Journal of Theoretical and Applied Information Technology
10th June 2013. Vol. 52 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

NEW STABILITY CONDITIONS FOR NEUTRAL SYSTEMS WITH DISTRIBUTED DELAYS

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ABSTRACT

The certain and uncertain neutral systems with distributed delays are investigated in this paper. The uncertainties under consideration are time-varying, but norm bounded. By the delay-dividing approach, firstly new asymptotic stability conditions for the certain neutral systems with distributed delays are given by Lyapunov method. Then the asymptotic stability conditions for the uncertain case are obtained subsequently. Numerical examples illustrate that the proposed criteria are effective and lead to less conservative results than existing ones.
Keywords: *Neutral Systems, Robust Stability, Distributed Delays, Asymptotic Stability*

1. INTRODUCTION

The problem of stability of time-delay systems of neutral type has received considerable attention in the last two decades. To start with, the delay-dependent stability criteria can be seen in [4, 8] by the model transformation method. Next, matrix decomposition method is applied to neutral systems in [1, 11]. This method divides the delayed terms into two groups (the stabilizing ones and destabilizing ones) and enables one to consider the stabilizing effect of part of the delayed terms, which caused less conservative results than before. Besides, discrete delay bi-decomposition approach is introduced in [13] on the basis of the results in [12].

However, for some systems, delay phenomena may not be simply considered as delays in the velocity terms or discrete delays in the states. Therefore, it is desirable to extend the system model to include distributed delays and stability analysis for them is of both practical and theoretical importance. In the recent papers by Chen & Zheng [14] and Han [11], a descriptor system approach (see Fridman [2] and [3]) has been used to investigate the stability of neutral systems with discrete and distributed delays. Han [11] rewrites the discrete-delay term and employs a decomposition technique [1]. Different from Han [11], Chen and Zheng [14] rewrite both the discrete-delay and the distributed-delay terms and apply Moon's inequality [16]. In addition, since a new form of Lyapunov functional including some triple-integral terms, discrete, neutral, and distributed-delay dependent criteria have been proposed, the augmented vector and triple-integral terms play key roles in the reduction of conservativeness [6].

In this paper, asymptotic stability of uncertain neutral systems with discrete and distributed delays are considered. The delay decomposition approach is used and a novel Lyapunov functional is proposed. Not only discrete delay interval but also neutral delay interval are divided, at the same time a triple-integral term is employed for distributed delays. By linear matrix inequality approach, new stability criteria for neutral systems with distributed delays are derived. The resultant stability criteria are less conservative. Numerical examples are given to show the reduced conservativeness.

The remainder of this paper is organized as follows: **Section 2** contains the problem statement and preliminaries; **Section 3** presents the main results; **Section 4** provides two numerical examples to verify the effectiveness of the results; **Section 5** draws a brief conclusion.

1.1 *Notations*

Throughout this paper, $*$ denotes the elements below the main diagonal of a symmetric block matrix, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m\times n}$ is for the set of all $m\times n$ matrices. A^T and A^{-1} denote the transpose and the inverse of a matrix A. The notation $X \geq Y$ (or $X > Y$) means

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

that X and Y are symmetric matrices, and that $X-Y$ is positive semidefinite (or positive definite). In this paper $\|*\|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, $\|A\|$ is its spectral norm. *i.e.*

$$\|A\| = \sup\{\|Ax\|: \ \|x\| = 1\} = \sqrt{\lambda_{\max}(A^T A)}$$

where $\lambda_{\max}(A)$ (or $\lambda_{\min}(A)$) means the largest (or smallest) eigenvalue of the matrix A. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider a class of neutral systems with discrete and distributed delays which are described by the following:

$$\dot{x}(t) - D(t)\dot{x}(t-\tau) = A(t)x(t) + B(t)x(t-h) + C(t)\int_{t-r}^{t} x(s)ds \qquad (1)$$

$$x(t) = \varphi(t), \quad t \in [-\rho, 0] \qquad (2)$$

where $x(t)$ is the state, $\tau > 0,\ h > 0$ and $r \ge 0\ (r \ne \tau, h)$ are constant neutral, discrete and distributed delays. $\rho = \max\{\tau, h, r\}$ and the initial condition $\varphi(t)$ is a continuously differentiable vector-valued function. The continuous norm of $\varphi(t)$ is defined as

$$\|\varphi\|_c = \max_{t\in[-\rho,0]} |\varphi(t)|.$$

$A(t) \in \mathbb{R}^{n\times n}, B(t) \in \mathbb{R}^{n\times n}, C(t) \in \mathbb{R}^{n\times n}, D(t) \in \mathbb{R}^{n\times n}$ are uncertain matrices. We assume that uncertainties are norm-bounded and can be described as:

$$\begin{aligned} A(t) &= A + \Delta A(t) \\ B(t) &= B + \Delta B(t) \\ C(t) &= C + \Delta C(t) \\ D(t) &= D + \Delta D(t) \end{aligned} \qquad (3)$$

where A, B, C, D are known constant matrices; The admissible uncertainties are assumed to satisfy the following condition:

$$\left[\Delta A(t)\ \Delta B(t)\ \Delta C(t)\ \Delta D(t)\right] = HF(t)\left[E_A\ E_B\ E_C\ E_D\right] \qquad (4)$$

where H, E_A, E_B, E_C, E_D are known constant matrices with appropriate dimensions and $F(t)$ is an unknown and time-varying matrix satisfying:

$$F^T(t)F(t) \le I, \quad \forall t \qquad (5)$$

Throughout this paper, we assume that the matrix $D(t)$ is Schur stable. Considering $F(t) = 0$, we get the nominal neutral system with distributed delays as follows:

$$\dot{x}(t) - D\dot{x}(t-\tau) = Ax(t) + Bx(t-h) + C\int_{t-r}^{t} x(s)ds \qquad (6)$$

