

POINT CLOUD REGISTRATION FOR A NON-DEFORMABLE OBJECT USING SURFACE CURVATURE FEATURES

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ABSTRACT

Recently, many applications require 3D information of an object (i.e. point cloud) such as 3D image scanning, 3D surface reconstruction and etc. However many researches on point cloud registration face many challenges especially for increasing the accuracy of point cloud matching. This research employs surface curvature features in discrete surfaces. A surface curvature feature is a pointer of ridges that are invariant to rigid body transformations. In this paper, a new algorithm of point cloud registration for non deformable object is proposed. This algorithm employs surface curvature features estimated by fitting k-nearest neighbor of local point to hyperbolic paraboloid equation.

The proposed algorithm is implemented with Iterative Closest Point (ICP) technique and quantitatively evaluated and compared with common techniques for point cloud registration. Experimental results demonstrate that the proposed technique approximately 63% faster and 23% more accurate than iterative closest point with angular invariant feature (ICP-AIF) registration techniques. These results are obtained by testing the proposed frame work with noise, different pose of point cloud and overlapped area.

Keywords: *Registration, Point Cloud, Surface Curvature Feature.*

1. INTRODUCTION

Recently, 3D computer models of real world objects have been widely used for virtual reality applications such as business (real estate, architecture), education (electronic museums, multimedia books) and entertainment (3D interactive games, movie)[10]. It is also to construct a virtual environment of a real environment for Industrial Robotics and inverse engineering purposes[19].

To build a 3D model required the coordinates of the object surface is collected from a 3D acquisition equipment such as a 3D scanner[14] or by using multiview 3D image reconstruction technique [13]. Due to its high density, the data point is often referred to as point cloud. A point cloud of a single view can not accommodate the entire shape of the object due to self occlusion and the limitation of the field of view of the 3D scanner[12]. To build the entire object required some data points captured from multiple viewpoints to obtain the entire surface [10]. Furthermore, point cloud is registered to obtain a complete 3D model. The

registration of point clouds can be defined as the process of estimating the rigid transformation that places point clouds in to a common coordinate system[2].

To register point cloud it is started by data matching method to establishing correspondences between two point clouds followed by transformation estimation[11]. Iterative closest point algorithm (ICP)[4] is a common algorithm registration that widely used by researchers. ICP and its invariant [5][3][6][15] is based on metric distance techniques to find the closest point to establish correspondence. This techniques can lead the registration algorithm converge to a local minima because the closest points is not exactly represent compatible point on the original surface.

Some researches using features such as color[10], curvature[9], spin image[17] that have extracted from a set of point interest previously selected in both point cloud to find the compatible points. Two points are compatible if the value of the associated features lies below a threshold.

This paper proposes a novel registration algorithm for non-deformable object based on

surface curvature matching to find the closest point. Non-deformable object is a special case of a system of particles where in the distances between all particles remains unchanged.

In the proposed algorithm, the surface curvature is represented by surface curvature feature is extracted from a local surface. To find the closest compatible point, we match the points by finding the nearest surface curvature feature in one point cloud to another point cloud.

2. POINT CLOUD REGISTRATION AND SURFACE CURVATURE FEATURE

2..1. Point Cloud Registration

Multiple point clouds that obtained from 3D range acquisition equipment are in the local coordinate system with respect to the range finder. The purpose of point clouds registration is to find the relative position and orientation one point cloud to another point cloud.

Given two point set $P = \{\vec{p}_1, \dots, \vec{p}_N\}$ and $Y = \{\vec{y}_1, \dots, \vec{y}_N\}$ where N the number point. If \vec{p}_i is the i th data point of the whole data point set P and \vec{y}_i denote the i th data point of the whole data of point set Y which $\{\vec{p}_i, \vec{y}_i\}$ is pair point matching.

