Journal of Theoretical and Applied Information Technology
20th May 2013. Vol. 51 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

VIBRATION CONTROL OF PIEZOELECTRIC FLEXIBLE STRUCTURE USING ROBUST CONTROL METHODOLOGY

JUNFENG HU, GUIYANG XU

School of Mechanical & Electrical Engineering, Jiangxi University of Science and Technology, Ganzhou 341000, China

E-mail: hjfsuper@126.com, 997413919@qq.com

ABSTRACT

Active vibration control of a smart flexible cantilever beam was studied by using mode theory and μ synthesis method. The actuator is piezoceramic patch, the sensor is strain gauge. The finite element method and experimental modal test was applied to obtain the dynamic model of the smart structure. Taking account into uncertainty of the external disturbance and measurement noise, and uncertainty of natural frequency, damping ratio and actuator parameters of the system, μ synthesis approach was applied to design the vibration controller by selecting mode displacement as evaluation signal and selecting appropriate weighting function according to amplitude and frequency characteristics of the actual signal. A H_∞ controller with same weightings was also designed and implemented for comparison analysis. The performances of the controller are verified experimentally in this study. Experimental results showed that μ-controller can provide good disturbance rejection and is more robust to parameter variations than H_∞ controller. And the proposed controllers can effectively suppress the vibration response of the flexible beam.

Keywords: *Smart Structure, Active Vibration Control, Finite Element Method, μ Synthesis, Experimental Modal Test*

1. INTRODUCTION

In precision and aerospace industry, many researches on lightweight and miniaturized structures have been carried out to improve structural performances. Among the researches, passive structures using composite material are typically known as one of the effective methods. However, the traditional passive structures are very sensitive to change of internal load condition and external environment condition which can even cause sudden destruction of structures. Therefore, in order to satisfy stringent requirements for precision control and lightweight miniaturization, smart materials such as shape memory alloys, piezoceramics, and magnetorheological fluids are frequently adopted for smart structures. The performance requirements of future space structures, jet fighters and concept automobiles have brought much interest to the area of smart structures. A smart structure can be defined as a structure with bonded or embedded sensors and actuators as well as an associated control system, which enable the structure to respond simultaneously to external stimuli exerted on it and then suppresses undesired effects or enhance desired effects. Among various smart structures, those with piezoelectric patches have received much attention in recent years, due to the fact that piezoelectric materials have simple mechanical properties, small volume, light weight, large useful bandwidth, efficient conversion between electrical and mechanical energy, good ability to perform vibration control and ease of integration into metallic and composite structures [1-3].

Different control techniques have been investigated in the control of smart structure. Abreu conducted experimental work for the vibration control of flexible beam by using piezoelectric sensors and actuators with Linear Quadratic Gaussian (LQG) controller [4]. There are many classical strategies that can be used when the mathematical model is available, for instance pole allocation and optimal control. However, if the model has uncertainties these methods are not indicated. There are many robust techniques in structural control literature. Li investigated two control strategies for robust vibration control of

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

parameter uncertain systems [5]. Mayhan combined intelligent control and smart materials to produce an adaptive and robust controller to dampen the fundamental vibration mode of the system in the presence of modeling uncertainties [6]. Zhang et al. studied the active vibration control problem for the high-speed flexible mechanisms all of whose members were considered as flexible by using complex mode method and robust H_∞ control scheme [7-8]. Kawabe utilized neural networks (NN) theory for active control in a longitudinal cantilevered-beam system by simulation and experiment. It is found that fairly satisfactory active damping effect using the NN controller is obtained [9]. But the random disturbance and measurement noise of the actual system were not considered by these currently proposed vibration control strategies. The issue of robustness against external disturbances was not addressed, and therefore the proposed vibration controllers cannot be effectively applied to the smart structure under the random uncertain disturbances.

Very few attempts have been made toward the application of robust control methodology to control vibrations in lightweight flexible structures. Robust vibration control methodology has received much attention due to their wide applications, and led to a rapid development of various control strategies such as the LQG control and the fuzzy control, etc. In recent years, a great deal of attention has been paid to the H_∞ control because it not only provides a unified and general control framework for all control structures, but also yields a controller with guaranteed margins. However, H_∞ control models all uncertainties as a single complex full block, which results in a rather conservative design. Under such circumstances, the μ synthesis technique, which involves the use of H_∞ optimization for synthesis and structured singular value for analysis, has been developed. During the controller design, an issue to be considered is the process of the mathematical model between the disturbance force and the manipulator, which is always assumed to be known. In engineering practice, however, it is difficult to obtain this model even though system identification or theoretical approach. Hence, how to design a μ-controller without requiring a mathematical model between the disturbance force and structure is of great interest in the application of μ-synthesis technique.

We aim here to deal with the active vibration reduction problem in flexible structure with uncertainties through designing reasonable μ-controller. In this paper, the vibration control of a flexible beam is investigated by using μ synthesis and experimental modal test method, and taking into account the random disturbance uncertainty, modal parameter uncertainty and measurement noise.

The paper is organized as follows. In section 2, a dynamic model of a flexible beam bonded with piezoelectric actuators and strain gauge sensors is constructed by using finite element method. In section 3, the μ controller is proposed. In section 4, experimental identification of the flexible cantilever beam is performed to obtain its modal parameters. And the experimental validation test is performed based on the dSPACE DS1103 platform. The conclusions are given in section 5.

2. DYNAMIC MODELING OF SMART STRUCTUE

The modeling of smart structure with piezoelectric actuators and sensors has been a subject of intense research for a long time and is only briefly described here.

