

ADAPTIVE CONTROL USING MULTIPLE MODELS WITHOUT SWITCHING

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ABSTRACT

The multiple models adaptive control based on switching or switching and tuning have been developed extensively during the past twenty years, which can adapt rapidly to any unknown but constant operating environment. However, the stability of the system with multiple models cannot be easily established. Towards this goal, this paper proposes a novel multiple models adaptive controller. Unlike the prior results, the controller developed here do not require a switching scheme to guarantee the most appropriate model to be switched into the controller design. So the analysis of the stability of the closed-loop can be established easily. Besides, the N identification models are fixed, which is superior to the all adaptive identification models. Simulation results proposed to illustrate the effectiveness of the developed multiple models adaptive controller.

Keywords: Robust adaptive control, Multiple model control, Nonlinear system, Transient performance, Stability

1. INTRODUCTION

The adaptive control theory was firstly proposed in 1960s to cope with the control of linear time invariant systems with unknown parameters, which now achieves satisfactory closed-loop objective specified in meaningful engineering terms when the plant parametric uncertainty is small[1]. However, changes in operating conditions; failure or degradation of component; or unexpected changes in system dynamics may all violate the assumption of small uncertainty, particularly parametric uncertainty[2]. The impacts mentioned above always result in large and oscillatory responses or even instable when using the classical adaptive control methods.

To deal with such situations, adaptive control employing switching has been proposed[3-6]. Both fixed model and adaptive model have been used to identify the characteristics of plants, and numerous methods are currently available for controlling such plant satisfactorily. However, the methods mainly focus on the linear time invariant plants[4,5,7-9]. The multiple model adaptive controller for nonlinear system is firstly considered in [10], which using a direct parameter update law guarantees the stability of the closed-loop system. Then, ciliz[11] propose a different nonlinear multiple model adaptive control which require the condition of persistence of excitation, so that the unknown

parameter can be evaluated at the very beginning. Recently, an indirect multiple model adaptive control was developed in [12] which also demonstrated the global asymptotic stability of the close-loop switching system. In our prior work[13], the location of the multiple model controller has been investigated. However, from a computational point of view, multiple adaptive models are inefficient because of the need to update their parameter vectors at every instant. Fixed models do not have this drawback, but can represent exactly only a finite number of environments.

In this paper, a novel multiple model adaptive control was considered for the nonlinear system in parameter-strict-feedback form. The controller used multiple fixed model to determine the location of the unknown parameter instead of finding the most suitable identification model to switch to, and an additional adaptive controller to guarantee the convergence of the closed-loop. Besides, the multiple model adaptive controller removes the switching which brings about discontinuous control signal and in turn results in complicated procedure to analyze and synthesize the controller of the closed-loop system. The approach developed here in which the multiple model controller are used to play a significantly larger role in the decision making role, results in substantial improvement in the transient performance.

2. PROBLEM FORMULATION

Consider the multiple model adaptive control of the following nonlinear parameter-strict-feedback (PSF) system:

$$\begin{cases} \dot{x}_i = x_{i+1} + \varphi_i^T(\bar{x}_i)\theta, 1 \leq i \leq n-1 \\ \dot{x}_n = \beta(x)u + \varphi_n^T(x)\theta \\ y = x_1 \end{cases}$$

(1)

where $\bar{x}_i = [x_1, \dots, x_i]^T \in \mathbb{R}^i$ and $x \in \mathbb{R}^n$ are the state, $u \in \mathbb{R}$ is the control input, $\theta \in \mathbb{R}^p$ is an unknown parameter vector belonging to a known compact set S . The function $\varphi_i(\bar{x}_i)$ and $\beta(x)$ are known smooth functions with $\beta(x) \neq 0, \forall x \in \mathbb{R}^n$. The focus of this paper is to improve the transient performance in the presence of large parametric uncertainties, and at the same time, assure the stability of the closed-loop system.

One easily way to improve the transient performance may be choosing sufficiently large high-frequency parameters in the conventional backstepping adaptive control design. Unfortunately, the control efforts can also be very large simultaneously^[12]. Alternately, in cope with this difficulties, “switching” or “switching and tuning” have emerged as the leading methods during the last decade^[14-18], which have some advantages as having rapid adaptation to large, abrupt parameter change. Unfortunately, it has also some disadvantages as i) the information provided by every identification model is not used efficiently; ii) the procedure of identification and control are coupled.