Point cloud registration problem is to find rigid transformation parameters $[R, \vec{t}]$ with R is rotation matrix and \vec{t} is translation vector which minimize a cost function in the least square distance metrics following:

$$\min_{R, \vec{t}} \sum_{i=1}^N \|(R \cdot \vec{p}_i + \vec{t}) - \vec{y}_i\| \tag{1}$$

2.2. Surface Curvature

On a Non-Deformable-object, surface curvature is an important feature because of its invariance with respect to the rigid transformation[1]. Surface curvature is defined as the curvature of a curve on the surface passing through a point. The maximum curvature (k_1) and the minimum curvature (k_2) of the curve is referred as the principal curvature.

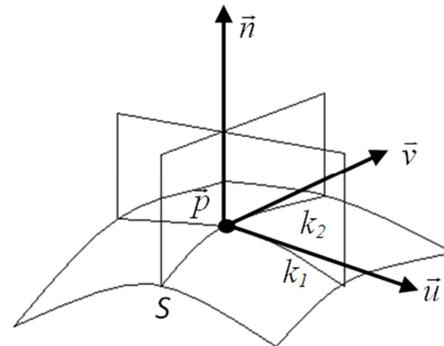


Fig 1. The view of curvature direction

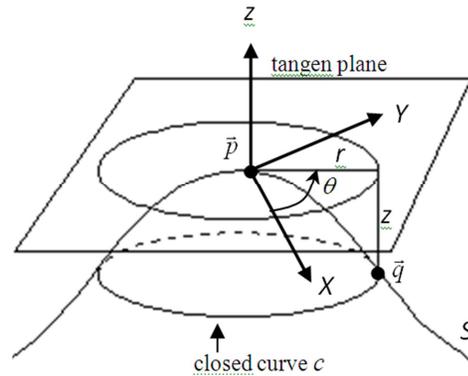


Fig 2. The Relation Between Angle And Z Coordinate Of Closest Curve C, With Which Surface Curvature Feature Can Be Constructed.

The associated vector of the principal curvature is called as principal directions (\vec{u} and \vec{v}). Fig.1 shows the Principal directions and normal vector at a point \vec{p} on surface S construct a local coordinate at point \vec{p} [18].

2.3. Surface Curvature Feature Definition

Suppose S is a local surface which centered at point \vec{p} and the normal vector of surface S pointing to positive Z axis. If \vec{p} is located at origin and \vec{q} is a point that lie on the local surface S . Fig.2 shows, in cylindrical coordinates system \vec{q} is represented as graph function $z = f(r, \theta)$ which r is the radius of the point to the central point and θ is the angle between the reference direction on the chosen plane and the line from the origin to the projection of \vec{q} on the plane. If r is set to α , it will be obtained a closed curve c on local surface S within α . Sampling those z coordinate of the closed curve c at $\theta = \theta_1, \theta_2, \dots, \theta_k$ one lap will obtain the relation between the angle and the z coordinates of the closed curve on local surface S at radius α .

This relationship is defined as the surface curvature feature of the point \vec{p} which denoted by $SCF(\vec{p}) = (z_1, z_2, \dots, z_k)$.

3. REGISTRATION POINT CLOUD USING SURFACE CURVATURE FEATURE.

3.1. Surface Normal Estimation

Geometry properties such as normal vectors and principal direction have an important role within the proposed frame work. This geometry property is used to construct local coordinate system in the whole point in every point cloud.

This section describes the estimation of normal vectors for each point. Our work adopt Thürme, et al [16] for finding normal vector of a point.

The Normal vector at a point \vec{p}_i is estimated by calculating the angle-weighted average of the normal vector of the triangle arranged by the point \vec{p}_i and its neighbors.

If $Q_i = \{\vec{q}_{i1}, \vec{q}_{i2}, \dots, \vec{q}_{ik}\}, \vec{q}_{ij} \in P, \vec{q}_{ij} \neq \vec{p}_i$ is k neighboring points are connected directly to the point \vec{p}_i the surface normal vector is

$$\vec{n}_i = \frac{1}{k} \sum_{j=1}^k w_j \frac{(\vec{q}_{ij} - \vec{p}_i) \times (\vec{q}_{i,j+1} - \vec{p}_i)}{\|\vec{q}_{ij} - \vec{p}_i\| \|\vec{q}_{i,j+1} - \vec{p}_i\|} \quad (3)$$

where w_j is the weight which is the angle two successive tangent vectors.