In this paper, we will firstly consider the asymptotic stability for the nominal system (6) and obtain the promising results for it. Then we will extend to analyze the system of (1) and deduce the homologous conclusions. In order to analyze the asymptotic and robust stability of the neutral system with distributed delay, the following inequalities and integral inequalities are required. They are stated in the lemmas given below.

Lemma 2.1: [5] Given a symmetric block matrix described by the following :

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$$

where S_{11}, S_{12} and S_{22} are proper dimensions. Then the following three conditions are equivalent:

$$(i) S < 0$$
$$(ii) S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$$
$$(iii) S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$$

Lemma 2.2: [5] Let $U \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{R}^{n\times n}$, then we have

$$x^T UVx \le \|UV\| x^T x \le \|U\|\|V\| x^T x, \quad \forall x \in \mathbb{R}^n$$

Lemma 2.3: [9] For any constant matrix $H \in \mathbb{R}^{n\times n}$, $H > 0$ and scalar $\gamma > 0$, vector function

$$\omega: [0,\gamma] \to \mathbb{R}^n$$

such that the integrations $\int_0^\gamma \omega^T(s)H\omega(s)ds$ and $\int_0^\gamma \omega^T(s)ds$ are well defined, then the following inequality holds:

$$-\int_0^\gamma \omega^T(s)H\omega(s)ds \le -\frac{1}{\gamma}\left[\int_0^\gamma \omega^T(s)ds\right] H \left[\int_0^\gamma \omega(s)ds\right]$$

Lemma 2.4: [7] For any constant matrix

$$H \in \mathbb{R}^{n\times n},\ H = H^T > 0$$

scalar $\gamma > 0$ vector function $\omega: [0,\gamma] \to \mathbb{R}^n$ such that the integrations in the following are well defined, then the inequality holds:

$$-\int_{-\gamma}^{0}\int_{t+\theta}^{t} \omega^T(s)H\omega(s)dsd\theta \le -\frac{1}{\gamma^2}\left[\int_{-\gamma}^{0}\int_{t+\theta}^{t} \omega^T(s)dsd\theta\right] H \left[\int_{-\gamma}^{0}\int_{t+\theta}^{t} \omega(s)dsd\theta\right]$$

Lemma 2.5: [10] For given matrices $Q = Q^T$, M and N with appropriate dimensions, then

$$Q + MF(t)N + N^T F^T(t) M^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \le I$, if and only if there exists a scalar $\delta > 0$, such that

$$Q + \delta^{-1} MM^T + \delta N^T N < 0$$

The objective of this paper is to further reduce the conservatism of the stability conditions for uncertain neutral systems with distributed delays to ensure a larger maximum upper bound on the delay.

Journal of Theoretical and Applied Information Technology
10th June 2013. Vol. 52 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

3. MAIN RESULTS

In this section, we study the asymptotic stability for the system (1) and (6) based on time domain approach. A new Lyapunov-Krasovskii functional is utilized and the proposed stability criterion is discrete-, distributed- and neutral-delay-dependent.

3.1 *Results on The Nominal Neutral System*

For the asymptotic stability for system (6), we have the following result.

Theorem 3.1: For given scalars τ, h and r, the neutral system with discrete and distributed delays described by (6) is asymptotically stable, if there exist matrices

$$P=[P_{ij}]_{5\times5}>0, Q=[Q_{ij}]_{2\times2}>0$$
$$V=\left[V_{ij}\right]_{2\times2}>0, R=\left[R_{ij}\right]_{2\times2}>0, W=\left[W_{ij}\right]_{2\times2}>0$$
$$X_1>0, X_2>0, Y_1>0, Y_2>0, Z_1>0, Z_2>0, Z_3>0$$

with appropriate dimensions such that

$$\Theta+\Lambda_1^TP\Lambda_2+\Lambda_2^TP\Lambda_1+L^TML<0 \qquad (7)$$

where

$$\Theta=\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} & 0 & 0 & 0 & \Theta_{110} \\ * & \Theta_{22} & \Theta_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Theta_{33} & 0 & 0 & \Theta_{36} & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_{44} & \Theta_{45} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & 0 & \Theta_{57} & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Theta_{88} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Theta_{1010} \end{bmatrix}$$

