ACCURACY SYNTHESIS OF A 3-UCR PARALLEL ROBOT USING GENETIC ALGORITHMS

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ABSTRACT

The genetic algorithm is implemented to solve the problem of accuracy synthesis for a 3-UCR (Universal-Cylindrical-Revolute) parallel robot. Accuracy synthesis is one challenging topic for the parallel robot in the precision applications. First, the kinematic model and error model are built for the 3-UCR parallel robot. Then, the accuracy synthesis model based on manufacturing cost is formulated. The synthesis problem is converted to an optimization problem taking the tolerances of all error sources as design parameters, the manufacturing cost as objective and the accuracy requirements as the constraint condition. Finally, the genetic algorithm is used to solve this constraint and nonlinear optimization problem. The simulation of a 3-UCR parallel support for the telescope secondary mirror shows that genetic algorithm is an appropriate tool to achieve the accuracy synthesis based on manufacturing cost. This approach is practical and effective, and the synthesis results can satisfy the need of pose accuracy. The synthesis result is more economical and practical than accuracy synthesis based on sensitivity which cost twice as much.

Keywords: *3-UCR Parallel Robot, Error Model, Accuracy Synthesis, Manufacturing Costs, Genetic Algorithm*

1. INTRODUCTION

The kinematics accuracy is the chief factor for the parallel robots in the precision or ultra-precision applications, such as mirror mount of the telescope, micro surgical robot in the clinical medicine, positioning device in industry and other fields [1-2]. Different factors, such as a poor kinematic design, manufacturing tolerances, installation errors, link offsets, actuation, measurement and control error, may affect the kinematics accuracy. It appears so important to develop reliable and effective tools to analysis the sensitivity of different error sources, and optimally distribute the allowable tolerances for each error source under the given pose accuracy since the earlier mechanism design stages. This process is accuracy synthesis and it's an important and challenging topic.

Although tremendous efforts have been made and some remarkable results achieved in the accuracy synthesis for six DoF parallel robots [3-5], few effective results [6-9] have been achieved for the three DoF parallel robots which is much more

difficult because of the kinematic constraints and couple. A 3-UCR parallel robot could be implemented to tip, tilt and focus the secondary mirror allowing the image defects to be corrected. Here, the accuracy is a key factor and it's greatly valuable to conduct the accuracy synthesis.

A simple approach of accuracy synthesis could be implemented based on the error sensitivity, but the cost and difficulty in actual manufacturing could not be considered. There must be greater significance for practical applications if we shall take the manufacturing costs into consideration. However, the accuracy synthesis based on manufacturing costs is a nonlinear optimization problem which is difficult to obtain the solution using the ordinary optimization approach.

In this paper, the error model and the accuracy synthesis model of the 3-UCR parallel robot are built. Then, the genetic algorithm is adopted to solve the accuracy synthesis problem. Finally, a numerical simulation is given to verify the efficiency and practicability.

2. ERROR MODEL OF 3-UCR PARALLEL ROBOT

2.1 Kinematics Model of 3-UCR Parallel Robot

A 3-UCR parallel robot and its coordinate systems are shown as Figure 1. The 3-UCR Parallel Robot is composed of a fixed base and a moving platform connected by three symmetrical variable length branches. Each branch chain is attached to the moving platform by a revolute joint, to the fixed base by a universal joint and is driven by a prismatic actuator in the middle. The attachment points of the three branches form 2 equilateral triangles $a_1a_2a_3$ and $A_1A_2A_3$ with circumradiuses r_p and r_B. Using the screw theory in [10], one can know that the robot has one translation and two rotations degrees of freedom.
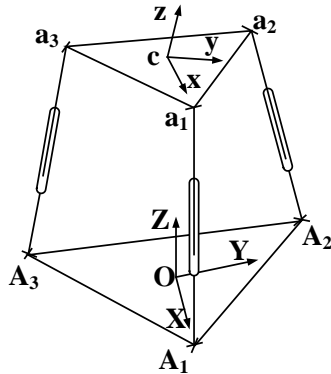

Fig.1. Coordinate Systems of a 3-UCR Parallel Robot

The fixed coordinate system OXYZ is placed in the fixed base with the center of triangle $A_1A_2A_3$ as original point, OA_1 as X-axis and A_3A_2 parallel to Y-axis. Similarly, the moving coordinate system c-xyz is placed in the center of the moving platform, where x-axis pointed to a_1 from c, y-axis parallel to a_3a_2. The unit vectors of the 3 expansion chains are n_1, n_2, n_3, and the lengths are l_1, l_2, l_3. If each universal joint is represented by two orthogonal revolute pairs R_{i1}, R_{i2}, each cylindrical joint is represented by one prismatic pair r_i and one revolute pair R_{i3}, each revolute joint is marked as R_{i4}, then the geometrical relationships are the following: $R_{i1} \perp A_iO$, $R_{i2} \perp R_{i1}$, $r_i \perp R_{i2}$, $R_{i3} \parallel r_i$, $R_{i4} \perp R_{i3}, R_{i4} \perp a_ic$.