$$w_j = \cos^{-1} \left(\frac{\|(\vec{q}_{ij} - \vec{p}_i) \times (\vec{q}_{i,j+1} - \vec{p}_i)\|}{\|\vec{q}_{ij} - \vec{p}_i\| \|\vec{q}_{i,j+1} - \vec{p}_i\|} \right) \quad (4)$$

3.2. Principal Curvature and Principal Direction Estimation.

The principal curvature and principal direction is calculated by decomposition of eigenvalue and eigenvector of hessian matrix of the local surface of point pi [7]. This local surface is estimated by fitting of k nearest neighborhood of \vec{p}_i to hyperbolic paraboloid function :

$$z = f(x, y) = \frac{a}{2}x^2 + \frac{b}{2}y^2 + cxy + dx + ey + g \quad (5)$$

The hessian matrix of (5) is

$$H = \begin{bmatrix} a & c \\ c & b \end{bmatrix} \quad (6)$$

If λ_1 and λ_2 are the eigenvalue of matrix hessian H which $|\lambda_1| > |\lambda_2|$ then the maximum and the

minimum curvature of the local surface S are λ_1 and λ_2 respectively. The eigenvector of matrix hessian H are the principal direction of local surface $S(\vec{u}, \vec{v})$.

3.3. Point Cloud Matching Using Surface Curvature Feature.

Given two point set $P = \{\vec{p}_1, \vec{p}_2, \dots, \vec{p}_N\}$ and $P' = \{\vec{p}'_1, \vec{p}'_2, \dots, \vec{p}'_N\}$. If \vec{p}_i is the i th of the whole data point set P and \vec{p}'_j is the j th data point of the whole data point set P . Let $SCF(\vec{p}_i) = (z_{i1}, z_{i2}, \dots, z_{ik})$ and $SCF(\vec{p}'_j) = (z'_{j1}, z'_{j2}, \dots, z'_{jk})$ are surface curvature feature related to the point \vec{p}_i and point \vec{p}'_j respectively. The point matching strategy is finding the closest point with respect of the surface curvature feature distance. The surface curvature distance of $SCF(\vec{p}_i)$ to $SCF(\vec{p}'_j)$ is :

$$d(SCF(\vec{p}_i), SCF(\vec{p}'_j)) = \sqrt{\sum_{k=1}^K (z_{ik} - z'_{jk})^2} \quad (7)$$

Correspondence $\{\vec{p}_i, \vec{p}'_{c(i)}\}$ between two point sets P and P' is computed based on $c(i)$.

$$c(i) = \arg \min_{j \in \{1..N'\}} \left(\sum_{i=1}^N d(SCF(\vec{p}_i), SCF(\vec{p}'_j)) \right) \quad (8)$$

3.4. Rigid transformation estimation

Rigid transformation of two point sets $\{\vec{p}_i\}_{i=1}^N$ and $\{\vec{p}'_{c(i)}\}_{i=1}^N$ is estimating by calculated the matrix rotation R and the vector translation \vec{t} that minimize the distance error between $\{\vec{p}_i\}_{i=1}^N$ and $\{\vec{p}'_{c(i)}\}_{i=1}^N$ as follow:

$$(\mathbf{R}, \vec{t}) = \arg \min_{\mathbf{R}, \vec{t}} \left(\sum_{i=1}^N \|\mathbf{R}\vec{p}_i + \vec{t} - \vec{p}'_{c(i)}\| \right) \quad (9)$$

Where areas, we solved the rotation matrix R and the translation vector \vec{t} using Singular Value Decomposition (SVD) as described in [8].

3.5. Proposed Registration Algorithm.

In this paper, we propose 3D registration using surface curvature feature. Our registration

algorithm is based on standard iterative closest point (ICP) algorithm proposed by Besl[4] but unlike the standard ICP, our algorithm registration using surface curvature feature to find the coordinate closest compatible point.