$$\Lambda_1=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$\Lambda_2=\begin{bmatrix} A & 0 & 0 & 0 & B & D & 0 & 0 & 0 & C \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 & 0 & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
\Theta_{11}&=Q_{11}+Q_{12}A+A^TQ_{21}+R_{11}+R_{12}A+A^TR_{21}\\
&\quad+V_{11}-X_1-X_2+W_{11}-Y_1-Y_2+Z_1+r^2Z_2-r^2Z_3\\
\Theta_{12}&=V_{12}+X_2\\
\Theta_{13}&=X_1\\
\Theta_{14}&=W_{12}+Y_2\\
\Theta_{15}&=Y_1+Q_{12}B+R_{12}B\\
\Theta_{16}&=Q_{12}D+R_{12}D\\
\Theta_{110}&=rZ_3+Q_{12}C+R_{12}C\\
\Theta_{22}&=V_{22}-V_{11}-X_2\\
\Theta_{23}&=-V_{12}\\
\Theta_{33}&=-Q_{11}-V_{22}-X_1\\
\Theta_{36}&=-Q_{12}\\
\Theta_{44}&=W_{22}-W_{11}-Y_2\\
\Theta_{45}&=-W_{12}\\
\Theta_{55}&=-R_{11}-W_{22}-Y_1\\
\Theta_{57}&=-R_{12}\\
\Theta_{66}&=-Q_{22}\\
\Theta_{77}&=-R_{22}\\
\Theta_{88}&=-Z_1\\
\Theta_{1010}&=-Z_2-Z_3
\end{aligned}$$

$$L=\left[A\ 0\ 0\ 0\ B\ D\ 0\ 0\ 0\ C\right]$$

$$M=Q_{22}+R_{22}+\tau^2X_1+\frac{\tau^2}{4}X_2+h^2Y_1+\frac{h^2}{4}Y_2+\frac{r^4}{4}Z_3$$

Proof: Define a legitimate Lyapunov functional candidate as

$$V(x(t))=V_1(x(t))+V_2(x(t))+V_3(x(t))+V_4(x(t)) \quad (8)$$

where

$$V_1(x(t))=\eta^T(t)P\eta(t) \qquad (9)$$

$$\begin{aligned}
V_2(x(t))&=\int_{t-\tau}^{t}\left[x^T(s)\ \dot{x}^T(s)\right]Q\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
&+\int_{t-\frac{\tau}{2}}^{t}\left[x^T(s)\ x^T(s-\frac{\tau}{2})\right]V\begin{bmatrix}x(s)\\ x(s-\frac{\tau}{2})\end{bmatrix}ds \qquad (10)\\
&+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(\tau X_1)\dot{x}(s)dsd\theta\\
&+\int_{-\frac{\tau}{2}}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(\frac{\tau}{2}X_2)\dot{x}(s)dsd\theta
\end{aligned}$$

$$\begin{aligned}
V_3(x(t))&=\int_{t-h}^{t}\left[x^T(s)\ \dot{x}^T(s)\right]R\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
&+\int_{t-\frac{h}{2}}^{t}\left[x^T(s)\ x^T(s-\frac{h}{2})\right]W\begin{bmatrix}x(s)\\ x(s-\frac{h}{2})\end{bmatrix}ds \qquad (11)\\
&+\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(hY_1)\dot{x}(s)dsd\theta\\
&+\int_{-\frac{h}{2}}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(\frac{h}{2}Y_2)\dot{x}(s)dsd\theta
\end{aligned}$$

$$\begin{aligned}
V_4(x(t))&=\int_{t-r}^{t}x^T(s)Z_1x(s)ds\\
&+\int_{-r}^{0}\int_{t+\theta}^{t}x^T(s)(rZ_2)x(s)dsd\theta \qquad (12)\\
&+\int_{-r}^{0}\int_{\theta}^{0}\int_{t+u}^{t}\dot{x}^T(s)(\frac{r^2}{2}Z_3)\dot{x}(s)dsdud\theta
\end{aligned}$$

where P,Q,R,V,W,X_i,Y_i,Z_j, (i=1,2 and j=1,2,3) are defined as in *Theorem* 3.1.

Journal of Theoretical and Applied Information Technology
10th June 2013. Vol. 52 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\eta^T(t)=\left[x^T(t)\ x^T(t-\tau)\ x^T(t-h)\ \int_{t-\tau}^{t}x^T(s)ds\ \int_{t-r}^{t}x^T(s)ds\right]$$

The time derivative of $V(x(t))$ along the trajectory of system (8) is given by

$$\dot V(x(t))=\dot V_1(x(t))+\dot V_2(x(t))+\dot V_3(x(t))+\dot V_4(x(t)) \quad (13)$$

In fact, $\dot V_1(x(t))$, $\dot V_2(x(t))$, $\dot V_3(x(t))$ and $\dot V_4(x(t))$ can be computed and estimated as follows:

$$\begin{aligned}\dot V_1(x(t))&=\dot\eta^T(t)P\eta(t)+\eta^T(t)P\dot\eta(t)\\&=\xi^T(t)(\Lambda_1^TP\Lambda_2+\Lambda_2^TP\Lambda_1)\xi(t)\end{aligned} \quad (14)$$

where Λ_1 and Λ_2 have been defined before,

$$\begin{aligned}\xi^T(t)=\Big[&x^T(t)\ x^T(t-\tfrac{\tau}{2})\ x^T(t-\tau)\ x^T(t-\tfrac{h}{2})\ x^T(t-h)\ \dot x^T(t-\tau)\\&\dot x^T(t-h)\ x^T(t-r)\ \int_{t-\tau}^{t}x^T(s)ds\ \int_{t-r}^{t}x^T(s)ds\Big]\end{aligned}$$

