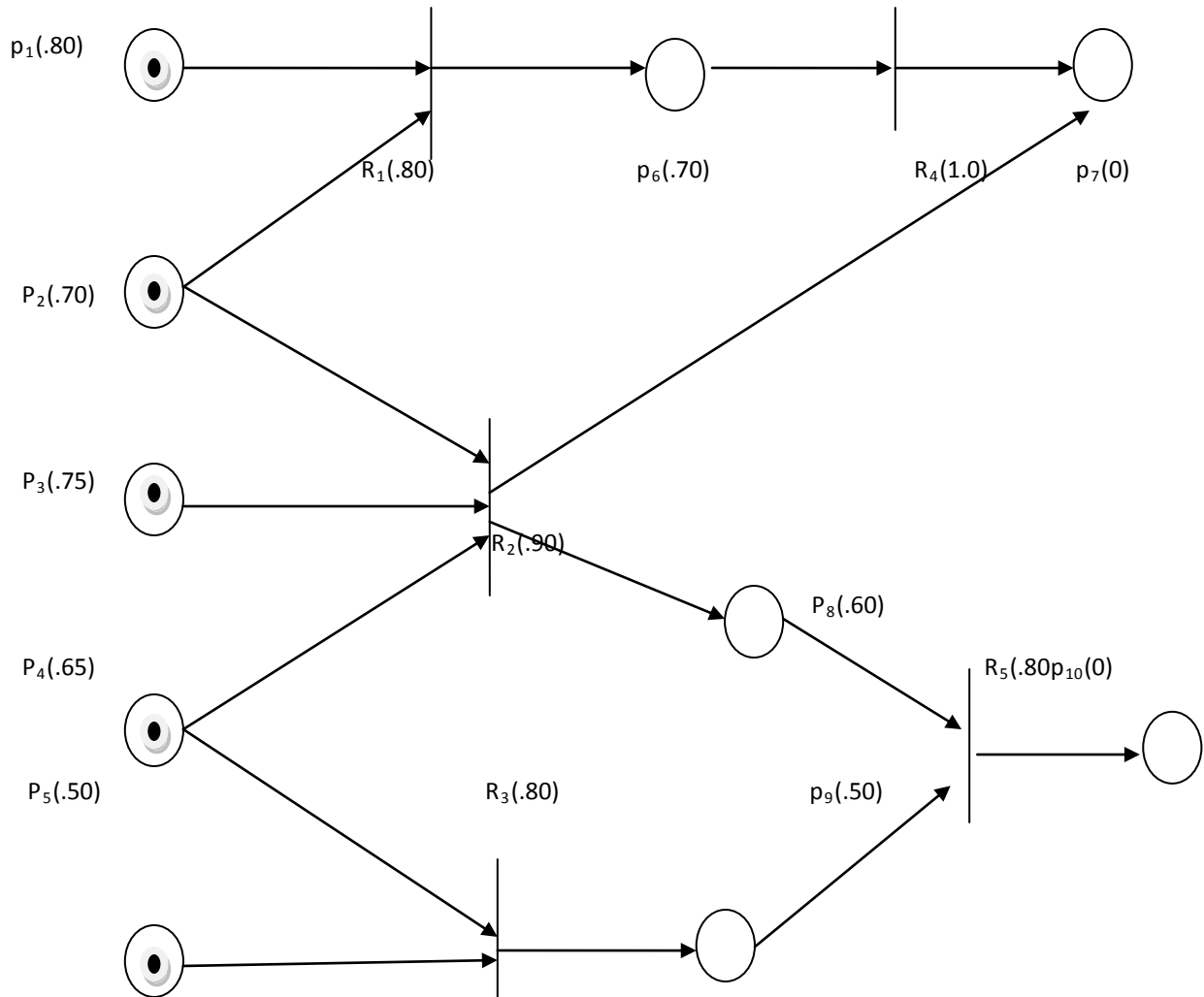


Figure 4



2.3 Some of the salient features of FRPNs

- a) Number of tokens in a place cannot be greater than one since token is associated with degree of truth between zero and one .
- b) FRPNs are always conflict-free sets.
- c) On transition firing, tokens are not removed from the input places. This is because the evolution of the rules means the only the truth propagation of the propositions. In other words, the truth of proposition will not disappear because of the rule reasoning.

3. PRELIMINARY ON INTUITIONISTIC FUZZY SETS

Intuitionistic Fuzzy Sets (IFS) is the generalization of fuzzy sets [2, 3] to represent vague and uncertainty.

