

A 3D MODEL ASYMMETRIC WATERMARKING ALGORITHM BASED ON OPTIMIZATION STATISTICS

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ABSTRACT

The popularity of 3D content is on the rise since it provides an immersive experience to viewers, which has motivated us to develop techniques for watermarking 3D models so that we can hide copyright information behind them. In this paper, a novel method suitable for 3D models asymmetric watermarking applications is proposed based on optimization statistics. Through choosing the vertexes, we are able to obtain the embedded watermark that has the least modified to topology transform of the 3D geometry model, and then project the watermark to the space that has the least mean square error value. So, we obtain that the robustness of the approach lies in hiding a watermark in the space that is least susceptible to the 3D model potential modification. Through analysis and constraint the conditions, we can obtain a high detection probability, a low false alarm probability. The robustness of our method is demonstrated by various attacks through computer simulation.

Keywords: 3D model watermarking, Asymmetry, Optimization statistics, Robustness, Security

1. INTRODUCTION

With the emerging of low cost 3D display devices, different kinds of 3D applications and the amount of 3D content are booming up, recently[1,2]. As an important technique for information security and information protection, digital watermarks have been widely applied into fields such as digital signature and authentication etc[3,4]. Digital watermark now has been widely applied into fields such as audio products, digital images and videos etc [5,6]. However, because of its own characteristics such as various representations, unset sequence of data, users could conduct geometry and topology attack, it is difficult to apply the other successful algorithms into the three-dimensional model[7-9].

2. THE CHOOSING AND SEQUENCING OF VERTEXES

To 3 dimensional models, the vertex is the most appropriate for embedding watermark, which is different from the other properties such as the easy changes of color and texture. We select and sequence the vertexes of the 3 dimensional geometrical models with the following methods.

Suppose 3 dimensional geometrical model is O , vertex $V_i \in O$, the corresponding coordinate position of V_i is $V_i = (v_{ix}, v_{iy}, v_{iz})^T$. Firstly calculate the vertexes number of O and the effective distances between V_i by the following equation,

$$N(V_i) = \{V_j \mid |V_j V_i| > 0, j = 1, 2, \dots, N_i\} \quad (1)$$

In the above equation $|V_j V_i|$ represents the distance of connection line between vertexes V_j and V_i (if there is no connection, then $|V_j V_i| = 0$), $N(V_i)$ represents the number of vertexes between V_i which connects with lines (excluding V_i). Suppose $D(V_i)$ represents the Euclidean distance of the lines which connect with V_i ,

$$D(V_i) = \sum_{V_j \in N(V_i)} \|V_i - V_j\|^2 \quad (2)$$

Then confirm the first selected and sequenced peak $V_{(1)}$ by the following equation,

$$V_{(1)} = \min_{V_i \in O} D(\hat{V}_i) \quad (3) \quad \left\{ w_i = (w_{ix}, w_{iy}, w_{iz}), 1 \leq i \leq n \right\} \quad \text{into the vertex } V,$$

Then remove peak $V_{(1)}$ and the peaks $V_{(1)}$ connecting with the effective lines. Then, continue the sequencing of vertexes by the following equation,

$$\|V_{(k-1)} - V_{(1)}\|^2 < \|V_{(k)} - V_{(1)}\|^2 \quad (4)$$

After each of sequencing, remove the selected vertexes and the vertexes connecting with the effective lines. (It is not deletion but the exclusion while selecting the next point so as to make sure that there is no connection relation between the points). If the Euclidean distance of two points is the same, then calculate the area and size of the triangle which is composed of the points by the following equation.

$$A(V_i) = \frac{1}{2} \sum_{V_j \in N(V_i)} \|V_i - V_j\| \|V_i - V_{(j+1)}\| \times \sin(\overline{V_i V_j}, \overline{V_i V_{(j+1)}}) \quad (5)$$

The point with the least area should be high on the list.