The novel approach developed in this paper addresses all the above concerns as: i) the information provided by all the models is utilized efficiently; ii) the identification and control are decoupled which can facilitate the procedure during the controller design.

3. MULTIPLE MODEL CONTROLLER DESIGN

For the convenience of understanding the controller design, we firstly give the classical adaptive controller.

3.1 Classical Adaptive Controller Design

In this section, we present the classical adaptive controller for the unknown parameter plant (1) as

$$u = \alpha_n(x, \hat{\theta}, y_r, \dots, y_r^n) / \beta(x) \quad (2)$$

$$\dot{\hat{\theta}} = \sum_{i=1}^n w_i z_i \quad (3)$$

where y_r is the reference signal to be tracked, and

α_n is recursively designed as

$$z_i = x_i - \alpha_{i-1}(\bar{x}_{i-1}, \hat{\theta}, y_r, \dots, y_r^{i-1}) \quad (4)$$

$$\alpha_i = -z_{i-1} - c_i z_i + y_r^i - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \quad (5)$$

$$+ \sum_{k=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{k-1}} y_r^k \right)$$

$$w_i(x_1, \dots, x_i, \hat{\theta}, y_r, \dots, y_r^{i-1}) = \varphi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_k \quad (6)$$

with c_i being high-gain design parameters, and

$$\alpha_0 = y_r, z_0 = 0.$$

Proof: we choose the candidate Lyapunov function as

$$V = \frac{1}{2} \sum_{i=1}^n z_i^2 + \frac{1}{2} (\theta - \hat{\theta})^2. \quad (7)$$

The time derivative of V , computed with (2)-(6), is given by

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^n c_i z_i^2 + \sum_{i=1}^n w_i^T (\theta - \hat{\theta}) z_i - (\theta - \hat{\theta})^T \dot{\hat{\theta}} \\ &= -\sum_{i=1}^n c_i z_i^2 \end{aligned} \quad (8)$$

which implies the boundedness of the error $(\theta - \hat{\theta})$ and the states of $z_i, 1 \leq i \leq n$, and which in turn indicates the boundedness of the states of $x_i, 1 \leq i \leq n$ and control u .

However, when it comes to the multiple model adaptive control, the well-established results from the classical adaptive control cannot be used directly. Our multiple model adaptive controller contains two parts: one is the nonlinear parametrized controller, the other is N parallel operating identification models. For improving the transient performance, it is necessary to distribute the initial estimate values of the unknown parameter $\{\theta_j(0)\}_{j=1}^N$ uniformly in the compact set S to which the unknown parameter θ belongs. Therefore at least one $\theta_j(0)$ is close to θ .

3.2 Design Of Nonlinear Multiple Model Adaptive Controller

The controller design is somewhat different from the existed approaches. Firstly, the multiple fixed models used are not to find the best identification model to be switched, hence there is no switching. The switching can improve the transient performance, and at the same time make it difficulty to develop the controller. Secondly, the

controller used multiple fixed models to determine the most likely location of the unknown parameter. Thirdly, in order to guarantee the convergence of the closed-loop system, an additional adaptive controller is introduced.

The nonlinear parametrized controller is given as

$$u = (\sum_{j=1}^N \gamma_j \alpha_{n,j} + \alpha_{n,a}) / \beta(\mathbf{x}) \quad (9)$$

where $\alpha_{n,j}, \alpha_{n,a}$ are recursively designed as

$$z_i = x_i - \sum_{j=1}^N \gamma_j \alpha_{i-1,j} - \alpha_{i-1,a} \quad (10)$$

$$\alpha_{i,j} = -z_{i-1} - c_i z_i + y_r^i - w_{i,j}^T \theta_j(0) + \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1,j}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1,j}}{\partial y_r^{k-1}} y_r^k) \quad (11)$$

$$\alpha_{i,a} = -\sum_{j=1}^N (\gamma_j w_{i,j}^T \hat{\theta}_{i,j} - \dot{\gamma}_j \alpha_{i-1,j} - \frac{\partial \alpha_{i-1,a}}{\partial \hat{\theta}_{i-1,j}} \dot{\hat{\theta}}_{i-1,j}) + \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1,a}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1,a}}{\partial y_r^{k-1}} y_r^k) \quad (12)$$

$$w_{i,j} = \varphi_i - \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1,j}}{\partial x_k} \varphi_k + \frac{\partial \alpha_{i-1,a}}{\partial x_k} \varphi_k) \quad (13)$$