Definition 1 (IFS): Given an universal set X, an intuitionistic fuzzy set (IFS) .A is a set of ordered triples,

$$A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \} \tag{1}$$

Where $\mu_A(x) : x \rightarrow [0,1]$, $\gamma_A(x) : x \rightarrow [0,1]$

Representing membership and non-membership functions respectively of A .Moreover for every $x \in X$ in A, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$, holds and $(\mu_A(x) , \gamma_A(x))$ is called intuitionistic fuzzy number. For every common fuzzy subset Aon X, we define $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$, $\pi_A(x)$ is intuitionistic index of X in A. Obviously, for every $x \in X$, $0 \leq \pi_A(x) \leq 1$,and for common fuzzy subset A on X,

$$\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0, \forall x \in X$$

Definition 2: Let A and B be two intuitionistic fuzzy subsets on X ,then

$$A+B = \{ (x, \mu_A(x) + \mu_B(x) - \mu_A(x). \mu_B(x) , \gamma_A(x). \gamma_B(x)) \mid \forall x \in X \} \tag{2}$$

$$A.B = \{ (x, \mu_A(x). \mu_B(x) , \gamma_A(x) + \gamma_B(x) - \gamma_A(x). \gamma_B(x)) \mid \forall x \in X \} \tag{3}$$

4. REVIEW OF IFPN

4.1. Basic IFPN

IFPN model is defined as 7-tuples as follows:

$$IFPN=(P,T,D,I,O,\tau,\theta) \tag{4}$$

where

$P = \{p_1, p_2, p_3 \dots p_n \}$ is a finite set of places

$T = \{t_1, t_2, t_3 \dots t_m \}$ is a finite set of transitions

$D = \{d_1, d_2, d_3, \dots d_n \}$ is a finite set of propositions attached to each place

$I = \{a_{ij} \} : P \times T \rightarrow [0,1]$ is an $n \times m$ weighted input matrix defining the directed arcs from p_i to t_j , then $a_{ij} = w_{ij}$ (weight attached to the arc from p_i to t_j) or else $a_{ij} = 0$, for $i=1, \dots, n, j=1, \dots, m$

$O = \{b_{ij} \} : T \times P \rightarrow [0,1]$ is $m \times n$ output matrix defining directed arcs from transitions to places. If there is a directed arc from t_j to p_i ,then $b_{ij} = c_j$ or else $b_{ij} = 0$, where $c_j = (C\mu_j , C\gamma_j)$ is an intuitionistic fuzzy number , $C\mu_j$ means the support degree of t_j and $C\gamma_j$ means non-support degree of t_j , $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$

$\tau = (\tau_1 , \tau_2 , \dots \tau_m)$ represents the threshold vector whose components represent the condition of the rule that a transition can be fired, $\tau_j = (\alpha_j , \beta_j)$, $0 \leq \alpha_j + \beta_j \leq 1$ holds, α_j and β_j both non-negative representing the threshold support / non-support degree.

$\theta : P \times T \rightarrow [0,1]$ is an associate function which assigns a token value to each place .It is a fuzzy value of each proposition.The initial state vector $\theta^0 = (\theta_1^0 , \theta_2^0 , \dots , \theta_n^0)$ where $\theta_i^0 = (\theta\mu_i^0 , \theta\gamma_i^0)$

is an intuitionistic fuzzy number .Moreover $P \cap T \cap D = \emptyset$, $|P| = |D| = n$, $|T| = m$

4.2 Conversion from FPN to IFPN

We mapped a FPN structure to IFPN structure by introducing the knowledge of intuitionistic characters of fuzzy principles. We



introduce both the membership and non-membership of propositions, rules and the threshold of rules by intuitionistic fuzzy numbers. The conversion is shown in Fig 4 from the FPN presented in Figure 5.

4.3 Knowledge Representation of IFPN

Fuzzy production rules can be suitably modeled through intuitionistic fuzzy Petri nets.