3. ANALYZING AND CALCULATION OF THE VERTEX DISTURBANCE'S MEAN SQUARE ERROR

3.1 Obtaining the Error Energy

Mark the selected point sequence as $V_{(p)} = (v_{(1)}, v_{(2)}, \dots, v_{(n)})^T$,

$$\left\{ v_{(i)} = (v_{(i)x}, v_{(i)y}, v_{(i)z})^T \right\}, \quad 1 \leq i \leq n,$$

$v_{(i)x}, v_{(i)y}, v_{(i)z}$ represents the coordinate position of $V_{(i)}$. In order to realize the least mapping of attacks to the embedding watermarks, we firstly need to simulate the attackers to conduct kinds of attacking manipulation the 3D model which has been embedded with watermarks. More specifically, we firstly need to embed the watermark w ,

$$w = (w_1, w_2, \dots, w_n)^T,$$

$$V_{(p)}^M = V_{(p)} + w \quad (6)$$

Conduct kinds of attacking manipulation including rotation and simplification manipulation to the 3 dimensional geometric model M which has been embedded with watermarks (the vertex after selection sort $V_{(p)}^M = (v_{(1)}^M, v_{(2)}^M, \dots, v_{(n)}^M)^T$. We

obtain a group of changed 3 dimensional geometric model, select one of them arbitrarily and mark it as O^s . Similarly the vertex sequence for O^s , after the same selection and sequencing is

$V_{(p)}^G = (v_{(1)}^g, v_{(2)}^g, \dots, v_{(n)}^g)^T$. The error energy is,

$$\|e\|^2 = \sum_{i=1}^n \left(\begin{aligned} & (v_{(i)x}^g - v_{(i)x}^M)^2 + (v_{(i)y}^g - v_{(i)y}^M)^2 \\ & + (v_{(i)z}^g - v_{(i)z}^M)^2 \end{aligned} \right) \quad (7)$$

The above equation only considers the separate event in terms of random error, for more common sense; please refer to the following equation,

$$E(\|e\|^2) = E \left(\sum_{i=1}^n \left(\begin{aligned} & (v_{(i)x}^g - v_{(i)x}^M)^2 + \\ & (v_{(i)y}^g - v_{(i)y}^M)^2 + (v_{(i)z}^g - v_{(i)z}^M)^2 \end{aligned} \right) \right) \quad (8)$$

In the above equation, $E(\cdot)$ is the mathematical expectation operation. According to linear algebra, the right side of (8) is equal to the calculation of the following equation,

$$\begin{aligned} & \sum_{i=1}^n \left(\begin{aligned} & (v_{(i)x}^g - v_{(i)x}^M)^2 + \\ & (v_{(i)y}^g - v_{(i)y}^M)^2 + (v_{(i)z}^g - v_{(i)z}^M)^2 \end{aligned} \right) \quad (9) \\ & = \text{trace} \left((V_{(p)}^G - V_{(p)}^M)^T (V_{(p)}^G - V_{(p)}^M) \right) \end{aligned}$$

In the above equation, $\text{trace}(\cdot)$ is the matrix trace operation. According to algebra theory [Literature 5, P55], we could get the following equation,



$$\begin{aligned} \text{trace} \left(\left(V_{(p)}^G - V_{(p)}^M \right)^T \left(V_{(p)}^G - V_{(p)}^M \right) \right) &= w = Gw_0 \quad (16) \\ \text{trace} \left(\left(V_{(p)}^G - V_{(p)}^M \right) \left(V_{(p)}^G - V_{(p)}^M \right)^T \right) & \end{aligned} \quad (10)$$

Therefore we could get the following equation,

$$E \left(\|e\|^2 \right) = \text{trace} \left(E \left(\left(V_{(p)}^G - V_{(p)}^M \right) \left(V_{(p)}^G - V_{(p)}^M \right)^T \right) \right) \quad (11)$$

3.2 Obtaining the Space Embedded with Watermarks

The value of equation (11) is the average error energy but we are more concerned about how to realize the least mapping energy from the error energy to the embedding watermarks. For this reason, solve the following homogeneous equation.