$$\dot{\hat{\theta}}_{i,j} = z_i \gamma_j w_{i,j} \quad (14)$$

with c_i being high-gain design parameters, and γ_j

is nonnegative values satisfying $\sum_{j=1}^N \gamma_j = 1$, which

can be calculated from

$$\gamma_j = \begin{cases} (1/J_j) / \sum_{j=1}^N (1/J_j) & J_j \neq 0, 1 \leq j \leq N \\ 1 & J_j = 0, 1 \leq j \leq N \end{cases} \quad (15)$$

where J_j is the performance indices of the form

$$J_j(t) = \lambda e_j^2(t) + \beta \int_0^t e_j^2(\tau) d\tau, \lambda \geq 0, \beta > 0, \quad (16)$$

Since the fixed models can represent exactly only a finite number of environments, an additional adaptive model is needed to improve accuracy asymptotically.

However, the system can be controlled only by one controller and then varying, so we do the multiple model controller on the system directly is impossible. In order to facilitate the multiple model adaptive controller, it is necessary to find an extra equivalent system to develop the controller, i.e. we should identify the system firstly.

3.3 Multiple Identification Models

We will run in parallel N identification models with the same structure which takes the different initial parameter values $\{\theta_j(0)\}_{j=1}^N$ uniformly distributed in the compact set S to which the unknown parameter belongs. Therefore at least one $\theta_j(0)$ is close to θ . We introduce the filters as follows:

$$\dot{\xi}_0 = (\mathbf{A}_0 - \lambda \Xi(\mathbf{x}) \Xi^T(\mathbf{x}) \mathbf{P})(\xi_0 - \mathbf{x}) + f(\mathbf{x}, u), \xi_0 \in \mathbb{R}^n, \quad (17)$$

$$\dot{\xi} = (\mathbf{A}_0 - \lambda \Xi(\mathbf{x}) \Xi^T(\mathbf{x}) \mathbf{P}) \xi + \Xi(\mathbf{x}), \xi \in \mathbb{R}^{n \times p}, \quad (18)$$

where

$$f(\mathbf{x}, u) = [x_2 \quad \dots \quad x_n \quad \beta(\mathbf{x})u]^T,$$

$$\Xi(\mathbf{x}) = [\varphi_1(x_1) \quad \dots \quad \varphi_{n-1}(\bar{x}_{n-1}) \quad \varphi_n(\mathbf{x})]^T, \quad (19)$$

$\lambda > 0$ and \mathbf{A}_0 is a Hurwitz matrix such that the Lyapunov equation $\mathbf{P}\mathbf{A}_0 + \mathbf{A}_0^T \mathbf{P} = -\mathbf{I}$ has a positive definite solution P.

Define

$$\tilde{e} = \mathbf{x} - \xi_0 - \xi \theta,$$

(20)

$$e_j = \mathbf{x} - \xi_0 - \xi \theta_j(0), j = 1, \dots, N,$$

$$\tilde{\theta}_j(0) = \theta - \theta_j(0), j = 1, \dots, N. \quad (21)$$

$$(22)$$

It can be derived from (1), (17)-(22) that

$$\dot{\tilde{e}} = (\mathbf{A}_0 - \lambda \Xi(\mathbf{x}) \Xi^T(\mathbf{x}) \mathbf{P}) \tilde{e}, \quad (23)$$

(23)

$$e_j = \xi \tilde{\theta}_j(0) + \tilde{e}, j = 1, \dots, N. \quad (24)$$

(24)

Due to \tilde{e} converges to zero exponentially, Eq.(24) is called identification error equations.

3.4 Stability Analysis

Theorem 1: Suppose the multiple model adaptive controller (9)-(13) and adaptive law (14) presented in this note is applied to system (1). Then, for all initial conditions, all closed-loop states are bounded on $[0, \infty)$ and asymptotic tracking can be achieved, i.e., $\lim_{t \rightarrow \infty} z(t) = 0$ or $y(t) = y_r(t)$ as $t \rightarrow \infty$.