Rule R :

If d_1 and d_2 and ... d_n Then d_k (CF= μ)
 $\lambda_1, w_1, w_2, \dots, w_n$
 (5)

Where $d = (d_1, d_2, d_3, \dots, d_n)$ is the antecedent portion comprising of proposition d_k represent the consequent part, w_i ($i= 1, 2, \dots, n$) is the weigh of d_i to rule R_i . λ is the threshold value of R. μ is the confidence degree of R. Moreover all antecedent propositions, consequent propositions, λ, μ are intuitionistic fuzzy numbers. We may replace antecedent propositions d_1, d_2, \dots, d_n by places p_1, p_2, \dots, p_n and consequent proposition d_k by p_k . If is the token value of p_i is θ_i then the token value of p_k is $\theta_k = (\mu_k, \gamma_k)$ where,

$$\mu_k = (\sum_{i=1}^n w_i \mu_i) \times [1 + (C\mu - C\gamma) / 2]$$

$$\gamma_k = (\sum_{i=1}^n w_i \gamma_i) \times [1 + (C\gamma - C\mu) / 2]$$

(6)

4.4 Execution rules of IFPN

We use max algebra to introduce four operators of matrix algebra.

- (1) Addition operators $\oplus : C=A \oplus B \Leftrightarrow c_{ij} = \max(a_{ij}, b_{ij})$, Where A,B are $n \times m$ intuitionistic matrices
- (2) Multiplication operator $\otimes : C=A \otimes B \Leftrightarrow c_{ij} = \max_{1 < k < l} (a_{ik}, b_{kj})$ Where A is $n \times l$, B is $l \times m$, C is $n \times m$ matrices
- (3) Comparison operator $\odot : C = A \odot B \Leftrightarrow c_{ij} = \{1, \text{ if } a_{ij} \geq b_{ij}$
 $0, \text{ if } a_{ij} < b_{ij} \}$

- (4) Product operator $\ominus : C = A \ominus B \Leftrightarrow c_{ij} = a_{ij} \cdot b_{ij}$, where A,B,C are $n \times m$ intuitionistic fuzzy matrices.

Moreover if the elements of A, B, C are intuitionistic fuzzy numbers, the operations among the elements are in accordance with intuitionistic fuzzy logic such as

$$a_{ij} \cdot b_{ij} = (\mu(a_{ij}) \cdot \mu(b_{ij}), \gamma(a_{ij}) + \gamma(b_{ij}) - \gamma(a_{ij}) \cdot \gamma(b_{ij}))$$

$$\max(a_{ij}, b_{ij}) = (\max(\mu(a_{ij}), \mu(b_{ij})), \min(\gamma(a_{ij}), \gamma(b_{ij})))$$

and $a_{ij} \geq b_{ij} \Leftrightarrow \mu(a_{ij}) \geq \mu(b_{ij})$ and $\gamma(a_{ij}) \leq \gamma(b_{ij})$

5. IFPN ALGORITHM

Input: Given quantities are initial token vector θ^0 of places, the threshold vector τ of transition, the weighted input matrix $n \times m$ and the output matrix $m \times n$

Output: The final token vector θ and number of iterations k

Step 1: Set $k = 0$

Step 2: Compute the equivalent fuzzy token vector of input places of the transitions

$$E = I^T \times \theta^K$$

(7)

$$E \in \mathbb{R}^m, E = (e_i), e_i = (E\mu_i, E\gamma_i), i=1,2,3,\dots,m$$

is intuitionistic fuzzy number θ^K token vector at k^{th} iteration, θ^0 is known.

Step 3: Compare vector E with τ , then get rid of the item which can not touch off the transition

$$G = E \ominus (E \odot \tau)$$

(8)

$\tau_1 = (\alpha_i, \beta_i)$ and $E\mu_i \geq \alpha_i$ and $E\gamma_i \leq \beta_i$

place, let

$$S = O \otimes G \tag{9}$$

Then $g_i = e_i$ or else $g_i = 0$, $i = 1, 2, \dots, m$

Step 5: Compute $\theta^{K+1} = \theta^K \oplus S$
(10)

Step 4: Compute token value of the exporting

Step 6: If $\theta^{K+1} \neq \theta^K$, go to Step 2 else STOP

6. ILLUSTRATIVE EXAMPLE

Consider the rule which is shown in the following Fig 5:

R_1 : IF $P_1(.80)$ and $P_2(.70)$ THEN P_6 (CF(.8,.18) ($\tau_1 = (.25,.65)$)

R_3 : IF $P_4(.65)$ AND $P_5(.50)$ THEN P_9 (CF(.8,.15) ($\tau_3 = (.40,.55)$)

R_4 : IF $P_6(.70)$ THEN P_7 (CF(0,1) ($\tau_4 = (.30,.60)$)