Then we consider the time derivative of $V_2(x(t))$, we have

$$\begin{aligned}\dot V_2(x(t))=&\left[x^T(t)\ \dot x^T(t)\right]Q\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}-\left[x^T(t-\tau)\ \dot x^T(t-\tau)\right]Q\begin{bmatrix}x(t-\tau)\\ \dot x(t-\tau)\end{bmatrix}\\&+\left[x^T(t)\ x^T(t-\tfrac{\tau}{2})\right]V\begin{bmatrix}x(t)\\ x(t-\tfrac{\tau}{2})\end{bmatrix}-\left[x^T(t-\tfrac{\tau}{2})\ x^T(t-\tau)\right]V\begin{bmatrix}x(t-\tfrac{\tau}{2})\\ x(t-\tau)\end{bmatrix}\\&+\dot x^T(t)(\tau^2X_1+\frac{\tau^2}{4}X_2)\dot x(t)-\int_{t-\tau}^{t}\dot x^T(s)(\tau X_1)\dot x(s)ds\\&-\int_{t-\frac{\tau}{2}}^{t}\dot x^T(s)(\frac{\tau}{2}X_2)\dot x(s)ds\end{aligned}$$

Using *Lemma* 2.3 yields

$$\begin{aligned}\dot V_2(x(t))\le&\left[x^T(t)\ \dot x^T(t)\right]Q\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}-\left[x^T(t-\tau)\ \dot x^T(t-\tau)\right]Q\begin{bmatrix}x(t-\tau)\\ \dot x(t-\tau)\end{bmatrix}\\&+\left[x^T(t)\ x^T(t-\tfrac{\tau}{2})\right]V\begin{bmatrix}x(t)\\ x(t-\tfrac{\tau}{2})\end{bmatrix}-\left[x^T(t-\tfrac{\tau}{2})\ x^T(t-\tau)\right]V\begin{bmatrix}x(t-\tfrac{\tau}{2})\\ x(t-\tau)\end{bmatrix}\\&+\dot x^T(t)(\tau^2X_1+\frac{\tau^2}{4}X_2)\dot x(t)-x^T(t)(X_1+X_2)x(t)-x^T(t-\tau)X_1x(t-\tau)\\&-x^T(t-\tfrac{\tau}{2})X_2x(t-\tfrac{\tau}{2})+2x^T(t)X_1x(t-\tau)+2x^T(t)X_2x(t-\tfrac{\tau}{2})\end{aligned} \quad (15)$$

Similarly we obtain

$$\begin{aligned}\dot V_3(x(t))\le&\left[x^T(t)\ \dot x^T(t)\right]R\begin{bmatrix}x(t)\\ \dot x(t)\end{bmatrix}-\left[x^T(t-h)\ \dot x^T(t-h)\right]R\begin{bmatrix}x(t-h)\\ \dot x(t-h)\end{bmatrix}\\&+\left[x^T(t)\ x^T(t-\tfrac{h}{2})\right]W\begin{bmatrix}x(t)\\ x(t-\tfrac{h}{2})\end{bmatrix}-\left[x^T(t-\tfrac{h}{2})\ x^T(t-h)\right]W\begin{bmatrix}x(t-\tfrac{h}{2})\\ x(t-h)\end{bmatrix}\\&+\dot x^T(t)(h^2Y_1+\frac{h^2}{4}Y_2)\dot x(t)-x^T(t)(Y_1+Y_2)x(t)-x^T(t-h)Y_1x(t-h)\\&-x^T(t-\tfrac{h}{2})Y_2x(t-\tfrac{h}{2})+2x^T(t)Y_1x(t-h)+2x^T(t)Y_2x(t-\tfrac{h}{2})\end{aligned} \quad (16)$$

Subsequently, we consider $V_4(x(t))$ in the following:

$$\begin{aligned}\dot V_4(x(t))=&x^T(t)Z_1x(t)-x^T(t-r)Z_1x(t-r)+x^T(t)(r^2Z_2)x(t)\\&+\dot x^T(t)\left(\frac{r^4}{4}Z_3\right)\dot x(t)-\int_{t-r}^{t}x^T(s)(rZ_2)x(s)ds\\&-\int_{-r}^{0}\int_{t+\theta}^{t}\dot x^T(s)\left(\frac{r^2}{2}Z_3\right)\dot x(s)dsd\theta\end{aligned}$$

By *Lemma* 2.3 and *Lemma* 2.4

$$\begin{aligned}\dot V_4(x(t))\le&x^T(t)\left[Z_1+r^2Z_2-r^2Z_3\right]x(t)-x^T(t-r)Z_1x(t-r)\\&+\dot x^T(t)\left(\frac{r^4}{4}Z_3\right)\dot x(t)+2x^T(t)(rZ_3)\left[\int_{t-r}^{t}x(s)ds\right]\\&-\left[\int_{t-r}^{t}x^T(s)ds\right](Z_2+Z_3)\left[\int_{t-r}^{t}x(s)ds\right]\end{aligned} \quad (17)$$

