

AERO-ENGINE VARIABLE STRUCTURE GLOBAL FAST TERMINAL SLIDING MODE CONTROL METHOD

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ABSTRACT

One new kind of aero-engine control method based on the variable structure global fast terminal sliding mode control is proposed for the varying uncertainty nonlinear system of aero-engine. The aero-engine global fast terminal sliding mode variable structure controller has been designed by the in-depth study of VSTSMC theory. The simulation results show that the designed controller has good control effect and strong suppression of external interference, so that the controlled system has strong robustness in the entire control stage.

Keywords: *Global Fast Terminal Sliding Mode, Aero-engine, Variable Structure Control*

1. INTRODUCTION^[1,2,3]

Aero-engine is a strongly nonlinear dynamical system with complex structure, in terms of multi-input and multi-output control, very obvious uncertainties of complex objects. Aero-engine work scope and the parameter range is wide. The control method based on precise mathematical models is powerless. Therefore, to ensure that the design of aero-engine control system has strong robustness and stability, sliding mode control to provide an effective means.

Sliding Mode Control with fast response, the perturbation parameters of the controlled system and external interference has strong robustness, so it has been widely used. However, the strong robustness of Sliding Mode Control only reflected in the sliding stage, to the reach stage, the dynamic characteristics are very sensitive to the system parameter perturbation and external disturbance, even instability in the larger parameter perturbation and external disturbance; in addition, due to the sliding mode variable structure control for handling uncertainties switching control will allow system control the volume of the high-frequency buffeting, this high-frequency chattering easily inspire the system's unmodeled characteristics, thus affecting the system's control performance and high-frequency buffeting can be achieved without any executive agency, therefore, this sliding mode variable structure control can not be achieved in practical applications.

In this paper, the introduction of the global fast terminal sliding mode control in aero-engine control system, the control method for the design of the control law to the system state can be zero in finite time. It breaks through the ordinary sliding mode control gradually nearly convergence characteristics in linear sliding surface conditions. The dynamic performance of the system is better than the ordinary sliding mode control. And with respect to the linear sliding mode control, the global fast final sliding mode control without switching items, can effectively eliminate chattering. The global fast terminal sliding mode control brings a new development direction in the proposed sliding mode control theory.

2. VARIABLE STRUCTURE FAST TERMINAL SLIDING MODE CONTROL INTRODUCTION

Since the early 1960s, the former Soviet scholars Emelyanov et al had study variable structure control system, Variable structure control has formed a branch of control theory. Variable structure control system is different with the conventional control system in that the system "structure" may be provided according to the system state (deviation of its order countdown) transitions changes in an instant destination, forcing the system along a predetermined sliding mode state trajectory to variable structure control system with a strong robustness.



A traditional of fast Terminal sliding mode form is [4]

$$S = \dot{x} + \beta x^{\frac{q}{p}} = 0, \quad (1)$$

$x \in R$ is state variables, $\beta > 0, p, q (p > q)$ is positive odd.

By the formula (1)

$$\frac{dx}{dt} = -\beta x^{\frac{q}{p}}$$

That

$$dt = -\frac{1}{\beta} x^{-\frac{q}{p}} dx$$

Definite integral of the above equation get

$$\int_0^t dt = \int_{x_0}^0 -\frac{1}{\beta} x^{-\frac{q}{p}} dx$$

Thereby obtaining the time is from any initial state $x(0) \neq 0$ in the sliding mode of formula (1) to reach the balanced state $x = 0$

$$t_s = \frac{p}{\beta(p-q)} |x(0)|^{\frac{(p-q)}{p}} \quad (2)$$

The equilibrium state $x = 0$ is also called the Terminal attractor. As a result of the introduction of a non-linear part $\beta x^{\frac{q}{p}}$ to improve the convergence rate of convergence to equilibrium state, and the more far-from-equilibrium state, and faster convergence. However, Terminal sliding mode control on the convergence time may not be optimal mainly because the speed of convergence of Nonlinear sliding mode formula (1) is more slow than the linear sliding mode ($p = q$) the convergence rate when the system state close to a state of equilibrium. To this end, a new global fast Terminal sliding mode has been proposed.

2.1variable Structure Global Fast Terminal Sliding Mode Control Problem Description [4,5,6,7]

Considering the linear sliding mode with fast Terminal sliding mode, a new kind of global fast Terminal sliding mode is

$$s = \dot{x} + \alpha x + \beta x^{\frac{q}{p}} = 0 \quad (3)$$

$x \in R$ is state variables,

$\alpha, \beta > 0, p, q (p > q)$ positive odd.