R_2 : IF $P_2(.70)$ AND $P_3(.75)$ AND $P_4(.65)$ THEN P_7 AND P_8 (CF (.9,.01) ($\tau_2 = (.70,.20)$)

R_5 : IF $P_8(.60)$ AND $P_9(.50)$ THEN P_{10} (CF (.10,.80) ($\tau_5 = (.81,.10)$)

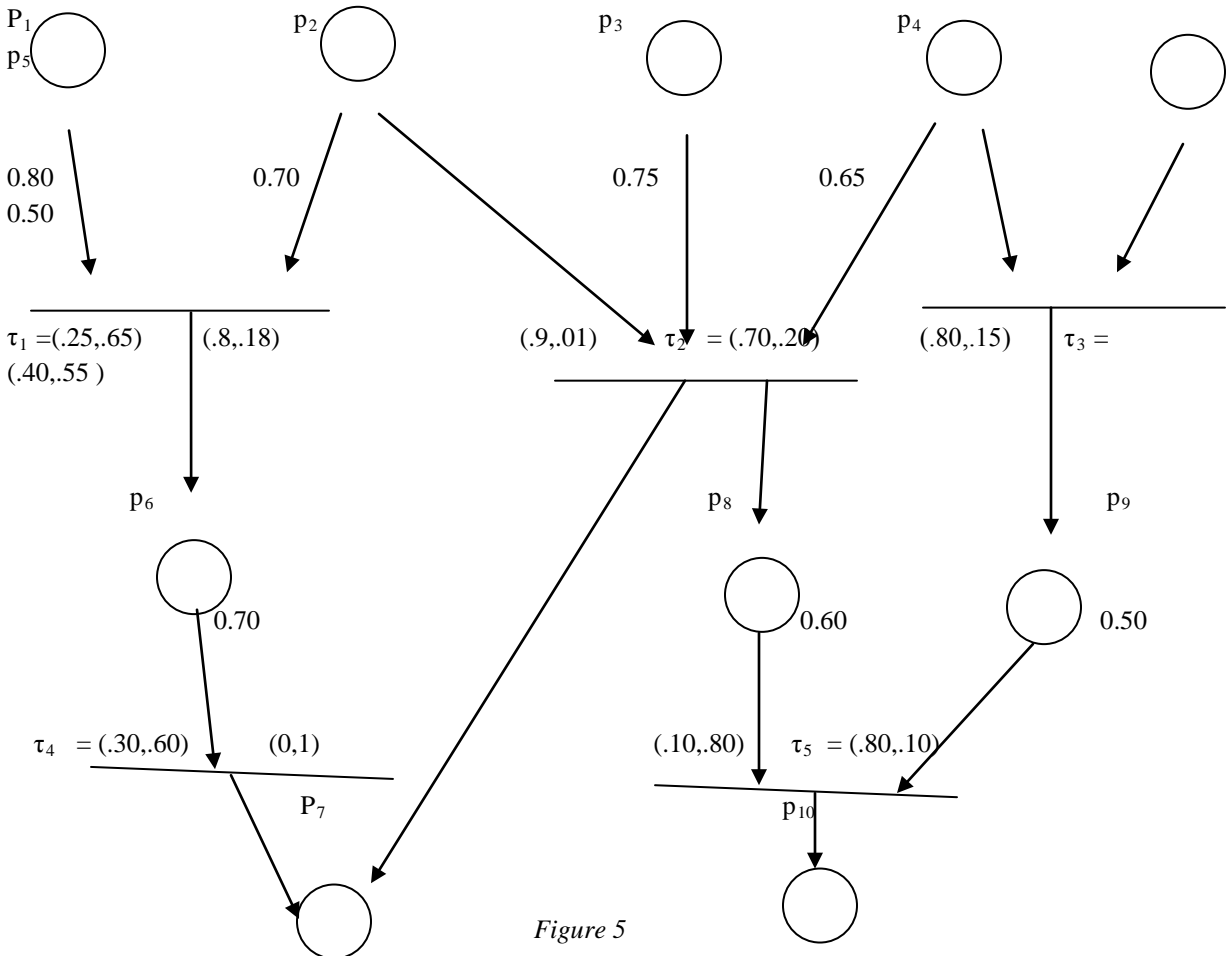


Figure 5

The input matrix

	R ₁	R ₂	R ₃	R ₄	R ₅
I = p ₁	.80	0	0	0	0
P ₂	.70	.70	0	0	0
P ₃	0	.75	0	0	0
P ₄	0	.65	.65	0	0
P ₅	0	0	.50	0	0
P ₆	0	0	0	.70	0
P ₇	0	0	0	0	0
P ₈	0	0	0	0	.60
P ₉	0	0	0	0	.50
P ₁₀	0	0	0	0	0