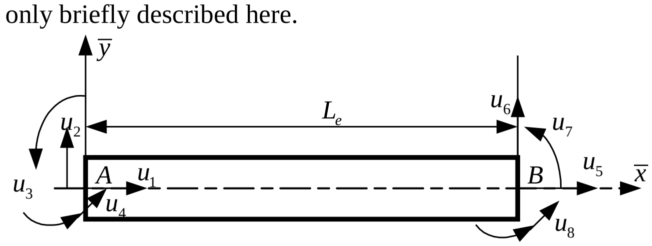

Figure 1: Planar beam element showing nodal degrees of freedom and coordinate systems

The flexible structure is modeled by using a two-node beam element. The beam element is shown in Fig. 1, which has two nodes with four degrees of freedom at each node; namely u_1, u_5, the longitudinal displacement, and u_2, u_6, the transverse displacement, and u_3, u_7, the slope, and u_4, u_8, the curvature. L_e is the length of element. The nodal displacement vector $\boldsymbol{u}$ with respect to reference frame $A-\bar{x}\bar{y}$ is expressed as

$$\boldsymbol{u} = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad u_6 \quad u_7 \quad u_8]^{\mathrm{T}} \quad (1)$$

The transverse and longitudinal displacement fields of two-node beam element are constructed using the quintic hermite and linear interpolation polynomials, respectively. V,

Journal of Theoretical and Applied Information Technology
20th May 2013. Vol. 51 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

W denotes longitudinal and transverse elastic displacement of arbitrary point, respectively. Subscript T denotes matrix transpose. They can be written by the following form

$$\begin{bmatrix} V(\bar{x},t) \\ W(\bar{x},t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{N}_1(\bar{x}) \\ \boldsymbol{N}_2(\bar{x}) \end{bmatrix} \boldsymbol{u} \qquad (2)$$

where $\boldsymbol{N}(\bar{x}) = \left[\boldsymbol{N}_1(\bar{x}) \quad \boldsymbol{N}_2(\bar{x}) \right]^{\mathrm{T}}$ is shape function, $\boldsymbol{u}$ is the nodal displacement vector.

The system dynamic equations can be obtained by using finite element method

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{C}\dot{\boldsymbol{U}} + \boldsymbol{K}\boldsymbol{U} = \boldsymbol{P} \qquad (3)$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are the systematic mass, damping, stiffness, respectively. $\boldsymbol{U}$, $\dot{\boldsymbol{U}}$, $\ddot{\boldsymbol{U}}$ are the generalized displacement, velocity, and acceleration vectors of the system, respectively. $\boldsymbol{P}$ is the systematic generalized disturbance force vector corresponding to vector $\boldsymbol{U}$.

The piezoelectric patches work as actuators are perfectly bonded on the upper and lower surfaces of the beam at the same location. For the modeling of PZT actuators, literature [10, 11] provides a detailed derivation of coupling of PZT actuators and a host beam. These bending moments induced by the actuators is given by

$$T_A = T_B = -d_{31} E_{\mathrm{p}} b_{\mathrm{p}} \left(t_{\mathrm{p}} + t_{\mathrm{a}} \right) \boldsymbol{V}_{\mathrm{in}} \qquad (4)$$

where d_{31} , E_{p} , b_{p} , t_{p} is piezoelectric constant, elastic module, width, and thickness of PZT patch, respectively. t_{a} is the thickness of the beam element, $\boldsymbol{V}_{\mathrm{in}}$ is the vector of input voltage to the piezoelectric actuators. The moments are assembled as a part of the external moments exerted on node.

Strain is the amount of deformation of a structure due to an applied force. Strain gauge is the most common sensor for measuring strain. A strain gauge's electrical resistance varies in proportion to the amount of strain placed on it. The deformations mainly include compression or tension and bending deformation for the flexible beam. The strain ε in $\bar{x}$ direction is given by

$$\varepsilon = \varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{B}} \qquad (5)$$

where ε_{L} is compression or tension strain and ε_{B} are longitudinal strain due to bending deformation, respectively. They can be given by

$$\begin{aligned} \varepsilon_{\mathrm{L}} &= \boldsymbol{N}_1'' \boldsymbol{u} = \boldsymbol{N}_1'' \boldsymbol{B}\boldsymbol{U}, \\ \varepsilon_{\mathrm{B}} &= -h\boldsymbol{N}_2'' \boldsymbol{u} = -h\boldsymbol{N}_2'' \boldsymbol{B}\boldsymbol{U} \end{aligned} \qquad (6)$$

where h is the distance between the neutral axis of the beam and the outer surface of the beam, $\boldsymbol{N}_1''$, $\boldsymbol{N}_2''$ are the first-order and the second-order differential of the shape function $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$, respectively, $\boldsymbol{B}$ denotes the transformation matrix. Thus, the exogenous perturbation and the control inputs have no direct effect on the measured outputs.

There are a piezoelectric actuators and s strain gage sensors on the flexible structure. Combining Eq. (3)~ Eq.(6), the dynamic equations of the structure equipped with piezoelectric actuators and strain gauge transducer can be expressed as

$$\begin{aligned} \boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{C}\dot{\boldsymbol{U}} + \boldsymbol{K}\boldsymbol{U} &= \boldsymbol{P} + \boldsymbol{D}_a \boldsymbol{V}_{\mathrm{in}} \\ \boldsymbol{y} &= \boldsymbol{D}_s \boldsymbol{U} \end{aligned} \qquad (7)$$

where $\boldsymbol{D}_{\mathrm{a}}$ is the systematic control matrix related to configuration of actuators, $\boldsymbol{D}_{\mathrm{s}}$ is the systematic output matrices determined by configuration of sensors, a a -by-1 vector, and $\boldsymbol{y}$ is the strain from the sensors, a s -by-1 vector.

It was shown that the dynamic response of the flexible structure is composed mainly of the lower modes. In order to control the lower modes, the physical-coordinate equations must be first transformed into modal coordinates. Here, we choose the first c order modes as control modes. Applying the modal theory, the normalized modal transformation is introduced by

$$\boldsymbol{U} = \boldsymbol{\psi}_{\mathrm{c}} \boldsymbol{\eta}_{\mathrm{c}} \qquad (8)$$

where $\boldsymbol{\psi}_{\mathrm{c}}$ is the controlled normalized modal matrix, $\boldsymbol{\eta}_c$ is controlled modal coordinate vectors. Substituting Eq. (8)into Eq.(7), the system dynamic equations are rewritten as

$$\begin{aligned} \ddot{\boldsymbol{\eta}}_{\mathrm{c}} + \boldsymbol{C}_{\mathrm{c}} \dot{\boldsymbol{\eta}}_{\mathrm{c}} + \boldsymbol{K}_{\mathrm{c}} \boldsymbol{\eta}_{\mathrm{c}} &= \boldsymbol{N}_{\mathrm{c}} + \boldsymbol{D}_{\mathrm{ac}} \boldsymbol{V}_{\mathrm{in}} \\ \boldsymbol{y} &= \boldsymbol{D}_{\mathrm{sc}} \boldsymbol{\eta}_{\mathrm{c}} \end{aligned} \qquad (9)$$