Substituting (14), (15), (16) and (17) into (13) and the time-derivative of $V(x(t))$ has new upper bound as follows:

$$\dot V(x(t))\le\xi^T(t)(\Lambda_1^TP\Lambda_2+\Lambda_2^TP\Lambda_1)\xi(t)+\xi^T(t)\Theta\xi(t)+\dot x^T(t)M\dot x(t)$$

Further consideration on $\dot x^T(t)M\dot x(t)$, we have

$$\dot V(x(t))\le\xi^T(t)\left[\Theta+\Lambda_1^TP\Lambda_2+\Lambda_2^TP\Lambda_1+L^TML\right]\xi(t)$$

Therefore, if there exist matrices

$$\begin{aligned}&P=[P_{ij}]_{5\times5}>0,Q=[Q_{ij}]_{2\times2}>0\\&V=\left[V_{ij}\right]_{2\times2}>0,R=\left[R_{ij}\right]_{2\times2}>0,W=\left[W_{ij}\right]_{2\times2}>0\\&X_1>0,X_2>0,Y_1>0,Y_2>0,Z_1>0,Z_2>0,Z_3>0\end{aligned}$$

with appropriate dimensions such that (7), then we have $\dot V(x(t))<0$ which guarantees system (8) is asymptotically stable. This completes the proof.■

Remark 3.1: By dividing the neutral delay interval $[-\tau,0]$ into $[-\tau,-\frac{\tau}{2}]$ and $[-\frac{\tau}{2},0]$, then different functional were chosen on each subinterval. We have performed a similar trick with discrete delay interval $[-h,0]$ and we can see that the division provides extra freedom for neutral delay terms and reduces the conservation.

Furthermore, asymptotic stability conditions for $r=0$ and $\tau\equiv h$ in the system (6) are obtained on the basis of *Theorem* 3.1. In this paper we will show them in the following *Corollary* 3.1 and *Corollary* 3.2.

Corollary 3.1: For given scalars τ,h and $r=0$, the neutral system with distributed delays described by (6) is asymptotically stable, if there exist matrices

$$\begin{aligned}&P=[P_{ij}]_{5\times5}>0,Q=[Q_{ij}]_{2\times2}>0\\&V=\left[V_{ij}\right]_{2\times2}>0,R=\left[R_{ij}\right]_{2\times2}>0,W=\left[W_{ij}\right]_{2\times2}>0\\&X_1>0,X_2>0,Y_1>0,Y_2>0\end{aligned}$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

with appropriate dimensions such that

$$\bar{\Theta}+\bar{\Lambda}_1^T P\bar{\Lambda}_2+\bar{\Lambda}_2^T P\bar{\Lambda}_1+\bar{L}^T\bar{M}\bar{L}<0 \tag{18}$$

where

$$\bar{\Lambda}_1=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$\bar{\Lambda}_2=\begin{bmatrix} A & 0 & 0 & 0 & B & D & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0\\ I & 0 & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{L}=[A\ 0\ 0\ 0\ B\ D\ 0\ 0]$$

$$\bar{M}=Q_{22}+R_{22}+\tau^2X_1+\frac{\tau^2}{4}X_2+h^2Y_1+\frac{h^2}{4}Y_2$$

$$\bar{\Theta}=\begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} & \bar{\Theta}_{13} & \bar{\Theta}_{14} & \bar{\Theta}_{15} & \bar{\Theta}_{16} & 0 & 0\\ * & \bar{\Theta}_{22} & \bar{\Theta}_{23} & 0 & 0 & 0 & 0 & 0\\ * & * & \bar{\Theta}_{33} & 0 & 0 & \bar{\Theta}_{36} & 0 & 0\\ * & * & * & \bar{\Theta}_{44} & \bar{\Theta}_{45} & 0 & 0 & 0\\ * & * & * & * & \bar{\Theta}_{55} & 0 & \bar{\Theta}_{57} & 0\\ * & * & * & * & * & \bar{\Theta}_{66} & 0 & 0\\ * & * & * & * & * & * & \bar{\Theta}_{77} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
\bar{\Theta}_{11}&=Q_{11}+Q_{12}A+A^TQ_{21}+R_{11}+R_{12}A+A^TR_{21}\\
&\quad+V_{11}-X_1-X_2+W_{11}-Y_1-Y_2\\
\bar{\Theta}_{12}&=V_{12}+X_2\\
\bar{\Theta}_{13}&=X_1\\
\bar{\Theta}_{14}&=W_{12}+Y_2\\
\bar{\Theta}_{15}&=Y_1+Q_{12}B+R_{12}B\\
\bar{\Theta}_{16}&=Q_{12}D+R_{12}D\\
\bar{\Theta}_{22}&=V_{22}-V_{11}-X_2\\
\bar{\Theta}_{23}&=-V_{12}\\
\bar{\Theta}_{33}&=-Q_{11}-V_{22}-X_1\\
\bar{\Theta}_{36}&=-Q_{12}\\
\bar{\Theta}_{44}&=W_{22}-W_{11}-Y_2\\
\bar{\Theta}_{45}&=-W_{12}\\
\bar{\Theta}_{55}&=-R_{11}-W_{22}-Y_1\\
\bar{\Theta}_{57}&=-R_{12}\\
\bar{\Theta}_{66}&=-Q_{22}\\
\bar{\Theta}_{77}&=-R_{22}
\end{aligned}$$