It's easy to get the coordinates of A_i under the fixed coordinate system, and the coordinates of a_i^e

and the unit vectors u_i^e for the axis of the revolute joints under the cxyz coordinate system.

Assuming $p_c = (X_c, Y_c, Z_c)^T$ is the coordinate of point c in the fixed coordinate system. Assuming the rotation matrix describing the orientation of cxyz with respect to OXYZ has a form by

$$R = \begin{pmatrix} x_l & y_l & z_l \\ x_m & y_m & z_m \\ x_n & y_n & z_n \end{pmatrix} \quad (1)$$

The homogeneous transformation from cxyz to OXYZ is defined as

$$g = \begin{bmatrix} R & p_c \\ 0 & 1 \end{bmatrix} \quad (2)$$

Under the O-XYZ coordinate system, a_1, a_2, a_3 and u_1, u_2, u_3 can be expressed as

$$\begin{bmatrix} a_i \\ 1 \end{bmatrix} = g \begin{bmatrix} a_i^e \\ 1 \end{bmatrix} \quad \begin{bmatrix} u_i \\ 0 \end{bmatrix} = g \begin{bmatrix} u_i^e \\ 0 \end{bmatrix} \quad i = 1,2,3 \quad (3)$$

The kinematic vector of branch $i, i = 1,2,3$ can be expressed as：

$$l_i n = p_c + Ra_i^e - A_i = p_c - A_i + s_i \quad (4)$$

Because of the revolute joints' constraints, the motion direction of each chain must be perpendicular to the axis of the revolute joint. That is to say, $n_i \cdot u_i = u_i^T n_i = 0$. Then three constraint equations can be drawn:

$$y_l X_c + y_m Y_c + y_n Z_c - r_B y_l = 0$$

$$x_l X_c + x_m Y_c + x_n Z_c + \frac{x_l}{2} r_B - \frac{y_m}{2} r_B = 0 \quad (5)$$

$$y_l X_c + y_m Y_c + y_n Z_c + \frac{y_l}{2} r_B - \frac{3x_m}{2} r_B = 0$$

There are several approaches to describe the rotation between two coordinate systems. For the constrained 3-dof parallel robots, the problem of kinematics and accuracy analysis could be complex under improper describing approaches. Here, the set of z-x-z Euler angles is taken and the parameters α, β, γ represent the rotation angles around the z-axis, x-axis and z-axis in turn. Then the rotation matrix can be expressed as

$$R_{z,x,z}(\alpha, \beta, \gamma) = R_Z(\alpha) R_X(\beta) R_Z(\gamma) \quad (6)$$

Consider the constraint equations (5), then we can get $\gamma = -\alpha$. The rotation matrix would be

$$R = \begin{pmatrix} c^2\alpha + s^2\alpha c\beta & s\alpha c\alpha(1-c\beta) & s\alpha s\beta \\ s\alpha c\alpha(1-c\beta) & s^2\alpha + c^2\alpha c\beta & -c\alpha s\beta \\ -s\alpha s\beta & c\alpha s\beta & c\beta \end{pmatrix} \quad (7)$$

where s represents sin and c represents cos.

The rotation matrix equals a rotation β around the line which has an angle α from the positive x-axis. Then, only 2 angle parameters can be used to define the orientation of the moving platform.

Then the constraint equations are converted to:

$$X_c = \frac{2s\alpha s\beta Z_c + r_B(\upsilon\beta - c^2\alpha s^2\beta - s^2 2\alpha\upsilon^2\beta)}{2c\beta}$$

$$Y_c = -\frac{2c\alpha s\beta Z_c + r_B s\alpha c\alpha\upsilon\beta(1-3c\beta - 4c^2\alpha\upsilon\beta)}{2c\beta} \quad (8)$$

where $\upsilon\beta = 1 - \cos\beta$.

From (8), X_c and Y_c depend on (α, β, Z_c). Here, parameter vector (α, β, Z_c) is taken to describe the pose of a 3-UCR parallel robot.