Given two set of point cloud $P = \{\vec{p}_1, \vec{p}_2, \dots, \vec{p}_N\}$ and $P' = \{\vec{p}'_1, \vec{p}'_2, \dots, \vec{p}'_{N'}\}$ which N and N' are the number of point in point sets P and P' respectively. Registration techniques is initialized by setting rotation matrix R_0 to unit $I_{3 \times 3} \in R^{3 \times 3}$, translation vector \vec{t}_0 to $(0,0,0)$, iteration index m to 0 and $P_0 = P$. The registration algorithm using surface curvature feature can be stated as follows:

Step1: Construct local coordinate system of each point. The local coordinate system of each point is constructed from the normal vector and the principal direction of each point. The estimation of normal vector and principal direction in a point is described on sub section 3.1 and sub section 3.2.

Step 2: Construct surface curvature feature vector for each point. Based on the local coordinate system that has calculated in Step 1, the surface curvature of point \vec{p}_i and \vec{p}'_j is calculated by finding the z coordinate of the point on local surface of point \vec{p}_i and \vec{p}'_j within radius r from z axis and angles $\theta_1, \theta_2, \dots, \theta_k$ from reference axis x . The surface curvature feature vector of point \vec{p}_i and point \vec{p}'_j are $SCF(\vec{p}_i) = (z_{i1}, z_{i2}, \dots, z_{ik})$ and $SCF(\vec{p}'_j) = (z'_{j1}, z'_{j2}, \dots, z'_{jk})$ respectively.

Step 3: Find pair matching point between two sets P and P' . The pair matching point $\{\vec{p}_{mi}, \vec{p}'_{mc(j)}\}$ is calculated by finding the closest distance between two surfaces curvature feature $SCF(\vec{p}_{mi})$ and $SCF(\vec{p}'_{mj})$ as described in sub section 3. 3.

Step 4: Estimated rotational and translational transformation matrix for $(m+1)$ iteration. The rotational matrix $R_{(m+1)}$ and transformation vector $\vec{t}_{(m+1)}$ is estimated by minimizing the distance error between $\{\vec{p}_{mi}\}_{i=1}^N$ and $\{\vec{p}'_{mc(i)}\}_{i=1}^N$ as described in sub section 3.4

Step 5: Apply the registration of each point in P_m :

$$\vec{p}_{(m+1)i} = R_{(m+1)}\vec{p}_{(m+1)i} + \vec{t}_{(m+1)i} \quad (10)$$

Step 6: Update correspondence. Correspondence $\{\vec{p}_{(m+1)i}, \vec{p}'_{(m+1)c(j)}\}$ is updated by selecting the pair

$\{\vec{p}_{mi}, \vec{p}'_{mc(j)}\}$ which is have distance less than the threshold t_d and surface curvature feature distance vector less than threshold t_c to remove false pair matching.

Step 6: Calculate the root mean square error as the cost function

$$X = \sum_{i=1}^{N_p} \left\| (R_{(m+1)}\vec{p}_{(m+1)i} + \vec{t}_{(m+1)}) - \vec{p}'_{(m+1)i} \right\|^2 \quad (11)$$

If ϵ less than threshold t then terminated the iteration else repeat step 4.

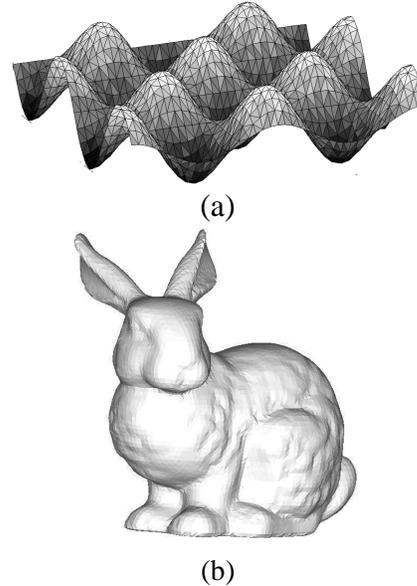


Fig. 3. Point cloud models of test object : (a) wave, (b) bunny

4. EXPERIMENTS AND RESULT

In this section, we evaluate the performance of proposed registration algorithm We is test the proposed algorithm on two objects wave and bunny object. The wave objects is a pair synthetic surfaces that are consisting 10000 vertex obtained from non-uniform sampling of 3D sinusoidal surface. Overlap area, pose, and noises of the wave objects can be adjusted to test the proposed algorithm performance in different setting.