Corollary 3.2: For given scalars $\tau\equiv h$ and r, the neutral system with distributed delays described by (6) is asymptotically stable, if there exist matrices

$$P=[P_{ij}]_{5\times5}>0, Q=[Q_{ij}]_{2\times2}>0, V=\left[V_{ij}\right]_{2\times2}>0$$
$$X_1>0, X_2>0, Y_1>0, Y_2>0, Z_1>0, Z_2>0, Z_3>0$$

with appropriate dimensions such that

$$\hat{\Theta}+\hat{\Lambda}_1^T P\hat{\Lambda}_2+\hat{\Lambda}_2^T P\hat{\Lambda}_1+\hat{L}^T\hat{M}\hat{L}<0 \tag{19}$$

where

$$\hat{L}=[A\ 0\ B\ D\ 0\ 0\ C]$$

$$\hat{M}=Q_{22}+\tau^2X_1+\frac{\tau^2}{4}X_2+\frac{r^4}{4}Z_3$$

$$\hat{\Lambda}_1=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & I & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & I & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$\hat{\Lambda}_2=\begin{bmatrix} A & 0 & B & D & 0 & 0 & C\\ 0 & 0 & 0 & I & 0 & 0 & 0\\ I & 0 & -I & 0 & 0 & 0 & 0\\ I & 0 & 0 & 0 & -I & 0 & 0 \end{bmatrix}$$

$$\hat{\Theta}=\begin{bmatrix} \hat{\Theta}_{11} & \hat{\Theta}_{12} & \hat{\Theta}_{13} & \hat{\Theta}_{14} & 0 & 0 & \hat{\Theta}_{17}\\ * & \hat{\Theta}_{22} & \hat{\Theta}_{23} & 0 & 0 & 0 & 0\\ * & * & \hat{\Theta}_{33} & 0 & 0 & 0 & 0\\ * & * & * & \hat{\Theta}_{44} & 0 & 0 & 0\\ * & * & * & * & \hat{\Theta}_{55} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \hat{\Theta}_{77} \end{bmatrix}$$

$$\begin{aligned}
\hat{\Theta}_{11}&=Q_{11}+Q_{12}A+A^TQ_{21}+V_{11}-X_1-X_2\\
&\quad+W_{11}-Y_1-Y_2+Z_1+r^2Z_2-r^2Z_3\\
\hat{\Theta}_{12}&=V_{12}+X_2\\
\hat{\Theta}_{13}&=X_1+Q_{12}B\\
\hat{\Theta}_{14}&=Q_{12}D\\
\hat{\Theta}_{17}&=Q_{12}+rZ_3\\
\hat{\Theta}_{22}&=V_{22}-V_{11}-X_2\\
\hat{\Theta}_{23}&=-V_{12}\\
\hat{\Theta}_{33}&=-Q_{11}-V_{22}-X_1\\
\hat{\Theta}_{34}&=-Q_{12}\\
\hat{\Theta}_{44}&=-Q_{22}\\
\hat{\Theta}_{55}&=-Z_1\\
\hat{\Theta}_{77}&=-Z_2-Z_3
\end{aligned}$$

3.2 *Results on The Uncertain Neutral System*

For the asymptotic stability for system (6), we have the following result.

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Theorem 3.2: For given scalars τ, h and r, the neutral system described by (1) with uncertainty described by (3), (4), (5) is asymptotically stable, if there exist matrices

$$P=[P_{ij}]_{5\times5}>0, Q=[Q_{ij}]_{2\times2}>0$$
$$V=\left[V_{ij}\right]_{2\times2}>0, R=\left[R_{ij}\right]_{2\times2}>0, W=\left[W_{ij}\right]_{2\times2}>0$$
$$X_1>0, X_2>0, Y_1>0, Y_2>0, Z_1>0, Z_2>0, Z_3>0$$

with appropriate dimensions such that

$$\begin{bmatrix} \Theta+\Lambda_1^T P\Lambda_2+\Lambda_2^T P\Lambda_1+\frac{1}{\varepsilon}\chi_2^T HH^T\chi_2+\varepsilon\chi_1^T\chi_1 & L^T M+\frac{1}{\varepsilon}\chi_2^T HH^T M \\ ML+\frac{1}{\varepsilon}MHH^T\chi_2 & -M \end{bmatrix}<0 \tag{20}$$

where

$$\chi_1=\left[E_A\ 0\ 0\ 0\ E_B\ E_D\ 0\ 0\ 0\ E_C\right]$$
$$\chi_2=\left[Q_{21}+R_{21}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]$$

and other notations have been defined in *Theorem* 3.1.