where $\boldsymbol{N}_{\mathrm{c}} = \boldsymbol{\psi}_{\mathrm{c}}^{\mathrm{T}} \boldsymbol{P}$, $\boldsymbol{D}_{ac} = \boldsymbol{\psi}_{\mathrm{c}}^{\mathrm{T}} \boldsymbol{D}_{\mathrm{a}}$, $\boldsymbol{D}_{\mathrm{sc}} = \boldsymbol{\psi}_{\mathrm{c}}^{\mathrm{T}} \boldsymbol{D}_{\mathrm{s}}$ $\boldsymbol{C}_{\mathrm{c}}, \boldsymbol{K}_{\mathrm{c}}$ is $c \times c$ diagram matrix, which is determined by system natural frequency and damping ratio.

For control synthesis, the system must be written as a system of first-order ordinary differential equations (ODEs). We can define the controlled state variables by the following form

$$\boldsymbol{X}_c = \left[\eta_1 \quad \cdots \quad \eta_c \quad \dot{\eta}_1 \quad \cdots \quad \dot{\eta}_c \right]^{\mathrm{T}} \qquad (10)$$

Due to the controlled mode number c , the number of controlled state variables is $2c$. Taking into account measurement noise $\boldsymbol{v}$, the state-space model for the system can be written as

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\begin{aligned} \dot{\boldsymbol{X}}_{c} &= \boldsymbol{A}_{c}\boldsymbol{X}_{c} + \boldsymbol{B}_{c}\boldsymbol{V}_{in} + \boldsymbol{N}_{c} \\ \boldsymbol{y} &= \boldsymbol{C}_{c}\boldsymbol{X}_{c} + \boldsymbol{v} \end{aligned} \qquad (11)$$

where

$$\boldsymbol{A}_{c} = \begin{bmatrix} \mathbf{0}_{c} & \boldsymbol{I}_{c} \\ -\boldsymbol{K}_{c} & -\boldsymbol{C}_{c} \end{bmatrix},\ \boldsymbol{B}_{c} = \begin{bmatrix} \mathbf{0}_{ac} \\ \boldsymbol{D}_{ac} \end{bmatrix},\ \boldsymbol{C}_{c} = \begin{bmatrix} \boldsymbol{D}_{sc} & \mathbf{0}_{sc} \end{bmatrix},$$

$\boldsymbol{I}_c$ is $c \times c$ unit matrix , $\mathbf{0}_{ac}$, $\mathbf{0}_{sc}$ are $c \times a$, $s \times c$ zero matrix, respectively.

3. ROBUST μ-CONTROLLER DESIGN

Robust control theory is exploited to develop a more simple and reliable controller for the flexible structure vibration control applications. The robust controller requires both nominal model and uncertainty model. So in order to make actual control become robust, it is necessary to provide an accurate and detail modeling of the uncertainties that is relevant to the structure model. Because the plant to be controlled is a flexible structure modal model is best one described its dynamic behavior.

In general, modal parameters of a flexible structure model are: resonance frequencies, damping ratios, modal vibration shape. Each of these modal parameters contains uncertainty. In addition, the truncated modes in the model reduction and signal noise in actuators and sensors also increase the uncertainty in the assumed model. The mathematical models for uncertainties established in robust theory are additive uncertainty. Since the natural frequencies and mode shapes of the system are functions of the time, Equations (11) are continuous time-varying state space representation of the mechanism system. Designing a controller with time-varying state-space matrices is out of the scope of this study. The prevailing practice in the analytical studies available is to assume constant state space matrices, i.e., constant natural frequencies and mode shapes for the entire motion cycle of the system. Although these results are not conclusive, similar assumption will be made in the controller design of current work.

Based on the model obtained in section 2, a controller is designed for the smart flexible beam. The goal of the controller is to attenuate the vibrations of the smart beam at its first two flexural frequencies and obtain gain stabilize the unmodeled high frequency modes. In H_∞ control design framework, the objective is to minimize the H_∞ norm of the weighted transfer functions from the input disturbance signals to the output error signals. The uncertainties in the plant model can be put in such a form that some of the disturbances and error signals correspond to the channels through which the nominal model interacts with a norm bounded uncertainty block. This generates the set of plants in which the true plant is assumed to exist. This framework is represented in Fig.2.

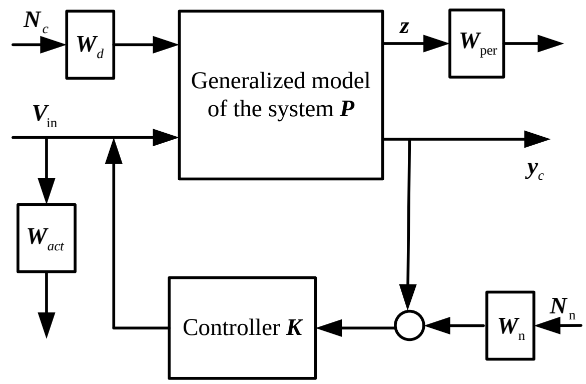

Figure 2: The augmented dynamic closed-loop system

From the perspective of robust control, the suppression of the vibration response of elastic mechanism is to inhibit the effect of the disturbance signal, such as modal force and measurement noise, on the output of the system. According to H_∞ control theory, that is, the better the control performance is, the smaller sensitivity function is. This is a minimum sensitivity control problem. According to the model of flexible manipulator, we can obtain the augmented dynamic closed-loop system as shown in Fig.2. In the Figure, $\boldsymbol{N}_c$ is the modal force, is treated as external disturbance term, and $\boldsymbol{W}_d$ is the weighted function of the signal. $\boldsymbol{N}_n$ is the measurement noise, and $\boldsymbol{W}_n$ is the weighted function corresponding to the noise signal. In general, in order to find a controller to suppress $\boldsymbol{N}_c$ in desired frequency bandwidth $\boldsymbol{y}_c$ should be weighted by a performance weighting function matrix. $\boldsymbol{z}$ represents the evaluation signal and $\boldsymbol{W}_{per}$ is the weighted function of the signal. $\boldsymbol{V}_{in}$ is the control voltage. In addition, the control voltage should be weighted by the weighting function matrix so as to prevent the saturation of controller output. In order to limit the voltage range, the weighted function $\boldsymbol{W}_{act}$ is used to penalize the signal. $\boldsymbol{y}_c$ is the measurement output of the system, the output strains from strain gages. Measurement noise represents electrical noise, sampling errors, drifting calibration, and other effects that impair measurement precision and accuracy.