By the formula (3)

$$x^{-\frac{q}{p}} \frac{dx}{dt} + \alpha x^{\frac{1-q}{p}} = -\beta \quad (4)$$

$$\frac{1-q}{p}$$

Make $y = x^{\frac{1-q}{p}}$, the formula (4) is

$$\frac{dy}{dt} + \frac{p-q}{p} \alpha y = -\frac{p-q}{p} \beta \quad (5)$$

As a first-order linear differential equations

$$\frac{dy}{dx} + P(x)y = Q(x)$$

general solution is

$$y = e^{-\int P(x)dx} \left(\int Q(x)e^{\int P(x)dx} dx + C \right)$$

That formula (6) 's solution is

$$y = e^{-\int_0^t \frac{p-q}{p} \alpha dt} \left(\int_0^t -\frac{p-q}{p} \beta e^{\int_0^t \frac{p-q}{p} \alpha dt} dt + C \right)$$

When $t = 0, C = y(0)$, The above equation becomes

$$y = -\frac{\beta}{\alpha} + \frac{\beta}{\alpha} e^{-\frac{p-q}{p} \alpha t} + y(0) e^{-\frac{p-q}{p} \alpha t}$$

when $x = 0, y = 0, t = t_s$, The above equation becomes

$$\frac{[\beta + \alpha y(0)]}{\beta} = e^{\frac{p-q}{p} \alpha t_s}$$

on the sliding mode, time from an arbitrary initial state $x(0) \neq 0$ convergence to equilibrium $x = 0$

$$t_s = \frac{p}{\alpha(p-q)} \ln \frac{\alpha x(0)^{\frac{p-q}{p}} + \beta}{\beta} \quad (7)$$

By setting α, β, p, q enables the system to reach a state of equilibrium in a finite time t_s .

By the formula (3)

$$\dot{x} = -\alpha x - \beta x^{\frac{q}{p}} \quad (8)$$

When the system state x away from zero, the convergence time mainly by the rapid Terminal attract child that is $\dot{x} = -\beta x^{\frac{q}{p}}$ determined; When the system state x close to a balanced state $x = 0$, the convergence time mainly by

$\dot{x} = -\alpha x$ decision, x exponentially fast Attenuation. Therefore, sliding mode formula (3) not only introduced Terminal attractor, the system state convergence in finite time, but also retains the fast when linear sliding mode close to the equilibrium in order to achieve the state of the system quickly and accurately convergence to equilibrium, so called sliding mode formula (3) as global fast sliding mode.



The global fast sliding mode control combines the advantages of traditional sliding mode control and Terminal sliding mode control in sliding mode design. In the same time the use of the concept of fast arrival while reaching the stage. The global fast sliding mode control has the following characteristics:

(1) The global fast sliding mode control system reach the sliding surface in finite time, so that the state of the system to rapidly converge to the equilibrium state in a finite time. System state converge to the equilibrium state time can be adjusted by selecting the parameters.

(2) global fast sliding mode control law is continuous, without switching items, which can eliminate the chattering phenomenon.

(3) global fast sliding mode control has good robustness to uncertainty and interference by choosing a small enough q/p , The state of the system can reach the sliding surface is small enough neighborhood of convergence to equilibrium along the sliding surface.

Consider under multivariate uncertainty system^[8]:

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) + Df(t) \quad (9)$$

$x(t) \in R^n$ is the system state variables,

$u(t) \in R^m$ is control vector,

$f(t) \in R^l$ is

Outside

interference; $A \in R^{n \times n}$, $B \in R^{n \times m}$, $D \in R^{n \times l}$ respectively is control object's nominal system matrix, nominal control matrix and disturbance allocation matrix;

$\Delta A(t)$, $\Delta B(t)$ respectively is the perturbation matrix of matrices A, B .

Assume

1. $\Delta A(t)$, $\Delta B(t)$, D all can be Lebesgue measured and bounded;

2. For an arbitrary $t \in \Omega$ $\Delta A(t)$, $\Delta B(t)$

Continuous;

3. $\Delta A(t)$, $\Delta B(t)$ uniformly bounded in Ω

Assume

1. (A, B) is fully controllable array on;

2. Without loss of generality, control matrix $B^T = [0 \quad B_2^T]$, and B_2 nonsingular.

Assume

1. Parameter perturbation $\Delta A, \Delta B$ meet:

$$\|\Delta A\| \leq \psi_a, \|\Delta B\| \leq \psi_b \quad (10)$$

2. External disturbance $f(t)$ meet:

$$\|f(t)\| \leq \psi_f \quad (11)$$

ψ_a, ψ_b, ψ_f are known positive number, the

text if not otherwise specified, $\|\bullet\|$ for matrix

terms are induced norm.