Proof: we choose the whole candidate Lyapunov function as

$$V = \frac{1}{2} \sum_{i=1}^n z_i^2 + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^N (\theta - \theta_j(0) - \hat{\theta}_{i,j})^2 \quad (25)$$

It is obvious that

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n z_i [x_{i+1} + \sum_{j=1}^N \gamma_j (w_{i,j}^T \theta - \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1,j}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1,j}}{\partial y_r^{k-1}} y_r^k)) \\ &\quad - \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1,a}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1,j}}{\partial y_r^{k-1}} y_r^k)] + \frac{\partial \alpha_{i-1,j}}{\partial \hat{\theta}_{i-1,j}} \dot{\hat{\theta}}_{i-1,j} \\ &\quad - y_r^i - \sum_{j=1}^N \dot{\gamma}_j \alpha_{i-1,j} - \sum_{i=1}^n \sum_{j=1}^N (\theta - \theta_j(0) - \hat{\theta}_{i,j}) \dot{\hat{\theta}}_{i,j} \\ &= \sum_{i=1}^n z_i [-z_{i-1} - c_i z_i - \sum_{j=1}^N \gamma_j w_{i,j}^T \theta_j(0) + \sum_{j=1}^N \gamma_j w_{i,j}^T (\theta - \hat{\theta}_{i,j}) \\ &\quad + z_{i+1}] - \sum_{i=1}^n \sum_{j=1}^N (\theta - \theta_j(0) - \hat{\theta}_{i,j}) \dot{\hat{\theta}}_{i,j} \\ &= - \sum_{i=1}^n c_i z_i^2 \leq 0 \end{aligned} \tag{26}$$

This implies the boundedness of the states of z_i , $\hat{\theta}_{i,j}, 1 \leq i \leq n, 1 \leq j \leq N$, and which in turn indicates the boundedness of the states of $x_i, 1 \leq i \leq n$ and control u . Then using the standard arguments of adaptive control theory, it follows that $z_i(t), 1 \leq i \leq n$ tend to zero asymptotically with time, i.e. $\lim_{t \rightarrow \infty} z_i(t) = 0$, and thus $\lim_{t \rightarrow \infty} z_i(t) = \lim_{t \rightarrow \infty} (y(t) - y_r(t)) = 0$. The proof is completed.

4. SIMULATIONS

Consider the following second-order nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + \theta_1 x_1 + \theta_2 x_1^2 \\ \dot{x}_2 = u \\ y(t) = x_1(t) \end{cases} \tag{27}$$

where $\theta_1 \in [1, 5]$, and $\theta_2 \in [1, 40]$ are unknown parameters. The output $y(t) = x_1(t)$ is to asymptotically track the reference signal $y_r(t) = \sin 2t$.

In simulation, the parametric controller is developed as (9)-(14), with $c_1 = c_2 = 4$, $\lambda = \beta = 1$. Since in (27), the unknown parameter appears only in the first equation, the filter can be constructed as [1] to reduce filter dynamic order

$$\begin{aligned} \dot{\xi}_0 &= -c(\xi_0 - x) + x_2, \xi_0 \in \mathbb{R}^1, \\ \dot{\xi} &= -c\xi + [x_1, x_1^2], \xi \in \mathbb{R}^{1 \times 2}, \end{aligned}$$

where $c=10$. The unknown parameter is $[\theta_1, \theta_2]^T = [4.4, 38.5]^T$; the multiple identification models is $N=200$ (for convenience to comparison with the case developed in [12]); the initial plant state is $[x_1(0), x_2(0)]^T = [0.5, -10]^T$; the same initial

filter states are $\xi_0 = 0.5, \xi = [0, 0]$, and the initial estimates of parameter for classical adaptive control and multiple model adaptive control are $\hat{\theta} = [1, 1]^T$ and $\theta_1(0) = [1, 1]^T, \dots, \theta_{200}(0) = [5, 40]^T$ respectively. Figs 1~8 depict the simulation results which demonstrate that the transient performance using multiple identification models is significantly superior to that using the classical adaptive control when there exist large initial estimation errors. Figs.1~2 show that the output using the classical adaptive control is not only with the unimaginable overshoot but also having a slower convergence rate. Figs.3~4 demonstrate that the control input using the classical adaptive control is unbearable large at the start period and is still larger when the plant undergoes 20 units of time.