In the design of the μ controller, we can choose the two different evaluation signals in accordance with the actual physical meaning of the

Journal of Theoretical and Applied Information Technology
20th May 2013. Vol. 51 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

controlled plant, the first type signal is the controlled mode displacement, and the other is the output strain signal from the sensor. And taking into account the range of input voltage of actuator, the two different evaluation signals z_1 and z_2 are represented by the following form

$$z_1 = \begin{bmatrix} \eta_c \\ V_{\text{in}} \end{bmatrix} = \begin{bmatrix} C_1 \\ 0_{N_a} \end{bmatrix} X_c + \begin{bmatrix} 0_c \\ I_{N_a} \end{bmatrix} V_{\text{in}} \tag{12}$$

where $C_1 = \begin{bmatrix} I_c & 0_c \\ 0_c & 0_c \end{bmatrix}$, I_c , I_{N_a} is $c \times c$ and $N_a \times N_a$ unit matrix, respectively. 0_c 、 0_{N_a} is $c \times c$ 、 $N_a \times N_a$ zero matrix. The external disturbance signal w includes the modal force signal and measurement noise signal, and it can denotes $w = [N_c, N_n]$. Combined equation(11) and equation(12), the state-space equation of the generalized controlled plant can be written as follow

$$\begin{aligned} \dot{X}_c &= A_c X_c + B_c V_{\text{in}} + N_c \\ y_c &= C_c X_c \\ z_1 &= \begin{bmatrix} \eta_c \\ V_{\text{in}} \end{bmatrix} = \begin{bmatrix} C_1 \\ 0_{N_a} \end{bmatrix} X_c + \begin{bmatrix} 0_c \\ I_{N_a} \end{bmatrix} V_{\text{in}} \end{aligned} \tag{13}$$

The determination of weighting function matrices $W_d(s), W_n(s), W_{\text{act}}(s), W_{\text{per}}(s)$ is a crucial step in the controller design. According to the amplitude and frequency characteristics of modal force and measurement noise signal, the weighting function $W_d(s)$ and $W_n(s)$ are set to respectively

$$\begin{aligned} W_d &= N_{\max} f_d(s) \\ W_n &= M_n \end{aligned} \tag{14}$$

where $N_{\max}$ is the maximum amplitude of the mode force signal, $f_d(s)$ denotes its characters in the frequency domain, M_n is the amplitude of the measurement noise signal.

The weighting function matrix $W_{\text{act}}(s)$ is selected as to prevent the saturation of controller output due to the limits of DS1103 D/A converter output. Suppose the maximum allowable value of the actuator $V_{\max}$, the frequency character of the signal f_{act} , the weighting function $W_{\text{act}}(s)$ is

$$W_{act} = \frac{1}{V_{\max}} f_{act}(s) \tag{15}$$

The introduction of $W_{\text{per}}(s)$ is to reduce the influence of disturbance on sensor outputs. In general, it is chosen with large amplitude so as to suppress the low-frequency vibration. There are two type evaluation signals. One is modal displacement and W_{per1} is its weighting function; the other is output strain and W_{per2} denotes the weighting function corresponding to the signal. They are given by

$$\begin{aligned} W_{per1} &= P_1 f_{per1}(s) \\ W_{per2} &= P_2 f_{per2}(s) \end{aligned} \tag{16}$$

where P_1 and P_2 is the weighting coefficient, $f_{\text{per1}}(s)$ and $f_{\text{per2}}(s)$ is chosen as a one-order diagonal matrix, and they can be given by

$$\begin{aligned} f_{per1}(s) &= diag\left(\frac{1}{s/\omega_1 + 1}, \cdots, \frac{1}{s/\omega_c + 1}\right) \\ f_{per2}(s) &= diag\left(\frac{1}{s/\omega_o + 1}, \cdots, \frac{1}{s/\omega_o + 1}\right) \end{aligned} \tag{17}$$

where $\omega_1 \sim \omega_c$ is chosen as the larger value than the first c controlled mode frequency, ω_o is a slightly larger value than the highest the controlled natural frequency value. That is, the output signal whose frequency is less than ω_o can be attenuated under the action of the controller.

In general, the design of a μ -control system includes the synthesis of the controller and the selection of weighting functions. The consideration of the physical system is crucial. The so-called physical system includes both the system to be controlled and the actuator/sensor configuration. Different selections of actuator/sensor locations lead to very different sensor/actuator transfer functions and accordingly affect the design of μ controllers. As is known, the actuator/sensor configuration determines the controllability and observability of the system, which has direct impact on the modes to be controlled and the control energy required. When piezoelectric actuators are used, they should be usually located at regions where the strain deformations of the most dominant modes are large enough. Ideally, sensors should be collocated (or at least very close to the actuators) to ensure the plant model described by the transfer function as a minimum phase system in the controlled frequency band. The number of sensors and actuators to be used depends on the number of modes to be controlled.

Combining equation(13)~(17), the transfer function of the generalized controlled plant P can be written by the following form

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{y}_c \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{V}_{\text{in}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{V}_{\text{in}} \end{bmatrix} \tag{18}$$

Suppose the controller is given by

$$\boldsymbol{V}_{\text{in}} = \boldsymbol{K}\boldsymbol{y}_c \tag{19}$$

Substituting equation (19) into equation (18), the closed-loop transfer function from $\boldsymbol{w}$ to $\boldsymbol{z}$ can be obtained

$$\boldsymbol{F}_l(\boldsymbol{P}, \boldsymbol{K}) = \boldsymbol{P}_{11} + \boldsymbol{P}_{12}\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{P}_{22}\boldsymbol{K})^{-1}\boldsymbol{P}_{21} \tag{20}$$

The controller $\boldsymbol{K}$ can obtained by solving two algebraic Riccati equations which the closed-loop system is stable and the norm of the closed-loop transfer function can reach to the minimum value.