Proof: We replace Θ, Λ_2, L with $\Theta(t), \Lambda_2(t), L(t)$ on the basis of result of *Theorem* 3.1, thus we have the inequality by *Lemma* 2.1.

$$\begin{bmatrix} \Theta(t)+\Lambda_1^T P\Lambda_2(t)+\Lambda_2^T(t)P\Lambda_1 & L^T(t)M \\ ML(t) & -M \end{bmatrix}<0 \tag{21}$$

Considering the uncertainty described by (3), (4), (5), we obtain the following matrix inequality

$$\begin{bmatrix} \Theta+\Lambda_1^T P\Lambda_2+\Lambda_2^T P\Lambda_1 & L^T M \\ ML & -M \end{bmatrix}$$
$$+\chi_1^T F^T(t)H^T\begin{bmatrix} \chi_2 & M \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} \chi_2^T & 0 \\ M & 0 \end{bmatrix}HF(t)\chi_1<0$$

By *Lemma* 2.5, we have

$$\begin{bmatrix} \Theta+\Lambda_1^T P\Lambda_2+\Lambda_2^T P\Lambda_1 & L^T M \\ ML & -M \end{bmatrix}$$
$$+\frac{1}{\varepsilon}\begin{bmatrix} \chi_2^T & 0 \\ M & 0 \end{bmatrix}HH^T\begin{bmatrix} \chi_2 & M \\ 0 & 0 \end{bmatrix}+\varepsilon\begin{bmatrix} \chi_1^T\chi_1 & 0 \\ 0 & 0 \end{bmatrix}<0$$

Therefore, we obtain the linear matrix inequality (20) which guarantees the uncertain neutral system with distributed delay is asymptotically stable. This completes the proof.■

We also obtain the corollaries when $r=0$ and $\tau\equiv h$ in the system (1) on the basis of *Theorem* 3.2.

Corollary 3.3: For given scalars τ, h and $r=0$, the neutral system described by (1) with uncertainty described by (3), (4), (5) is asymptotically stable, if there exist matrices

$$P=[P_{ij}]_{5\times5}>0, Q=[Q_{ij}]_{2\times2}>0$$
$$V=\left[V_{ij}\right]_{2\times2}>0, R=\left[R_{ij}\right]_{2\times2}>0, W=\left[W_{ij}\right]_{2\times2}>0$$
$$X_1>0, X_2>0, Y_1>0, Y_2>0$$

with appropriate dimensions such that

$$\begin{bmatrix} \bar{\Theta}+\bar{\Lambda}_1^T P\bar{\Lambda}_2+\bar{\Lambda}_2^T P\bar{\Lambda}_1+\frac{1}{\varepsilon}\bar{\chi}_2^T HH^T\bar{\chi}_2+\varepsilon\bar{\chi}_1^T\bar{\chi}_1 & \bar{L}^T M+\frac{1}{\varepsilon}\bar{\chi}_2^T HH^T M \\ M\bar{L}+\frac{1}{\varepsilon}MHH^T\bar{\chi}_2 & -M \end{bmatrix}<0 \tag{22}$$

where

$$\bar{\chi}_1=\left[E_A\ 0\ 0\ 0\ E_B\ E_D\ 0\ 0\right]$$
$$\bar{\chi}_2=\left[Q_{21}+R_{21}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]$$

and other notations have been defined in *Corollary* 3.1.

Corollary 3.4: For given scalars $\tau\equiv h$ and r, the neutral system described by (1) with uncertainty described by (3), (4), (5) is asymptotically stable, if there exist matrices

$$P=[P_{ij}]_{5\times5}>0, Q=[Q_{ij}]_{2\times2}>0, V=\left[V_{ij}\right]_{2\times2}>0$$
$$X_1>0, X_2>0, Y_1>0, Y_2>0, Z_1>0, Z_2>0, Z_3>0$$

with appropriate dimensions such that

$$\begin{bmatrix} \hat{\Theta}+\hat{\Lambda}_1^T P\hat{\Lambda}_2+\hat{\Lambda}_2^T P\hat{\Lambda}_1+\frac{1}{\varepsilon}\hat{\chi}_2^T HH^T\hat{\chi}_2+\varepsilon\hat{\chi}_1^T\hat{\chi}_1 & \hat{L}^T M+\frac{1}{\varepsilon}\hat{\chi}_2^T HH^T M \\ M\hat{L}+\frac{1}{\varepsilon}MHH^T\hat{\chi}_2 & -M \end{bmatrix}<0 \tag{23}$$

where

$$\hat{\chi}_1=\left[E_A\ 0\ E_B\ E_D\ 0\ 0\ E_C\right]$$
$$\hat{\chi}_2=\left[Q_{21}+R_{21}\ 0\ 0\ 0\ 0\ 0\ 0\right]$$

and other notations have been defined in *Corollary* 3.2.

4. NUMERICAL EXAMPLES

In this section, two numerical examples are given to illustrate the effectiveness and the improvement of the proposed method over some previous ones.