The bunny object is obtained from data repository of Stanford University that is consisting five different views. The amount of points of each view is shown in table 1 while each view pose is shown on Fig.4.

We called our algorithm registration as iterative closest point using surface curvature feature (ICP-SCF)

Table 1. The Number Of Point Each View Of Bunny Object

	Amount of point	Pose (degree)
View-1	40256	0
View-2	40097	45
View-3	30379	90
View-4	31701	270
View-5	35336	315

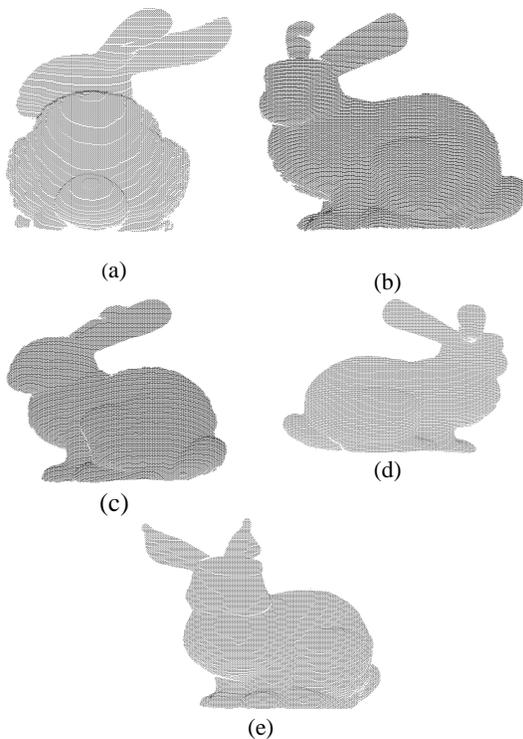


Fig.4. Pose of each view of the bunny object (a) 0°, (b)45°, (c)90°, (d) 270°, and (e) 315°

SCF). We compare our algorithm with original Iterative Closest Point Algorithm (ICP)[4] and ICP with angular invariant feature algorithm (ICP-AIF)[9]. We know the real conjugate points between point clouds. Therefore, we measure RMS distance error between two ground truth point sets. All registration result is plotted as graphic RMS registration error (ground truth) as a function of iteration number. The tests are performed on a computer with a processor "Intel" Core (TM) 2 Duo CPU, frequency 2 GHz and a memory of 2 GB.

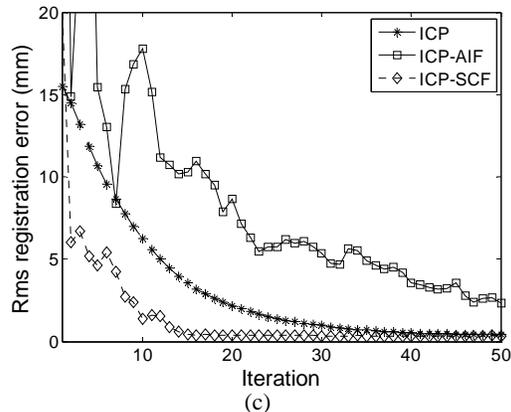
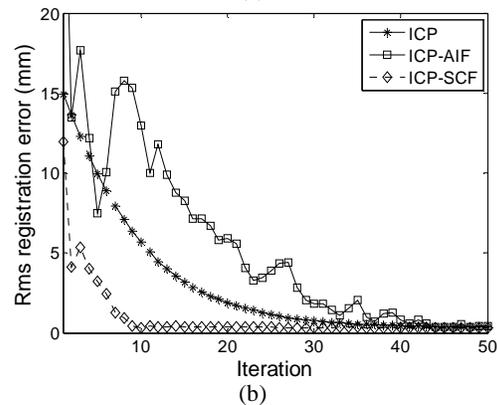
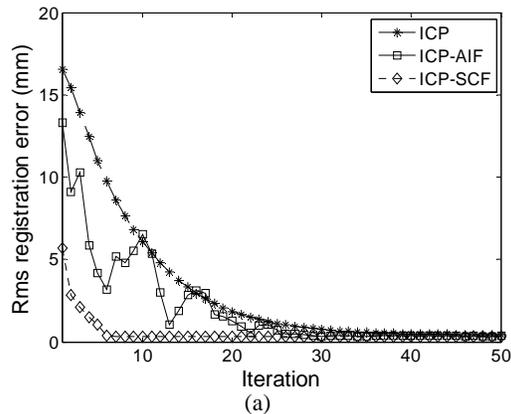