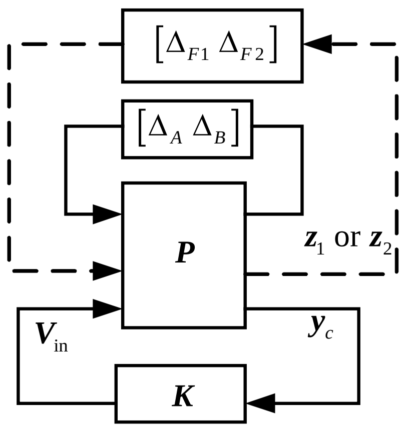

Figure 3: Block diagram of μ synthesis augmented model

Since it is difficult to establish the exact mathematical model of the dynamic system by using finite element method, the structural parameters such as natural frequency, damping ratio and the parameter relative to the actuator are not accurate. $\Delta\omega$, $\Delta\zeta$ and Δg are used to represent the uncertainty of these parameters, the uncertainty of coefficient matrix of state equations are described as

$$\begin{aligned} \Delta\boldsymbol{A}_c(\Delta\omega, \Delta\zeta) &= \boldsymbol{A}_c(\Delta\omega, \Delta\zeta) - \boldsymbol{A}_c \\ \Delta\boldsymbol{B}_c(\Delta g) &= \boldsymbol{B}_c(\Delta g) - \boldsymbol{B}_c \end{aligned} \tag{21}$$

where $\boldsymbol{A}_c(\Delta\omega, \Delta\zeta)$ and $\boldsymbol{B}_c(\Delta g)$ is the actual coefficient matrix, $\boldsymbol{A}_c$ and $\boldsymbol{B}_c$ is the nominal value, $\Delta\boldsymbol{A}_c(\Delta\omega, \Delta\zeta)$ and $\Delta\boldsymbol{B}_c(\Delta g)$ represent the bounded uncertainty of these parameters. In order to design the controller by using μ-synthesis method, $\Delta\boldsymbol{A}_c$ and $\Delta\boldsymbol{B}_c$ can be described as

$$\begin{aligned} &\Delta\boldsymbol{A}_c(\Delta\omega, \Delta\zeta) = \boldsymbol{D}_A\Delta_A(\Delta\omega, \Delta\zeta)\boldsymbol{E}_A \ \|\Delta_A(\Delta\omega, \Delta\zeta)\| \le 1 \\ &\Delta\boldsymbol{B}_c(\Delta g) = \boldsymbol{D}_B\Delta_B(\Delta g)\boldsymbol{E}_B \ \|\Delta_B(\Delta g)\| \le 1 \end{aligned} \tag{22}$$

where $\boldsymbol{D}_A$, $\boldsymbol{E}_A$, $\boldsymbol{D}_B$ and $\boldsymbol{E}_B$ are uncertainty structured constant matrix, respectively. $\Delta_A(\Delta\omega, \Delta\zeta)$ and $\Delta_B(\Delta g)$ is unknown uncertainty parameters which is a bounded perturbation.

According to the performance index introduced by above part, the μ-synthesis augmented model is shown as Fig.3. The dotted lines indicate the hypothetical perturbation $[\Delta_{F1}\ \Delta_{F2}]$. $[\Delta_A\ \Delta_B]$ is the actual perturbations. Thus, the diagonal block $\boldsymbol{\Delta}_p$ of uncertainty matrix is obtained to construct a standard μ-synthesis framework.

$$\boldsymbol{\Delta}_p = \begin{bmatrix} \Delta_{F1} & & & \\ & \Delta_{F2} & & \\ & & \Delta_A & \\ & & & \Delta_B \end{bmatrix} \tag{23}$$

Obviously, this is the standard μ-control problem, and the design can be based on the Matlab μ-toolbox, in which the *D-K* iteration is adopted to perform the synthesis procedure. *D-K* iteration is a two-step minimization process: the first step is a minimization of the H_∞ norm over all stabilizing controllers $\boldsymbol{K}$ while the scaling matrix $\boldsymbol{D}$ is held fixed and second step is a minimization over a set of scaling $\boldsymbol{D}$ while the controller $\boldsymbol{K}$ is held fixed.

4. EXPERIMENTAL INVESTIGATION

In this section, we shall experimentally evaluate the effectiveness of the proposed control method in the vibration control of a flexible beam.

4.1 Experimental setup

The length of the cantilever beam is 300 mm, its width and thickness are 20 mm, 1.5 mm, respectively. Its material is steel with elastic module, density and Poisson's ratio 2100 GPa, 7800 kg/m^3, 0.3, respectively. The beam is divided into 7 beam elements and 28 generalized coordinates shown as in the Fig. 4. The lengths of the elements are 50 mm, 40 mm, 40 mm, 40 mm, 40 mm, 40 mm, 50 mm, respectively. The symbols N, E, A, S denote node, actuator, sensor, respectively, in the Fig. 4. The actuators are made of PZT-5H piezoelectric ceramic, which its thickness is 0.5 mm, length is 40 mm and width is 20 mm, piezoelectric constant d_{31}, elastic

Journal of Theoretical and Applied Information Technology
20th May 2013. Vol. 51 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

module and density is $200 \cdot 10^{-12}$ m/V, 1200 GPa, 7650 kg/m^3, respectively. The sensors are resistance strain gage. The configuration of the actuators and sensors are shown in Fig.4. The three pairs actuator bonded on the link are located at elements E_2, E_4 and E_6. Two sensors S_1, S_2 are located at the midpoint of elements E_3 and E_5. Three actuators A_1, A_2, A_3 are located at elements E_2, E_4 and E_6, respectively.