2.2 Error Model of 3-UCR Parallel Robot

Now, we shall find out the relationship between the errors of the links and joints (input errors) and the errors of the moving platform (output errors). The input errors include: (1) the length errors δl_i caused by inaccurate measurements and backlash in the actuators, and (2) the position errors of the six joints $\delta p = [\delta a^e \ \delta A]$ in the moving and fixed platforms. The resulting output errors δx of the moving platform include the position and orientation errors as

$$\delta x = \begin{bmatrix} \delta p_c \\ \delta\theta \end{bmatrix} \quad (9)$$

where δp_c is the position error vector of point c, $\delta\theta = [\delta\theta_x \ \delta\theta_y \ \delta\theta_z]^T$ represents the orientation errors of the moving platform coordinate. We can get the error model of the 3-UCR robot.

$$\delta l = J_x \delta x + J_p \delta p \quad (10)$$

where δl represents the 3 length errors and δp represents the position errors of the 18 joints.

$$\delta l = \begin{pmatrix} \delta l_1 \\ \delta l_2 \\ \delta l_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \delta p = \begin{pmatrix} \delta a_1^e \\ \delta A_1 \\ \delta a_2^e \\ \delta A_2 \\ \delta a_3^e \\ \delta A_3 \end{pmatrix} \quad J_x = \begin{pmatrix} n_1^T & (s_1 \times n_1)^T \\ n_2^T & (s_2 \times n_2)^T \\ n_3^T & (s_3 \times n_3)^T \\ u_1^T & (s_1 \times u_1)^T \\ u_2^T & (s_2 \times u_2)^T \\ u_3^T & (s_3 \times u_3)^T \end{pmatrix} \quad (11)$$

$$J_p = \begin{pmatrix} n_1^T R & -n_1^T & 0 & 0 & 0 & 0 \\ 0 & 0 & n_2^T R & -n_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 & n_3^T R & -n_3^T \\ u_1^T R & -u_1^T & 0 & 0 & 0 & 0 \\ 0 & 0 & u_2^T R & -u_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 & u_3^T R & -u_3^T \end{pmatrix}$$

The accuracy synthesis is to find out the optimal distribution of the manufacturing tolerances among all the error sources under the given allowable pose accuracy of the moving platform. That is to say, we should calculate the boundary of δl and δp when the boundary of δx is given.

2.3 Error Models of Universal and Revolute Joints

Because of the consideration for manipulability dexterity and the imperfect manufacturing error, there exists an unavoidable relatively large clearance between revolute or universal joint members. Since the complexity of errors at each attachment point, we shall consider only the dominant joint clearance error for a simple analysis. The joint clearance error is a complex random variable and can be described by joint probability density function with independent parameters. Assume that the probability of the clearance character is equal at any time at any pose in the clearance space or on the boundary.
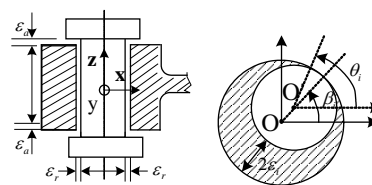

Fig.2 Clearance-affected Revolute Pair

The axial clearance of a revolute joint is usually small when compared with the radial clearance [3]. Assuming there is no relative deflection between the two adjacent components. Thereby clearance characteristic element is the projective point of the pin axe on the end face. And the clearance space is a circle with the radial clearance $r_{ri} \le \varepsilon_r$ as the radius and the projection point as centre. Based on

the condition of equal probability, the relative position deviation of the adjacent components caused by the clearance can be written as

$$\begin{cases} \delta x_i = r_{ri} \cdot \cos \alpha_i \\ \delta y_i = r_{ri} \cdot \sin \alpha_i \qquad i = 1,2,3 \\ \delta z_i = 0 \end{cases} \qquad (12)$$

where α_i obeys the uniform distribution in $(0, 2\pi)$ and r_{ri} obeys a normal distribution in $(0, \varepsilon_r)$.

The universal joint could be thought as two orthogonal revolute joints and the error model is similar. The error model is expressed in its own coordinate system and should be transferred to expressions in the fixed global coordinate system.

We can define the volume tolerance $\delta V_i = r_{ri}$ to calculate the manufacturing tolerance for revolute or universal joint. Then there is only one error parameter for a revolute joint and two parameters for a universal joint.

3. ACCURACY   SYNTHESIS   MODEL BASED ON MANUFACTURING COSTS

There must be greater significance for practical applications if we shall take the manufacturing costs function and the joints error models into consideration in accuracy synthesis.