The output matrix

	R ₁	R ₂	R ₃	R ₄	R ₅
O = p ₁	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
p ₂	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
p ₃	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
p ₄	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
p ₅	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
p ₆	(.8,.18)	(0,1)	(0,1)	(0,1)	(0,1)
p ₇	(0,1)	(.9,.01)	(0,1)	(0,1)	(0,1)
p ₈	(0,1)	(.9,.01)	(0,1)	(0,1)	(0,1)
p ₉	(0,1)	(0,1)	(.8,.15)	(0,1)	(0,1)
p ₁₀	(0,1)	(0,1)	(0,1)	(0,1)	(.1,.8)

$O(p_i, R_j) = \{ (0,1), \text{arc from } p_i \text{ to } R_j \}$
 CF, Otherwise }



The initial truth degree vector $\alpha^0 = (\alpha_1^0, \alpha_2^0, \dots, \alpha_n^0) : P \rightarrow (\mu, \gamma)$

Where μ is membership function and γ is non-membership function

The initial token value

$$\theta^0 = \alpha^0 = [(.8,.15), (.25,.7), (.35,.6), (.3,.65), (.5,.45), (0,1), (0,1), (0,1), (0,1), (0,1)]^T$$

The threshold of transition

$$\tau = [(.25,.65), (.2,.7), (.1,.85), (.3,.6), (.80,.10)]$$

7. CONVERSION OF IFPN TO FPN

We use the inverse transformation of IFPN to FPN. We mapped a IFPN structure to FPN structure by introducing the knowledge of fuzzy

$$P = \{ p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10} \}$$

$$R = \{ R_1, R_2, R_3, R_4, R_5 \}$$

According to IFPN algorithm, the reasoning result is:

$$\theta^1 = [(.8,.15), (.25,.7), (.35,.6), (.3,.65), (.51,.28), (.5,.45), (.23,.43), (.23,.43), (.2,.35), (0,1)]$$

$$\theta^2 = \theta^1 \quad (\alpha^2 = \alpha^1)$$

Due to $\theta^2 = \theta^1$, the inference is finished.

Hence $p_1 = (.8,.15)$ is the most probable causation and p_5 is the next.

principles. The conversion is shown in Fig 5 from the FPN presented in Figure 4.

	R ₁	R ₂	R ₃	R ₄	R ₅
O = p ₁	0	0	0	0	0
P ₂	0	0	0	0	0
P ₃	0	0	0	0	0
P ₄	0	0	0	0	0
P ₅	0	0	0	0	0
P ₆	1	0	0	0	0
P ₇	0	1	0	1	0
P ₈	0	1	0	0	0
P ₉	0	0	1	0	0
P ₁₀	0	0	0	0	1



$$C = \text{diag} (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5)$$

	R ₁	R ₂	R ₃	R ₄	R ₅
R ₁	0.8	0	0	0	0
R ₂	0	0.9	0	0	0
R ₃	0	0	0.8	0	0
R ₄	0	0	0	1	0
R ₅	0	0	0	0	0.8

	R ₁	R ₂	R ₃	R ₄	R ₅
I = p ₁	1	0	0	0	0
p ₂	1	1	0	0	0
p ₃	0	1	0	0	0
p ₄	0	1	1	0	0
p ₅	0	0	1	0	0
p ₆	0	0	0	1	0
p ₇	0	0	0	0	0
p ₈	0	0	0	0	1
p ₉	0	0	0	0	1
p ₁₀	0	0	0	0	0

$$I(p_i, R_j) = \begin{cases} 1, & \text{arc from } p_i \text{ to } R_j \\ 0, & \text{Otherwise} \end{cases}$$

8. CONCLUSION AND FUTURE SCOPE

A special type of FPN model has been extended to represent IFPN taking the advantage of both FPN and IFPN. It gives wider practical description and precision in reasoning. It is shown that the negation issue can also be addressed in IFPN. The matrix equation based format that is similar to PN's for knowledge representation in IFPN is suitably addressed. It has great potential to be used to solve other complex problems effectively.

We emphasize a lot of future works can be extended from the present analysis. We can convert IFPN to FPN and vice versa and study the related problems. We can use it for generalized IFPN to capture more practical use.

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