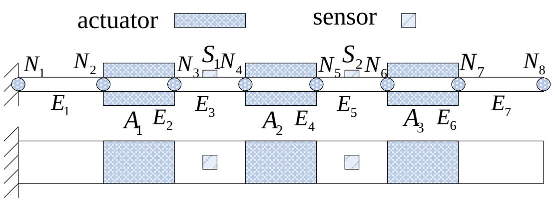

Figure 4: Configuration of the elements and nodes, actuator and sensor, N, E, A, S denote node, actuator, sensor, respectively

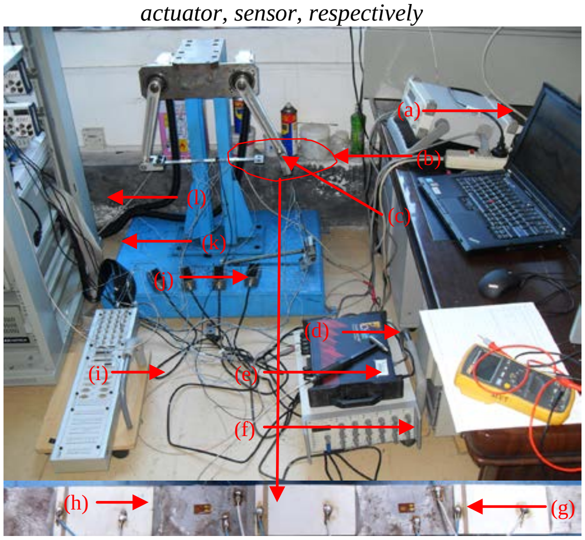

Figure 5: Experimental setup

(a) signal generator (b) cantilever beam (c) accelerometer (d) dynamic signal acquisition (e) hammer (f) dynamic strain gauge (g) strain gauge (h) piezoelectric patch (i) DS1103 connector panel (j) electric bridge box (k) voltage amplifier (l) industrial computer

Fig. 5 illustrates the experimental setup that consists of a flexible beam bonded on PZT and strain gauge. As the bolt is tighten, the rotational pair becomes a fixed pair, that is, the flexible link can be considered as a cantilever beam. The high-speed analog input and output ports are provided with dSPACE DS1103. The electric amplifier made in Harbin core tomorrow science and technology Co., Ltd is used to drive piezoelectric patch. Resistance strain gauges are made in ZEMIC Co., Ltd, its type is BE120-3AA (11), the resistance is $120\,\Omega$, and the sensitivity coefficient is 2.17. The strain gauge is connected to dynamic strain gauge through 1/4 electric bridge which is used to transform the strain signal to voltage.

4.2. Experimental modal test

Experimental modal analysis is a case of system identification where a priori model form consisting of modal parameters is assumed. Because it is hard to obtain damping ratio of structure by finite element method (FEM), the experimental modal test is a good method to get accurate natural frequency and damping ratio, which provide a basis for adjustment of the control model of the flexible structure. The setup of the experimental modal test is shown as Fig.5. Kistler 8690C50-type piezoelectric accelerometer is used as acceleration sensor. The ZonicBook/618E is the dynamic signal acquisition system. The eZ-Analyst software is real-time vibration analysis software equipped with a dynamic signal acquisition system which provides a real-time analysis capability in the frequency domain and time domain. ME'scopeVES software is used to be a post-processing test which is capable of analyzing mechanical and structural static and dynamic characteristics to obtain modal parameters.

Impulse hammer method is applied to perform experimental modal test. To excite the bending vibration, the cantilever beam was hit with a hammer at the specified excitation points. Using a hammer to produce a wide band of excitation, it can excite each mode in a wider frequency range. The locations of hammer and accelerometers are located at midpoints of element E_3 and elements $E_1 \sim E_7$, respectively. A miniaturized accelerometer PCB is sequentially placed at different locations. The tap position of the hammer is fixed, and the measurement points are 7 different positions. In order to eliminate measurement noise, the multiple measurements value of measurement point is averaged, the times of measurement is set to 5. The eZ-Analyst software is used to collect data of excitation point and all measurement points, to obtain the frequency response function of the measuring point, and then export the data format which ME'scopeVES can support. The ME'scopeVES software is used to identify the modal parameters of the flexible beam. The synthesis of the controller is based on a nominal model constructed by low-frequency modes. In the present case, the first four-order mode of the link is identified. The first four-order nature frequencies by using experimental identification and FEM are shown in Table 1, respectively, and the corresponding damping ratios are identified and

Journal of Theoretical and Applied Information Technology
20th May 2013. Vol. 51 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

tabulated in Table 1. As can be seen from the table, relative error of calculated and experimental values of natural frequency is close to 2-6 %, which indicates that the finite element model is not fully consistent in the actual system. That is to say, the model used to design controller is uncertain.

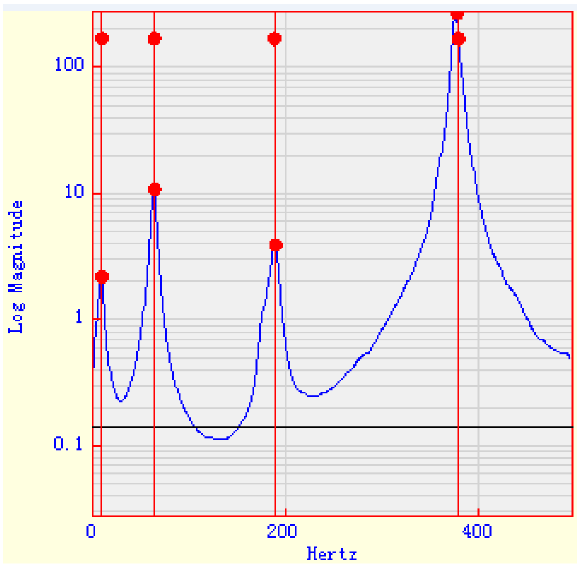

Figure 6: Fitting curve of frequency response data

Table 1: The first four-order natural frequency and damping ratio

Mode order	Natural frequency (Hz)		Relative error	Damping ratio (%)
	Calculation value	Experimental value		
1	10.5	10.8	1.52	1.52
2	73.8	72.6	2.37	2.37
3	180.9	189.6	4.56	1.24
4	366.7	387.5	5.37	0.96