Example1: Consider the uncertain system

$$A=\begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}\ B=\begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}$$
$$C=\begin{bmatrix} -0.12 & -0.12 \\ -0.12 & 0.12 \end{bmatrix}\ D=\begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}$$
$$H=I,\ E_A=E_B=E_C=E_D=0.1I,\ \tau=0.1$$

When $h=0.1$, by *Corollary* 3.4, the upper bound of r that guarantees the asymptotic stability of system (1) calculated by the method in this paper is 6.7. Applying the criteria in Chen & Zheng [14], Han [11] and Li & Zhu [15], we show the results in the following table 1:

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

	τ=0.1, $h=0.1$
Chen and Zheng.	6.2
Han.	6.4
Li and Zhu.	6.6
Corollary 3.4 in this paper	6.7

Table 1: The Allowed Upper Bound Of r

Example2: Consider the uncertain system

$$A=\begin{bmatrix}-3.4 & 0.2\\ 0.1 & -0.9\end{bmatrix}\quad B=\begin{bmatrix}-1.1 & 0.1\\ 0.1 & -1.2\end{bmatrix}$$

$$C=\begin{bmatrix}0.1 & -0.2\\ 0.1 & 0.3\end{bmatrix}\qquad D=\begin{bmatrix}d & 0\\ 0 & d\end{bmatrix}$$

$$H=I,\;\; E_A=E_B=E_C=E_D=0.2I,\;\;\tau=0.2$$

When $r=1.0, d=0.25$, applying the criteria in Li & Zhu [15], Sun et.al. [6] and *Theorem* 3.2 in this paper we can obtain the allowed upper bound of h. We show the results in the following table 2:

	τ=0.2, r =1.0
Li and Zhu.	0.70
Sun et.al.	0.72
Theorem 3.2 in this paper	0.79

Table 2: The Allowed Upper Bound Of h

Through these examples, it can be seen that our method is less conservative than the previous ones.

5. CONCLUSIONS

This paper is concerned with the stability of linear certain and uncertain neutral systems with distributed delays. Applying the delay-dividing approach, new discrete-, distributed- and neutral-delay-dependent stability criteria are presented based on the Lyapunov theory. Two numerical examples are given to respectively show that our method is less conservative than the previous ones.

ACKNOWLEDGEMENTS

This work was supported in part by the National Natural Science Foundation of China (61074033 and 61174061), National Key Scientific Research Project (61233003) and the Fundamental Research Funds for the Central Universities.

The authors would like to thank the associate editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality of the paper.

REFRENCES:

[1] A. Goubet, M. Dambrine and J. P. Richard, “Stability of Perturbed Systems with Time-Varying Delays”, *Systems & Control Letters*, Vol.31, 1997, pp. 155-163.

[2] E. Fridman, “New Lyapunov-Krasovkii Functionals for Stabiliy of Linear Retarded And Neutral Type Systems”, *Systems & Control Letters*, Vol.43, 2001, pp.309-319.

[3] E. Fridman and U. Shaked, “ An Improved Stabilization Method for Linear Time-delay Systems. *IEEE Transactions on Automatic Control*, Vol.47, 2002, pp.1931-1937.

[4] E. Fridman and U. Shaked, “Delay-dependent Stability and *H*∞ Control: Constant and Time-Varying Delays”, *International Journal of Control*, Vol.76, 2003, pp.48-60.

[5] H.B. Ji, “*Algebra Foundation of Control Theory*”, University of Science and Technology of China Press, Hefei, 2008, pp.213-215.

[6] J. Sun, J. Chen and G.P. Liu, “On Roubust Stability of Uncertain Neutral Systems with Discrete and Distributed Delays”, *American Control Conference*, 2009, pp.5469-5473.

[7] J. Sun, G. P. Liu and J. Chen, “Delay-dependent Stability and Stabilization of Neutral Time-delay Systems”, *International Journal of Robust and Nonlinear Control*, vol.19, 2009, pp.1364-1375.

[8] K. Gu and S. I. Niculescu, “Further Remarks on Additional Dynamics in Various Model Transformations of Linear Delay Systems”, *IEEE Transactions on Automatic Control*, Vol.46, 2001, pp.497-500.

[9] K. Gu, “An Improved Stability Criterion for Systems with Distributed Delays”, *International Journal of Robust and Nonlinear Control*, Vol.13, 2003, pp.819-831.

[10] L.H. Xie, “Output Feedback H_∞ Control of Systems with Parameter Uncertainty”, *International Journal of Control*, vol.63, 1996, pp.741-750.

[11] Q.L. Han, “A Descriptor System Approach to Robust Stability of Uncertain Neutral Systems with Discrete and Distributed Delays”, *Automatica*, Vol.40, 2004, pp.1791-1796.

[12] Q.L. Han, “A New Delay-dependent Stability Criterion for Linear Neutral Systems with Norm-bounded Uncertainties in All System Matrices”, *International Journal of Systems Science*, Vol.36, 2005, pp.469-475.

[13] Q.L. Han, “A Discrete Delay Decomposition Approach to Stability of Linear Retarded and Neutral Systems”, *Automatica*, Vol.45, 2009, pp. 517-524.

[14] W. H. Chen and W. X. Zheng, “Delay-dependent robust stabilization for uncertain neutral systems with distributed delays”, *Automatica*, Vol.43, 2007, pp.95-104.

[15] X.G. Li and X.J. Zhu, “ Stability Analysis of

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Neutral Systems with Distributed Delays", *Automatica*, Vol.44, 2008, pp. 2197-2201.

[16] Y. S. Moon, P. Park, W.H. Kwon and Y.S. Lee, Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control*, Vol.74, 2001, pp.1447-1455.