$$x^T E \left(\left(V_{(p)}^G - V_{(p)}^M \right) \left(V_{(p)}^G - V_{(p)}^M \right)^T \right) x = 0 \quad (12)$$

Conduct SVD division for

$$E \left(\left(V_{(p)}^G - V_{(p)}^M \right) \left(V_{(p)}^G - V_{(p)}^M \right)^T \right) \text{ and obtain,}$$

$$E \left(\left(V_{(p)}^G - V_{(p)}^M \right) \left(V_{(p)}^G - V_{(p)}^M \right)^T \right) = U \Sigma U^T \quad (13)$$

Matrix $U = (u_1, u_2, \dots, u_n)$, $U \in R^{n \times n}$ is orthogonal matrix (as

$E \left(\left(V^G - V^M \right) \left(V^G - V^M \right)^T \right)$ is symmetric

matrix so the two orthogonal matrixes after SVD division are the same), SVD

matrix $\Sigma = \text{diag} (\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2)$ and meet the

requirements as $\sigma_i^2 \geq \sigma_{i+1}^2 \geq 0, 1 \leq i \leq n$. Take (13) into (12) and obtain,

$$x^T U \Sigma U^T x = 0 \quad (14)$$

Because of the characteristic of orthogonal matrix, if select $x \in G$ according to the following principles,

$$\begin{cases} u_i \in G & \text{if } \sigma_i^2 = 0 \\ u_i \in H & \text{if } \sigma_i^2 \neq 0 \end{cases} \quad (15)$$

Then the mapping energy to space G is zero. Therefore if the selected w meets the following requirements,

Then the mapping error energy to w_0 is zero so as to realize the least mapping mean square error energy from 3 dimensional image manipulation to space G. In this way we could make sure of the extracting quality of the watermarks. In reality, as the number length of zero in SVD Σ could not meet the requirement of length for embedding watermarks (From the view of watermark security, the length of watermark must be large enough); we could choose the feature components with the relatively small SVD value to build up the feature space. Divide according to the following equation.

$$\begin{cases} \sigma_i^2 > \sigma^2 & \sigma_i^2 \text{ corresponding feature vectors } u_i \in H \\ \sigma_i^2 \leq \sigma^2 & \sigma_i^2 \text{ corresponding feature vectors } u_i \in G \end{cases} \quad (17)$$

σ^2 is the predefined threshold. Here we suppose $G \in R^{n \times (n-L)}, H \in R^{n \times L}$.

4. METHODS FOR WATERMARKING 3D MODELS

Conduct the selection and sequencing for the original 3 dimensional geometrical model, then we could get the vertex sequence as

$V_{(p)} = (v_{(1)}, v_{(2)}, \dots, v_{(n)})^T$. Let the embedding

watermark $W_0 \in R^{g_1 \times g_2}$ ($g_1 \times g_2 = n - l$), we alternate W_0 to W_1 [literature 10 , theorem 1 in], so,

$$UW_1V = W_1 \quad (18)$$

U and V are unitary matrixes . Then, we obtain the row coordinate

$cs(W_1), cs(W_1) \in R^{(n-l) \times l}$. Suppose matrix G ,

$G \in R^{n \times (n-l)}$ ($1 \leq l \leq n$) is a group of unit

orthogonal basis. G is the secret key, that is to say, while embedding the watermarks into the matrix,

G_2 and $cs(W_1)$ are public keys, namely the watermark detection matrix.

4.1 Embedding Watermarks

Mark the watermark as $W_1, W_1 \in R^{g_1 \times g_2}$,



$$W_2 = (cs(U_1 W_1 V), cs(U_1 W_1 V), cs(U_1 W_1 V))_{(n-l) \times 3} \quad (19)$$

Among $W_2 \in R^{(n-l) \times 3}$, unitary matrix $U_1 \in R^{g_1 \times g_1}$. Then we embed W_2 to the vertex of 3 dimensional geometrical model according to the following equation,

$$V_{(p)}^M = V_{(p)} + k_1 \times G \times W_2 \quad (20)$$

Construct the matrix G_2 as per the following:

$$G_2 = G \times (E_{g_2 \times g_2} \otimes U_2)^T \quad (21)$$

Among \otimes is Kronecker operator, $E_{g_2 \times g_2}$ is $g_2 \times g_2$ unit matrix. So, asymmetric extraction is realized.