4.3. Experimental results

Schematic diagram of vibration control experiment is shown as Fig. 7. The principal of the vibration control is described as follows. When the exogenous disturbances are exerted on the beam, the vibration response will be generated. The output of the strain gage is given as input to the dynamic strain gauge which filters out the noise contents. The conditioned sensor signal is given as analog input card through the electric bridge connector box. The vibration signal measured by a sensor is transformed into a voltage signal by a dynamic strain transducer, and through a low-pass filter and an A/D converter, the analog voltage signal is converted into a digital signal to the dSPACE controller board. And the control voltage applied to actuator can be obtained through the designed controller. As the control voltage from D/A port is relatively low, the voltage exerted on the piezoelectric patch must be amplified by the voltage amplifier to implement the vibration control. The control signal calculated by the dSPACE is converted into an analog signal by a D/A converter, and then is magnified 15 times by a voltage amplifier. Signals are then amplified and fed to a digital control system. The control algorithms are implemented using dSPACE DS1103 system with necessary Matlab/Simulink software installed in an industrial computer. The control algorithm is implemented using Simulink software and Real Time Workshop (RTW) is used to generate C code from the developed Simulink model. The C code is then converted to target specific code by real time interface (RTI) and target language compiler (TLC) supported by DS1103 controller board. Then we can design a vibration control experiment in real time by using ControlDestk software provided by dSPACE. The control objective is to minimize the output strain of two sensor outputs within the control bandwidth under the excitation of the disturbance force induced by external force.

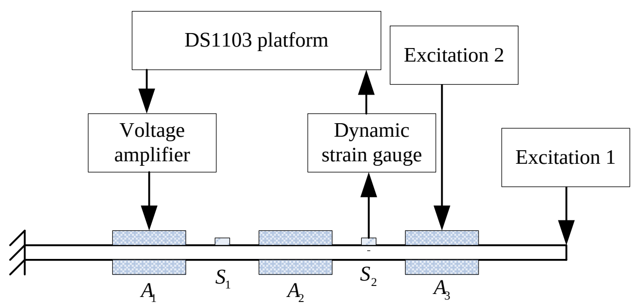

Figure7: Schematic diagram of experiment

The objective in the experimental studies is to control the first three vibration modes. In order to generate the disturbance force exerted on the beam, we use the actuator A_3 as excitation source. The actuators A_1, A_2 are applied to vibration control. The gain of the dynamic strain gauge is set to 1000, the bridge voltage 6 V, cutoff frequency of filter is set to 100 Hz. The sampling frequency of analog input and output port is set to 1000 Hz. The covariance of measurement noise is estimated as 10^{-4}. The sampling period of the controller is 1 ms. The real-time control time is set to 1 s. The schematic diagram of the feedback control system is depicted in Fig. 7. In order to experimentally investigate the control performance, the two excitation sources are exerted to the cantilever beam, respectively. The first excitation source called excitation 1 is the free-vibration one by

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

taping instantly the free end of the beam. The second excitation source named as excitation 2 is the forced one by applying a voltage to the piezoelectric ceramic patch A_3. The excitation voltage is generated by the signal generator which generates a white noise signal, amplified by voltage amplifier. And the covariance of the excitation signal is set to 120 V.

In the following section, the control effect of the controller is analyzed. In order to meet better the situation of the actual system, the measurement noise is considered in the experimental validation, and the signal is treated as white noise whose amplitude is set to 2×10^{-4}. Since the frequency band to control is between 0 and 100 Hz including the first two modes of the structure, the cut-off frequency of low-pass filter of dynamic strain gauge is set to 5 Hz. The weighting matrix of performance index function shown as equation(14), (15) and (16) are set as follows

$$\begin{aligned}
&W_d = 150, W_n = 1\times10^{-4}\times diag(1,1)\\
&W_{act} = \frac{1}{150}\times\frac{s+200}{s+600}\times diag(1,1)\\
&W_{per} = 500\times diag\left(\frac{75}{s+75},\frac{470}{s+470}\right)\\
&W_{per2} = 500\times diag\left(\frac{930}{s+930},\frac{930}{s+930}\right)
\end{aligned} \tag{24}$$

where $diag(\cdot)$ denotes diagonal matrix. The minimum and maximum value of input voltage is set to $\boldsymbol{V}_{\min}(k) = -150$ V and $\boldsymbol{V}_{\max}(k) = 150$ V.

The effectiveness and control performance of the four controllers are analyzed from the time domain point of view. The controller synthesis and analysis techniques described above are applied to the smart beam. Two H_∞ controllers and two μ controllers are designed according to above two performance index. In subsequent section, the H_∞ controller with modal displacement for evaluation signal is called controller1. The H_∞ controller with output strain for evaluation signal is called controller2. The μ controller with modal displacement for evaluation signal is called controller3, and the μ controller with output strain for evaluation signal is named as controller4. Figure 8 (a), (b) shows the output strain from two sensors without controller. Figure 9 and 10 (a), (b), (c), (d) represents the output strain from two sensors with controller1, controller2, controller3 and controller4, respectively. By comparing Fig.9 and 10 with Fig.8, we can see that the two output strains from sensors are reduced rapidly under the action of the four controllers. The results show that the four kinds of controllers can suppress vibration response of the flexible manipulator. By comparing the analysis of Figure 9, Figure 10(a), (b), the analysis results showed that two H_∞ controllers have the same control effect. Similarly, the two μ controllers also have the same control performance. However, the results of comparative analysis of Fig.9-10 (a), (b), (c), (d) show that the suppression of the output strain with μ controller is faster than one with H_∞, which it also indicates the control performance of μ controller is better than H_∞ controller. Figure 11 and 12 show the two actuator input voltages correspond to the controller 1, 2, 3, 4, respectively. From these diagrams, we can observe that these input voltages are with the available range. Through comparative analysis of the above two input voltages, the input voltage of the controller3 and controller4 are higher than the controller1 and controller2. This indicates that the μ controller can consume more energy to obtain better control performance.