Fig 5. Rms Registration Error (Ground Truth) Of Wave Object Vs Iteration (Count/Number) With Different Amounts Of Noise : (A) 2%,(B) 7% And (C) 10% Noise.

4.1. Registration Algorithm Test For Wave Object

We perform three different tests to the wave object: noise, overlapped areas, and pose test.

4.1.1 Noise test.

In the noise test, we set the overlap area of surfaces fix 90% and the pose is set by rotating one of the

surface 5 degree to x-axis and y-axis. We perform three tests by adding 2%, 7%, and 10% noise on each test to both surface. The registration results are plotted in Fig. 3, RMS registration as a function of iteration number. Fig. 5 shows that proposed algorithm convergent at 5, 10 and 13 iteration in 2%, 7% and 10% noise testing with 2 mm rms registration error however ICP algorithm convergent at 30 iteration with rms error registration 3 mm in all noise testing. The result of ICP-AIF registration shows that ICP-AIF

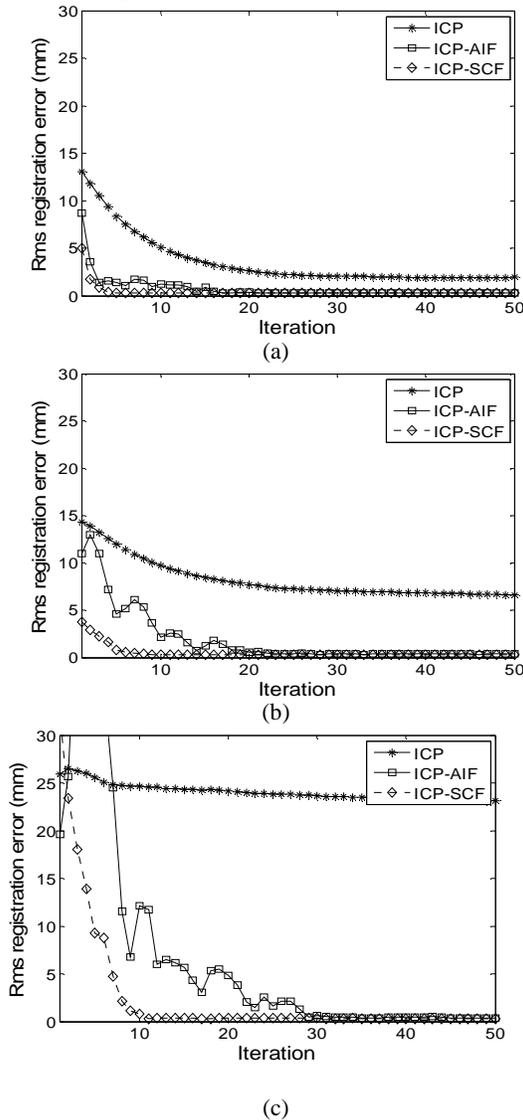


Fig 6. Rms Registration Error (Ground Truth) Of Wave Object Vs Iteration (Count/Number) With Different Overlap Area Rate : (A) 86%,(B) 68% And (C) 41% Of Overlapped Area Rate Between Two Surfaces.