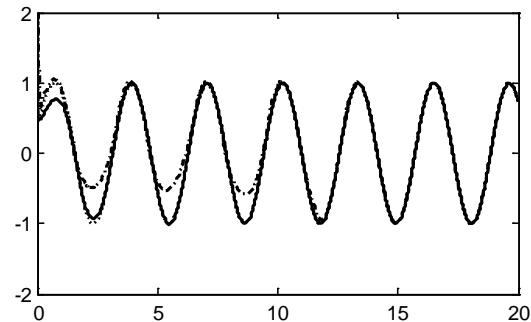


Figure1: Output $y(t)$: dash-dotted line for the classical adaptive control, dotted line for the case in [12], solid line for the no-switching multiple models adaptive control.

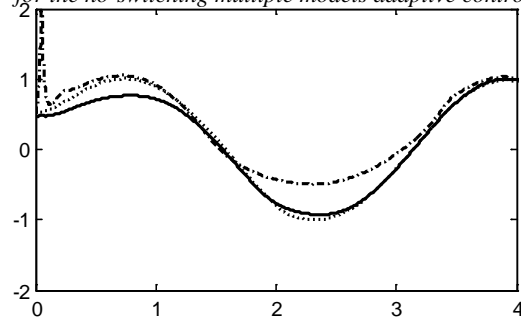


Figure2: Expanded time scale of Fig.1 on the time interval [0, 4]

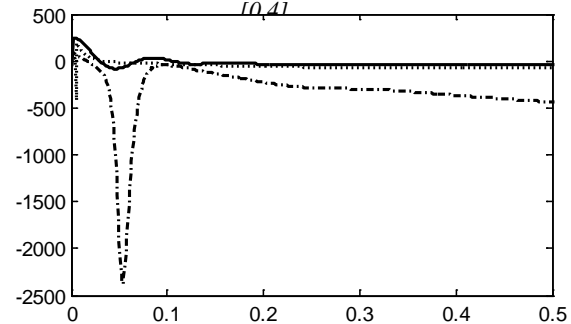


Figure3: Control $u(t)$ on the time interval [0 0.5]: dash-dotted line for the classical adaptive control, dotted line for the case developed in [12], solid line for the no-switching multiple models adaptive control.

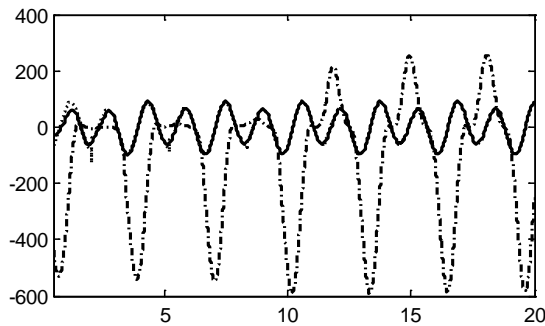


Figure4: Control $u(t)$ on the time interval [0.5 20].

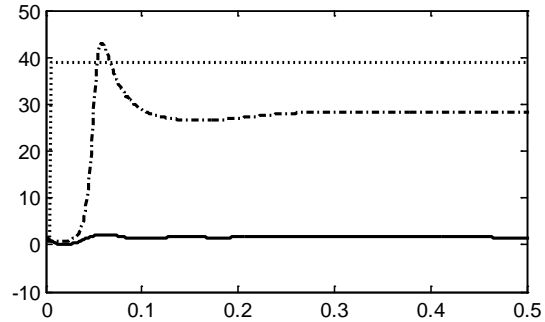


Figure8: Expand time scale of Fig.7 on the time interval [0 0.5]

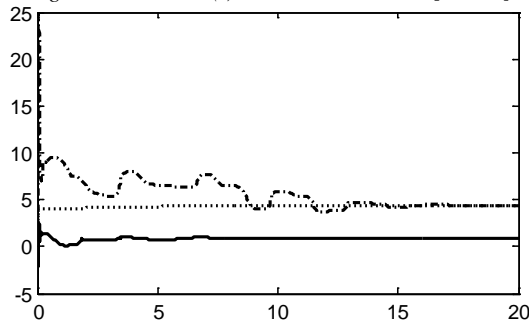


Figure5: Estimate parameter $\hat{\theta}_1(t)$: dash-dotted line for the classical adaptive control, dotted line for the case developed in [12], solid line for the no-switching multiple models adaptive control.