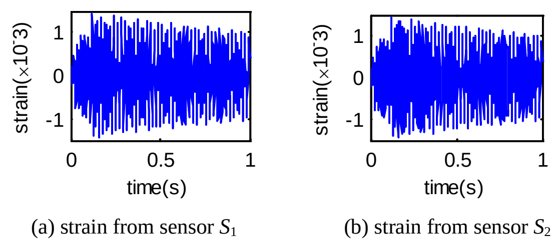

Figure 8: Strain of from two sensors in case of without controller

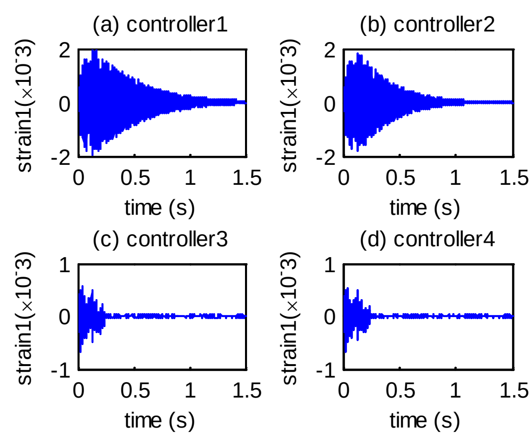

Figure 9: Strains from sensor S_1 with controller 1, 2, 3 and 4

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

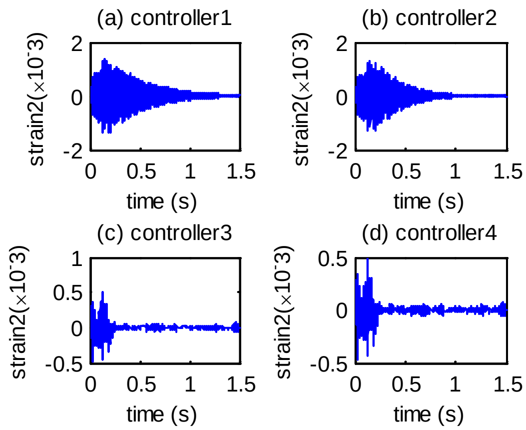

Figure 10: *Strains from sensor S_2 with controller 1, 2, 3 and 4*

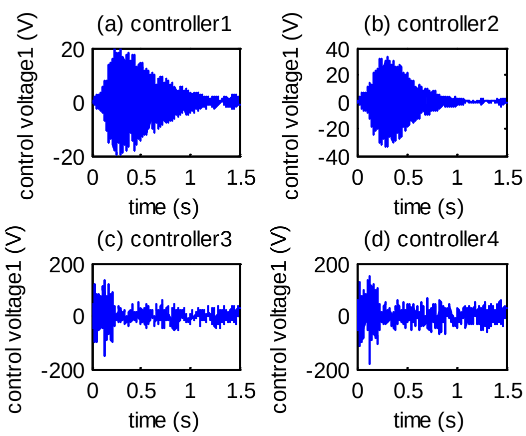

Figure 11: *Control input voltage for actuator A_1*

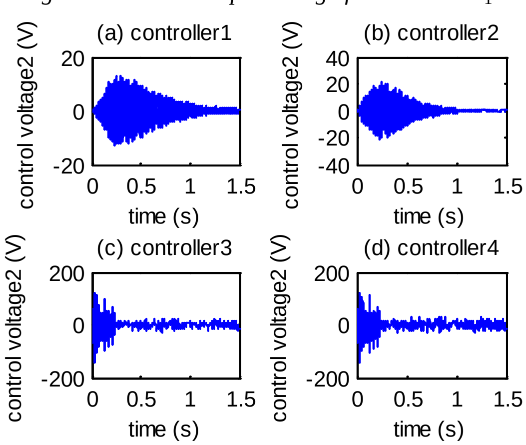

Figure 12: *Control input voltage for actuator A_2*

ACKNOWLEDGMENT

It is a project supported by the National Natural Science Foundation of China (No. 51265016), the Natural Science Foundation of Jiangxi Province (No. 20122BAB216029), the Education Fund of Jiangxi Provincial Department (No. GJJ12358) and the Research Fund of Jiangxi University of Science and Technology(No. jxxj12050).

REFRENCES:

[1] S.B. Choi, "Active structural acoustic control of a smart plate featuring piezoelectric actuators", *Journal of Sound and Vibration*, Vol. 294, 2006, pp. 421-429.

[2] K. Ma, "Adaptive nonlinear control of a clamped rectangular plate with PZT patches", *Journal of Sound and Vibration*, Vol. 264, No. 4, 2003,pp. 835-850.

[3] G. L. C. M. Abreu, J. F. Ribeiro, and V. Steffen, "Experiments on optimal vibration control of a flexible beam containing piezoelectric sensors and actuators", *Shock and vibration*, Vol.10,2003,pp. 283–300.

[4] J. F. Hu, D. C. Zhu, "Vibration control of smart structure using sliding mode control with observer", *Journal of Computer*, Vol. 7, No. 2, 2012, pp.411-418.

[5] J. F. Hu, "Vibration Control of a High-Speed Parallel Robot Using Discrete Variable Structure Control Methodology", *International Journal of Advancements in Computing Technology*, Vol. 4, No. 15, 2012, pp. 362-370.

[6] X. M. Zhang, C. J. Shao, and A. G. Erdman, "Active vibration controller design and comparison study of flexible linkage mechanism systems", *Mechanism & Machine Theory*, Vol. 37, 2002, pp. 985-997.

[7] X. M. Zhang, C. J. Shao, and S. Li, et al, "Robust H_∞ vibration control for flexible linkage mechanism systems with piezoelectric sensors and actuators", *Journal of Sound and Vibration*, Vol. 243, No. 1, 2001, pp. 145-155.

[8] H. Kawabe, N. Tsukiyama, and K. Yoshida, "Active vibration damping based on neural network theory", *Materials Science and Engineering A*, Vol. 442, 2006, pp. 547-550.

[9] L. Iorga, H. Baruh, and I. Ursu, "A Review of H_∞ robust control of piezoelectric smart structures", *Appl. Mech. Rev.*, Vol. 61, No. 4, 2008, pp. 1-16.

[10] X. Y. Wang, K. M. James, S. X. Guo, "Experimental identification and active control of configuration dependent linkage vibration in a planar parallel robot", *IEEE Transactions on Control Systems Technology*, Vol. 17, No. 4, 2009, pp. 960-969.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[11] N. Bel Hadj Ali, I.F.C. Smith, “Dynamic behavior and vibration control of a tensegrity structure”, *International Journal of Solids and Structures*, Vol. 47, No. 9, 2010, pp. 1285-1296.