Commonly tolerance is used to describe the accuracy of mechanical component. The smaller the tolerance is, the higher the manufacturing cost is. There are many mechanical cost-tolerance models [12], such as index model, power exponent model, negative square model, linear and index compound model, three and four polynomial model, etc., shown in the Figure 3.
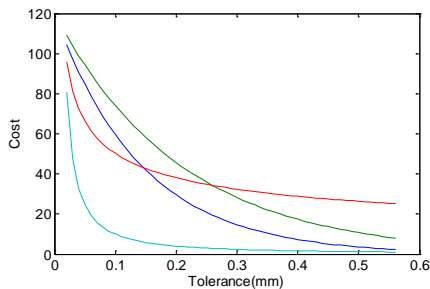

Fig.3 Tolerance-Manufacturing Cost Curve

Here, the index model is used:

$$f_i = a_{0i} e^{-a_{1i} x_i} \qquad (13)$$

where x_i is the boundary of each error source, the a_{0i}, a_{1i} are some constants.

Considering the applicability and economical, one should relax the tolerance boundary of error source while guaranteeing the pose error is under the given value. So we shall build an optimization model that takes the error sources values as design parameters, the total manufacturing cost as objective function, and the satisfaction of the pose errors as the constraint condition. Then the problem of accuracy synthesis of 3-UCR robot is transferred to a nonlinear constrained optimization problem:

$$\min f = \sum_{i=1}^{m} a_{0i} e^{-a_{1i} x_i} \qquad (14)$$
$$s.t. \ e_s < E$$

where $a_{0i}, a_{1i}, x_i, e_s, E$ are tolerance cost coefficient, bounds on error, pose error value obtained by branch chains error and the given boundary of the pose accuracy.

The problem is a multidimensional nonlinear constrained optimization problem, which is difficult to solve by ordinary optimization theory. The Genetic Algorithm (GA) is a good choice.

4. ACCURACY SYNTHESIS USING THE GENETIC ALGOTHIM

Genetic algorithm (GA), based on principles of natural selection and evolutionary genetics, is considered as one of the most popular optimization and search techniques. The main advantages of GA are its global optimization performance, the ease of distributing calculations among several processors, and it's simple and more applicable for the discontinuous problem than conventional gradient-based searching algorithms.

Given a problem for which a closed-form solution is unknown and difficult to obtain, the variables are coded as chromosomes (strings of binary or real numbers), and the possible solutions are represented as populations. The members of the final generation possess genetic information that corresponds to the best solution to the optimization problem. GA will gradually converge towards the best solution by using Darwinian-inspired natural evolutionary operations: selection, crossover, and mutation. Selection is processing to select the parent from a generation based on survival of the fittest (highest performance index). After the parent individuals are selected, the crossover process is then used to swap between two chromosomes by

specific probability. Mutation is the mechanism to prevent the algorithm from local optimal points by adding some random. The algorithm maintains a constant size of generation by selecting the fittest chromosomes from parents and children. The algorithm iteratively operates to converge.

The main implementation steps:

1. Initialize the population and set the parameters in the tolerance-cost expressions.

2. Calculate the fitness value of the population and decide whether it has reached the maximum cycle index. Output the results if reached and go to step 3 otherwise.

3. Selection: Choose the finest individual using roulette selection strategy.

4. Elitist: Elitist strategy is to allow some of the best chromosome from the current generation to carry over to the next, unaltered.

5. Crossover: Wright's heuristic crossover is implemented here, $g = \lambda g_1 + (1 - \lambda)g_2$.

6. Mutation. A random number in the range [a, b] is added to a selected gene where a, b are the highest and lowest values in the chromosome.

The idea of penalty for the constraint is adopted to avoid difficulty in the selection of fitness function. Then the fitness function is defined as:

$$f = \begin{cases} \sum_{i=1}^{m} a_{0i} e^{-a_{1i} x_i} , if \ e(x_i) < E \\ \sum_{i=1}^{m} a_{0i} e^{-a_{1i} x_i} + k, otherwise \end{cases} \qquad (15)$$

where k is a large constant.

If the calculated kinematic errors satisfy the given accuracy boundaries, then the fitness function equals the manufacturing cost function. Otherwise, an increment adds through the penalty term.

5. SIMULATION

Now, the accuracy synthesis of a 3-UCR parallel support for the telescope secondary mirror is considered. The GA is used to solve the synthesis based on manufacturing cost, and the results are compared to that based on error sensitive.