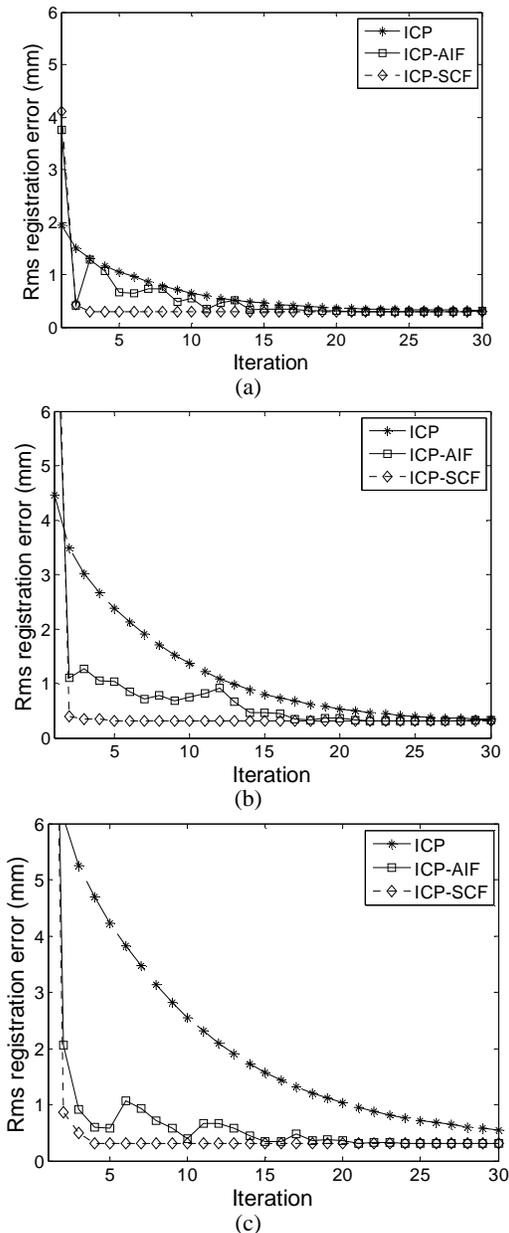


Fig 7. Rms registration error (ground truth) of wave object in different pose by rotating one of the surface to x-axis and follow to y-axis: (a) 4,(b) 10 and (c) 17.

convergent at 15 and 30 iteration in 2% and 7% noise test with registration error rms 3 mm. When ICP-AIF is tested with 10% noise, the algorithm failed to reach converge.

4.1.2 Overlapped area test

Overlapped area is part of the surfaces that have common area. In the overlapped area test, we test

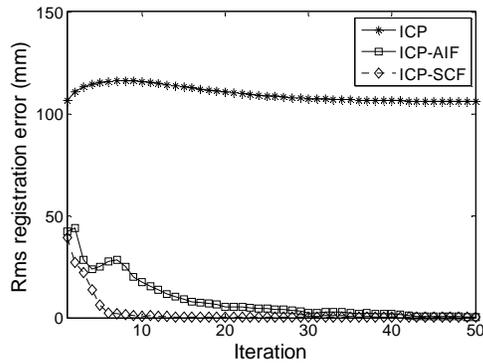


Fig 8. Rms Registration Error (Ground Truth) Result Of Object Vs Iteration (Count/Number) Of View-1 And View-2 Of Bunny Object.

the proposed algorithm to the wave object in three different percentage of overlapped area which are 86%, 68%, and 41%. We set noise rate fixed to 2% and set pose by rotating one of the surface 5 degree to the x axis and y-axis. The result is plotted in fig 6 as RMS registration error as the function of iteration number. Fig.5 shows that our proposed algorithm converge well at 5,10, and 15 iteration with rms error 2 mm in 86%, 68%, and 41% overlapped test while the ICP-AIF achieve convergence at 15, 23, and 29 iteration with rms registration error 3mm. Registration with ICP algorithm convergence well in 45 iteration in 86% overlap area test with the rms error (ground truth) 5mm. While in test with overlap less than 68% of the registration is fails.