4.2 Obtaining the Watermarks

While detecting a 3 dimensional model, firstly it is necessary to re-position the model into the original predefined coordinate axis, then re-sample (for the detailed part please refer to Part5). Then we could obtain the 3 dimensional geometric model O^{M1} , sequence and select the vertexes then we could get the vertex sequence as $V_{(p)}^{M1}$. After the above steps, then we could use the public key to extract the watermarks according to the following equation.

$$G_2^T \times (V_{(p)}^{M1} - V_{(p)}) = (cs(W_1^*), cs(W_1^*), cs(W_1^*)) \quad (22)$$

Conduct smoothing for the sake of increasing the robustness of the extracted watermark signals.

Suppose the row coordinate as $d = \frac{1}{3}(1, 1, 1)^T$ the

extracted watermark W_1^t is as follows,

$$cs(W_1^t) = (cs(W_1^*), cs(W_1^*), cs(W_1^*)) \times d \quad (23)$$

Then make the relevant conclusion. Although the attackers could analyze the division of the feature space and obtain matrix H . But he could not calculate G_1 on the basis of the public keys G_2 , $cs(W_1)$ and H , the attackers could not remove

the embedded watermarks so it is possible to make sure of the watermark's security in this way.

4.3 Detection Probability and False Alarm Probability

Use the relevant decision functions for detecting, make the projection of $V_{(p)}^{M*} - E(V_{(p)}^M)$ through

G_2^T and obtain,

$$\begin{aligned} G_2^T (V_{(p)}^{M*} - E(V_{(p)}^M)) d = \\ (c \times (cs(W_1), cs(W_1), cs(W_1)) + n_1) d \\ = c \times cs(W_1) + n_2 \end{aligned} \quad (24)$$

In the above equation, C is the constant for the projection of $V_{(p)}^{M*} - E(V_{(p)}^M)$

through G_2^T , $n_2 \in R^{(n-l) \times 1}$ is projection error.

Suppose detection decision function meets the following equations.

$$\left\{ \begin{array}{l} \|n_2\| \ll \|cs(W_1)\| \quad \text{if } V_{(p)}^{M*} - E(V_{(p)}^M) \text{ through} \\ \quad G_2^T \text{ including } cs(W_1) \\ |c| \rightarrow 0 \quad \text{if } V_{(p)}^{M*} - E(V_{(p)}^M) \text{ through} \\ \quad G_2^T \text{ no including } cs(W_1) \end{array} \right. \quad (25)$$

Use the relevant testing functions,

$$\begin{aligned} sim(cs(W_1), G_2^T \times (V_{(p)}^{M*} - V_{(p)})) d = \\ sim \left(cs(W_1), G_2^T \times \left(\begin{array}{l} (V_{(p)}^{M*} - E(V_{(p)}^M)) \\ + (E(V_{(p)}^M) - V_{(p)}) \end{array} \right) d \right) \\ = \frac{|cs(W_1)^T (G_2^T \times ((V_{(p)}^{M*} - E(V_{(p)}^M)) + (E(V_{(p)}^M) - V_{(p)})) d)|}{\|cs(W_1)\| \|G_2^T \times ((V_{(p)}^{M*} - E(V_{(p)}^M)) + (E(V_{(p)}^M) - V_{(p)})) d\|} \end{aligned} \quad (26)$$