2.2 The Variable Structure Global Fast Terminal Sliding Mode Control Law Design^[9]

A fast recursive structure sliding mode is expressed as

$$\dot{s}_1 = s_1 + \alpha_0 s_0 + \beta_0 s_0^{\frac{q_0}{p_0}}$$

$$\dot{s}_2 = s_2 + \alpha_1 s_1 + \beta_1 s_1^{\frac{q_1}{p_1}}$$

.....

$$\dot{s}_{n-1} = s_{n-1} + \alpha_{n-2} s_{n-2} + \beta_{n-2} s_{n-2}^{\frac{q_{n-2}}{p_{n-2}}} \quad (12)$$

$\alpha_i, \beta_i > 0$ and $q_i, p_i (q_i < p_i) (i = 0, 1, \dots, n-2)$ are positive odds.

The design globally Quick sliding mode Control law

$$u(t) = -b(X, t)^{-1} \left(f(x, t) + \sum_{k=0}^{n-2} \alpha_k s_k^{(n-k-1)} + \sum_{k=0}^{n-2} \beta_k \frac{d^{n-k-1}}{dt^{n-k-1}} s_k^{\frac{q_k}{p_k}} + \phi s_{n-1} + \gamma s_{n-1}^{\frac{p}{q}} \right)$$

(13)

$$s_0 = x_1$$

In the control law formula (13), The state of the system along

$$\dot{s}_{n-1} = -\phi s_{n-1} - \gamma s_{n-1}^{\frac{p}{q}}$$

reach the sliding surface $s_{n-1} = 0$ time $t_{s_{n-1}}$ is

$$t_{s_{n-1}} = \frac{p}{\phi(p-q)} \ln \frac{\phi [x_1(0)]^{\frac{p-q}{p}} + \gamma}{\gamma} \quad (14)$$

$\phi, \gamma > 0$, p and $q (q < p)$ are positive odds.

3. AERO-ENGINE VARIABLE STRUCTURE CONTROL SYSTEM DESIGN AND SIMULATION

Simulation to the design engine control system based variable structure terminal sliding mode control. The model used is a turbofan engine linear small deviations model, the mathematical model in the form of normalized is

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases}$$

$$X = [n_L n_H m_f A_e]^T, U = [\dot{m}_f \dot{A}_e]^T,$$

$$Y = [n_L T_4^*]^T.$$

Assuming beginning of control, low-pressure rotor speed n_L , high pressure rotor speed n_H , fuel supply m_f , last nozzle area A_e have a

small amount of deviation, That the system initial state point

$X_0 = [0.05 \ 0.04 \ 0.03 \ 0.03]^T$. Control law parameters are

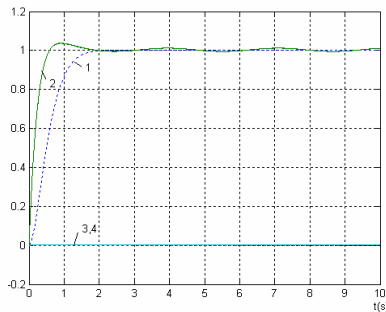
$$\alpha_0 = 2, \beta_0 = 1, p_0 = 9, q_0 = 5, \\ \phi = 10, \gamma = 10, p = 3, q = 1.$$

We want the adjust time not more than three seconds. According to the design requirements, select the sliding mode that the motion hope pole set $\{-3, 3\}$, the system in the hope regulation time that Basic realization of the perfect tracking.

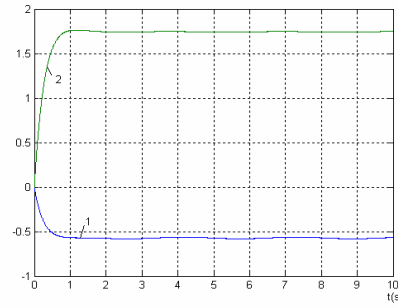
On this basis, added to the system the mismatched interference. The interference of the applied sinusoidal interference $0.8 \sin(2t)$,

$D = [1 \ 1 \ 1 \ 1]^T$, a simulation result is shown in Figure 3.1. Visible system can well suppress interference.