4.1.3 Pose test

To test the algorithm in different pose we rotate one of the surface to the x-axis and to y-axis by 4,10, and 17 degree. Noise and surface overlapped area is set fixed 2% and 80% respectively. The result is plotted in Fig.7 RMS registration error as function of the number of iteration. Our proposed algorithm convergent at 1,2, and 4 iteration with rms registration error 0.4 mm at 4, 10, and 17 degree pose test while ICP-AIF convergent at 5,

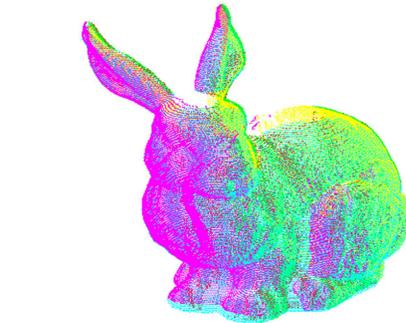


Fig. 9 The Result Of Registration All Views Of Bunny Object. Each View Is Transformed Relatively To View-1.

20, and 25 iteration with RMS error registration 0.45. The result of ICP registration test shows that in 4 degree pose test ICP convergent well at 20 iteration with rms error registration 0.45 while at 10 and 17 degree pose test, ICP convergent at more than 30 iteration.

4.2. Registration Algorithm Test For Bunny Object

We use five views of the bunny object to test our proposed algorithm which each view is shown in Fig.8. Each view have relative pose of 45° except View-3 and View-4 have relative pose of 90°. We test all of view by registering the nearest pair view. Fig.8 is the result of View-1 and View-2 registration of bunny object. It is shows that the ICP-SCF achieve convergence at 5 iterations with rms error registration 0.317 mm however ICP-AIF algorithm achieve convergence at 30 iterations with RMS registration error 0.336. The registration result of ICP algorithm shows that the algorithm fail to achieve convergence. This condition occurs in other registration test. All of result test is summarized in Table 3. We don't include the result of ICP registration because the ICP algorithm fail to achieve convergence when it tested to the bunny object.

The registration result of all view of bunny object is shown in Table 3. We compare the iteration convergence amount, convergence time, Rms error registration and processing time after 50 iteration between ICP-SCF and ICP AIF.

Proposed algorithm convergent in average

Table 3 Result Of Bunny Objects Registration

Registration	Iteration Convergence		Convergence Time (sec)		RMS Error Registration (mm)		Processing Time After 50 iteration (sec)	
	ICP-AIF	ICP-SCF	ICP-AIF	ICP-SCF	ICP-AIF	ICP-SCF	ICP-AIF	ICP-SCF
View 1-View 2	43	28	0.100	0.032	0.336	0.317	0.109	0.045
View 2-View 3	47	43	0.053	0.050	0.326	0.339	0.054	0.053
View 3-View 4	48	49	0.047	0.039	0.294	0.275	0.279	0.053
View 4-View 5	63	26	0.065	0.019	0.443	0.299	0.047	0.032
Average	50.25	36.5	0.066	0.035	0.397	0.307	0.122	0.045

36.6 iteration or in 0.035 second with RMS registration error average 0.307 mm while ICP-AIF algorithm convergent in average 50.25 iteration or in 0.066 second with rms registration error average 0.397 mm. Processing time average after 50 iteration of the proposed algorithm takes 0.045 second while ICP-AIF takes 0.122 second. It shows that our algorithm 63% faster and 23% more accurate than ICP-AIF.

Fig.9 shows the result of all views of bunny object. Each view is transformed relatively to View-1 as the result of registration process.

5. CONCLUSION

This research proposes an algorithm for point cloud registration base on surface curvature features estimated by fitting k-nearest neighbor of local point to hyperbolic paraboloid equation. The proposed algorithm is implemented with Iterative Closest Point (ICP) technique so called ICP-SCF. In experiment we compare ICP-SCF vs ICP and ICP-AIF then testing them with noise, rigid transformation and overlapped area. In the experiment, while ICP algorithm fail to achieve convergence, ICP-SCF demonstrates approximately 63% faster and 23% more accurate than ICP-AIF.

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