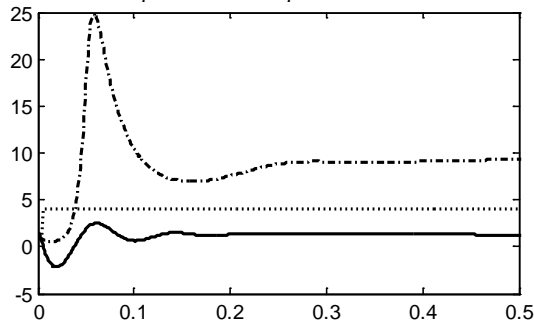


Figure6: Expand time scale of Fig.5 on the time interval [0 0.5]

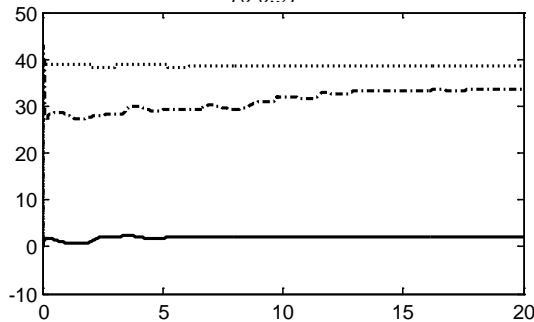


Figure7: Estimate parameter $\hat{\theta}_2(t)$: dash-dotted line for the classical adaptive control, dotted line for the case developed in [12], solid line for the no-switching multiple models adaptive control.

Figs.5~8 depict the estimates of parameter using the classical adaptive control are also inferior. From Figs.1~8, the multiple model adaptive control developed in [12] has nearly the same quality as the no-switching multiple model adaptive control proposed in this note. That phenomenon depends mainly on the large number of identification models, which can assure that the estimates of parameter can be find quickly with small error. While the large number of identification models is a burden on the computation and so is not easy to realize. Therefore, it is necessary to find another multiple model adaptive controller with less identification models and meanwhile retains the advantages mentioned above. Fortunately, we find the approach and call it the no-switching multiple model adaptive control. The advantage of the proposed approach will be simulated later as another example.

Next, we give another simulation with the choice as before but except that $N=15$, i.e., we reduce the identification models to 15. Figs.9~10 show that the output using the multiple model adaptive control developed in [12] is inferior to the output using no-switching multiple model adaptive control proposed in this note. The reason is due to the fact that the identification models are not so many to guarantee the estimates of the parameter can be found with the expected small errors. From the Fig.1 and Fig.9, we can find that the output with $N=200$ identification models developed in [12] can track the reference signal after 4 units of time, but the same quality needs at least 12 units of time when the identification models is reduced to 15. But when it comes to the approach presented in this note, the same quality can also be retained.

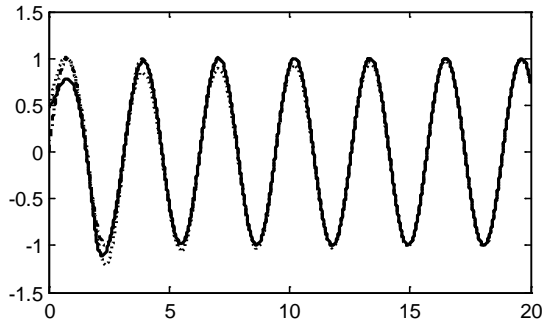


Fig.9 Output $y(t)$: dash-dotted line for the reference signal, dotted line for the case developed in [12] but with $N=15$, solid line for the no-switching multiple models adaptive control with $N=15$.

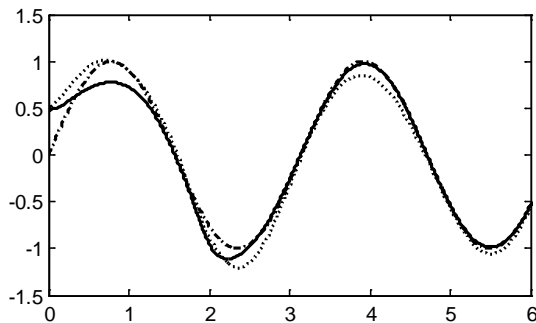


Fig.10 Expanded time scale of Fig.9 on the time interval [0 6].

5. CONCLUSIONS

In this note, a novel multiple model adaptive controller was developed for a class of nonlinear systems. The multiple model technique was used to describe the most appropriate model at different environments. By designing a blending instead of switching scheme, some models close to the real plant can be selected quickly, so that the transient performance can be improved significantly. Unlike previous results, we do not require a switching scheme to guarantee the most appropriate model to be switched into the controller design which can simplify the analysis of the stability of the closed-loop system. Besides, the global asymptotic stability of the closed-loop system is proved.

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