The coordinate system of the 3-UCR parallel robot is shown as Figure 1. Assuming the circum-radius of top and bottom platforms are $r_p = 0.1m$ and $r_B = 0.14m$. The required workspace is:

$\alpha \in [-5°, 5°], \theta \in [-5°, 5°], Z_c \in [0.255, 0.305]m$

The allowable position accuracy of the moving platform is $5 \times 10^{-5} m$.

Considering the kinematic joint's error model, the error sources can be expressed by three length errors, nine revolute joints errors. Therefore the number of errors has sharply reduced. The cost parameters for length error are $a_0 = 1.9, a_1 = 0.5$, and for revolute joint error are $a_0 = 1.1, a_1 = 0.6$.

First, GA is used to complete the accuracy synthesis of the 3-UCR parallel robot. The cost convergence curve is shown in Figure 4. The initial cost is 2.7320 and the final value is 2.1837.
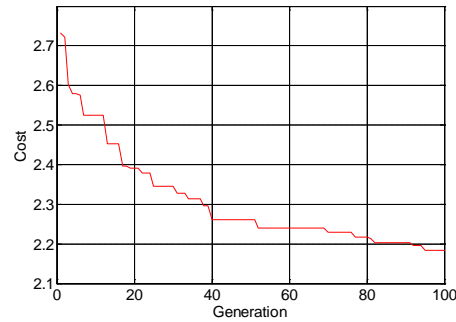

Fig.4 Convergence of Manufacturing Cost Using GA

Next, the Monte Carlo statistics method is used to check the viability of accuracy synthesis. At each pose in the workspace, 1000 groups of random input errors are produced and then the number of times that the output errors exceed the allowable error boundary is calculated. The exceeding rates for all required orientations at $Z_c = 0.305m$ are showed in Figure 5. Though the values are random, all of them are under 10% . Similarly, the exceeding rates can be calculated for all z position. The values are random and none of them exceed 10% . That indicates the synthesis results can satisfy the accuracy requirement.
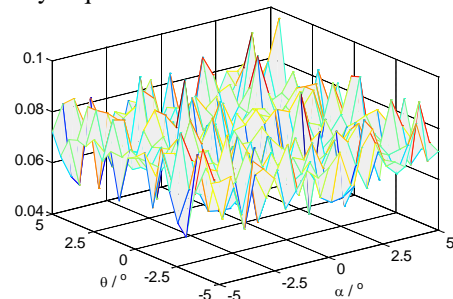

Fig.5 Verification Curve Using the MC Method

Finally, the synthesis results are compared to that based on error sensitivity. Table 1 shows the

maximum allowance manufacturing tolerances of all error sources which are calculated using the GA. The synthesis results based on sensitivity are given in Table 2.

Table 1: The Maximum Allowance Manufacturing Tolerance using GA ($10^{-5} m$)

	Branch 1	Branch 2	Branch 3
Lengths	4.94	4.81	4.96
U joints	2.94	2.93	2.97
	2.95	2.96	2.94
R joints	2.98	2.93	2.89

Table 2: Accuracy Synthesis Results based on Error Sensitivity ($10^{-5} m$)

	Branch 1	Branch 2	Branch 3
Lengths	1.38	1.38	1.38
U joints	2.21	2.19	2.17
	4.29	4.34	4.31
R joints	4.30	4.37	4.32

The calculated results show that the accuracy synthesis based on manufacturing costs has much stricter constants on the kinematic joints than that based on sensitivity. Compared to the joint error, it's more costly to reduce the length error which includes the actuator errors, the sensor errors, the controller errors, and so on. And the manufacturing cost value is 4.2391 for accuracy synthesis base on error sensitive which is nearly twice as much as that using GA. So, the accuracy synthesis using GA which is based on manufacturing costs has more practical significance.

6. CONCLUSION

In this paper, the relationship between the pose errors and the length errors and the joint errors was derived and the accuracy synthesis model was built for a 3-UCR parallel robot. Then the accuracy synthesis problem was converted to an optimization problem, and genetic algorithm was implemented to solve this multidimensional nonlinear constrained optimization problem.

The simulation using the 3-UCR parallel support for the telescope secondary mirror showed that genetic algorithm is an appropriate tool to achieve the accuracy synthesis based on manufacturing cost. The synthesis results could satisfy the need of pose accuracy. The percent of output errors exceeding the allowable error boundary at any pose was under 10%. The synthesis results were more economical than synthesis based on sensitivity which cost twice as much.

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