Detection probability: If the projection of $V_{(p)}^{M*} - E(V_{(p)}^M)$ through G_2^T contains $cs(W_1)$, then according to (25), we could obtain the following equation,

$$\begin{aligned} & \text{sim}\left(\text{cs}(W_1), G_2^T \times \left(V_{(p)}^{M*} - E\left(V_{(p)}^M\right)\right)\right) \\ &= \text{sim}\left(\text{cs}(W_1), \left((1+c)\text{cs}(W_1) + n_2\right)\right) \\ &\approx \frac{|1+c| \left|\text{cs}(W_1)^T \text{cs}(W_1)\right|}{|1+c| \left\|\text{cs}(W_1)^T\right\| \left\|\text{cs}(W_1)\right\|} = 1 \end{aligned} \quad (27)$$

False Alarm Probability: If the projection of $V_{(p)}^{M*} - E\left(V_{(p)}^M\right)$ through G_2^T does not contain $\text{cs}(W_1)$, according to (25), we could obtain the following equation,

$$\begin{aligned} & \text{sim}\left(\text{cs}(W_1), G_2^T \times \left(V_{(p)}^{M*} - E\left(V_{(p)}^M\right)\right)\right) \\ &= \text{sim}\left(\text{cs}(W_1), \left((1+c)\text{cs}(W_1) + n_2\right)\right) \\ &\approx \frac{|c| \left|\text{cs}(W_1)^T \text{cs}(W_1)\right|}{|c| \left\|\text{cs}(W_1)\right\| + \left\|\text{cs}(W_1)\right\| \|n\|} \approx 0 \end{aligned} \quad (28)$$

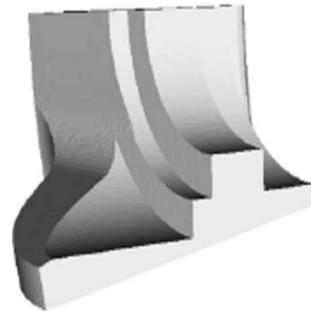
5. RE-POSITIONING AND RE-SAMPLING OF 3 DIMENSIONAL MODEL

The issue of re-positioning the 3 dimensional model has been given in several algorithms in [11]. The paper adopts the positioning algorithm in Literature [12] and adds a degree of freedom, that is to say, excluding constant scaling, after re-positioning the 3 dimensional model, the paper uses the grid re-sampling method, selects and sequences the vertexes of the 3 dimensional models (For more please refer to part 2).

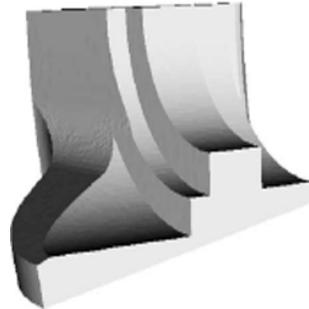
6. SIMULATION EXPERIMENT

In the experiment, the selected 3D geometric model is shown as Figure 1(a), the model has 6457 vertexes and 12946 surfaces. In the experiment, the data size of 3D model is all 3D-Studio (3ds). According to the previous analysis, we need to sequence and select the vertexes. After the selection and sequencing, there are totally 1024 selected vertexes. (For the sake of convenient selection of binary image). Conduct row expansion to the binary image (32×32) dimensional (Considering the 3D model's ability of embedding data and obtain the 1024 dimensional row coordinate). Embed it into

the 1024 vertexes which have been selected and sequenced. Then conduct geometric transform and topological transforms such as rotation ($\pm 1^0, \pm 2^0$), 3D rotation, cutting and noise attack etc to the 3D model embedded with watermarks, after this we could obtain 216 transformed 3D model. Select and sequence the vertexes of these 216 3D models and calculate the mathematical expectation for the 216 selected and sequenced vertex lists. Then select one of them arbitrarily and calculate the delta between the selected vertex and the mathematical expectation. Calculate the delta's covariance and obtain (1024×1024) dimensional covariance matrix. Conduct SVD division to the covariance matrix and obtain the unitary matrix of 1024 dimensional. In the 1024 dimensional space of unitary space, select the 900 dimensional feature component with the least corresponding value as the watermark embedding space (the watermark embedding space should not be too small otherwise the secret key could be easily decoded and attacked). Select the binary image of 30×30 (for Figure 1(c) is the watermark which is going to be embedded, firstly build up the full column rank and full row rank for the binary image, then transform by equation(19). Embed the watermark into the robustness space and form the 3D model with watermarks as Figure (1) b,