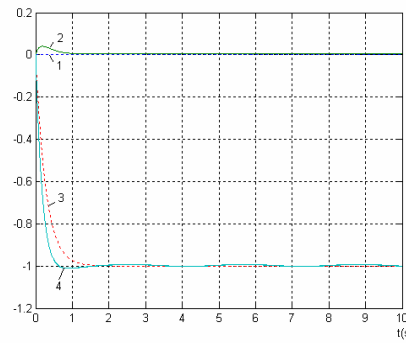
Simulation results show that: when the system parameter perturbation are not met uncertainties matching conditions, by the adjustment of the sliding mode parameter matrix, such that the closed-loop system is stable and the basic tracking. But in larger parameter perturbations, the adjustment time will become too long, the overshoot increases. Figure 3.2 show the response curve. When parameter perturbation ΔA amplitude up to 25%. As can be seen from the diagram, the adjustment time of the system up to more than 4 seconds, the overshoot. After Simulation study, when the parameter perturbation does not exceed 21% of the system's resistance to parametric capabilities are strong, the error can be controlled in less than 2%.



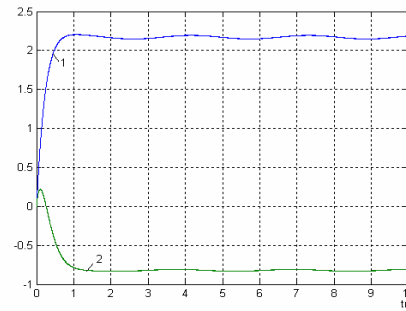
3.1a $R_{n_L} = 1, R_{T_4} = 0, 1 - R_{n_L}, 2 - n_L, 3 - R_{T_4}, 4 - T_4^*$



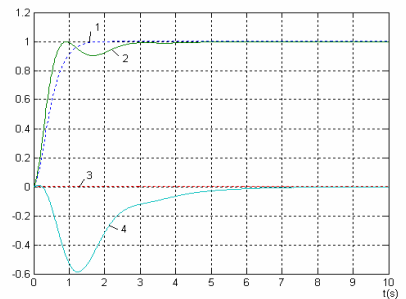
3.1b $R_{n_L} = 1, R_{T_4} = 0, 1 - m_f, 2 - A_e$



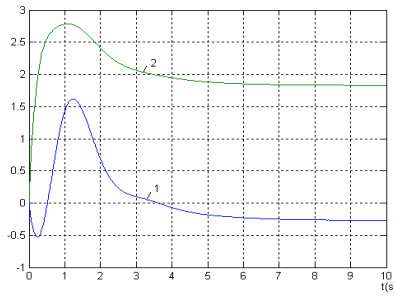
3.1c $R_{n_L} = 0, R_{T_4} = -1, 1 - R_{n_L}, 2 - n_L, 3 - R_{T_4}, 4 - T_4^*$



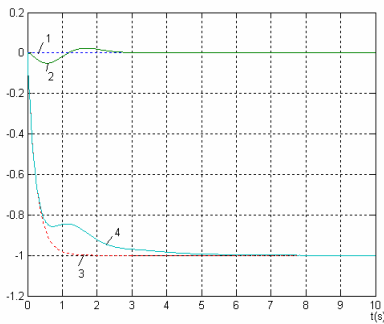
3.1d $R_{n_L} = 0, R_{T_4} = -1, 1 - A_e, 2 - m_f$



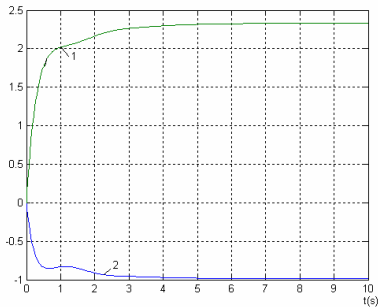
3.2a $R_{n_L} = 1, R_{T_4} = 0, 1 - R_{n_L}, 2 - n_L, 3 - R_{T_4}, 4 - T_4^*$



$$3.2b \quad R_{n_L} = 1, R_{T_4^*} = 0, 1 - m_f, 2 - A_e$$



$$3.2c \quad R_{n_L} = 0, R_{T_4^*} = -1, 1 - R_{n_L}, 2 - n_L, 3 - R_{T_4^*}, 4 - T_4^*$$



$$3.2d \quad R_{n_L} = 0, R_{T_4^*} = -1, 1 - A_e, 2 - m_f$$

4. CONCLUSION

For aero-engine is a complex, time-varying uncertainties, strongly nonlinear systems. the paper proposes a global variable structure fast terminal sliding mode control method of an aero-engine. By the method in the aero-engine nonlinear control applications, the results show that the paper designs the global fast terminal sliding mode variable structure control performance is good, a strong suppression of outside interference, the accused have robustness in entire control stage. It can overcome the uncertainty of the aviation engine, not only no steady-state error but also eliminate the

control effect the buffeting phenomenon, as the aero-engine nonlinear controller is effective.

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