(a) The original 3D geometric model (6475 vertexes and 12946 surfaces)

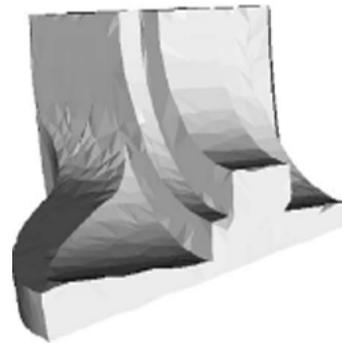


(b) 3D geometric model embedded with watermarks (6475 vertexes and 12946 surfaces)



(c) The extracted watermark(30×30)

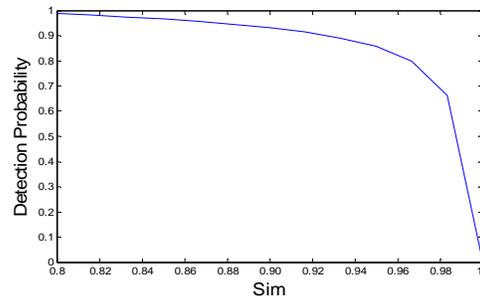
Figure 1: The model used in the experiment watermark



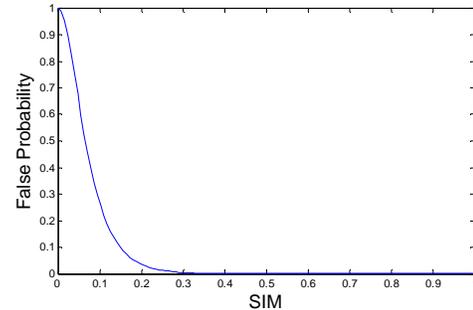
(b) Simplification attack (1/4)

Figure 2: The 3 dimensional model after simplification attack

After conducting several attacks to the 3D models with and without watermarks, make the relevant testing and we could get the testing probability and false alarm probability as Fig 3. Seen from Picture we could know that the algorithm has high detection probability and low false alarm probability. In the meanwhile, we have also conducted the simplification attack testing to the algorithm and the 3D model after attacks is shown as Picture2. We conduct the geometric attack and topological attack to 3D model with watermarks and the result of the experiment is shown as Table 1. The result has proven our theoretical analysis. Due to the size of this paper, here we only list the table of several main indicators for the experiment.

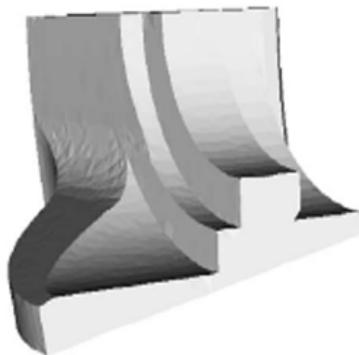


(a) Detection probability



(b) False alarm probability

Figure 3: Detection Probability and False Alarm probability



(a) Simplification attack(1/2)

Table 1: Test and Blind Attack Test.

Attack Type	Sim Mean	Sim Variance
Noise (Density 0.04)	0.9592	0.0322
Noise (Density 0.02)	0.9731	0.0138
Noise (Density 0.01)	0.9921	0.0017
Rotation ($\pm 2^0$)	0.9811	0.0112
3 d rotation ($\pm 2^0$)	0.9601	0.0024
Smooth	0.9978	0.0010
Simplification	0.9873	0.0169
simplification attack	0.9689	0.0247
cut attack(1/16))	0.9610	0.0139
shift	0.9911	0.0091
remeshing	0.9731	0.0136



7. CONCLUSION

According to the theoretical analysis and experiment result, we have proven that the asymmetric 3D watermark algorithm based on the division of feature space is feasible. It has high level of security and high performance of robustness.

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