VIBRATION CONTROL OF PIEZOELECTRIC FLEXIBLE STRUCTURE USING ROBUST CONTROL METHODOLOGY

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ABSTRACT

Active vibration control of a smart flexible cantilever beam was studied by using mode theory and μ synthesis method. The actuator is piezoceramic patch, the sensor is strain gauge. The finite element method and experimental modal test was applied to obtain the dynamic model of the smart structure. Taking account into uncertainty of the external disturbance and measurement noise, and uncertainty of natural frequency, damping ratio and actuator parameters of the system, μ synthesis approach was applied to design the vibration controller by selecting mode displacement as evaluation signal and selecting appropriate weighting function according to amplitude and frequency characteristics of the actual signal. A $H_\infty$ controller with same weightings was also designed and implemented for comparison analysis. The performances of the controller are verified experimentally in this study. Experimental results showed that μ -controller can provide good disturbance rejection and is more robust to parameter variations than $H_\infty$ controller. And the proposed controllers can effectively suppress the vibration response of the flexible beam.

Keywords: Smart Structure, Active Vibration Control, Finite Element Method, μ Synthesis, Experimental Modal Test

1. INTRODUCTION

In precision and aerospace industry, many researches on lightweight and miniaturized structures have been carried out to improve structural performances. Among the researches, passive structures using composite material are typically known as one of the effective methods. However, the traditional passive structures are very sensitive to change of internal load condition and external environment condition which can even cause sudden destruction of structures. Therefore, in order to satisfy stringent requirements for precision control and lightweight miniaturization, smart materials such as shape memory alloys, piezoceramics, and magnetorheological fluids are frequently adopted for smart structures. The performance requirements of future space structures, jet fighters and concept automobiles have brought much interest to the area of smart structures. A smart structure can be defined as a structure with bonded or embedded sensors and actuators as well as an associated control system, which enable the structure to respond simultaneously to external stimuli exerted on it and then suppresses undesired effects or enhance desired effects. Among various smart structures, those with piezoelectric patches have received much attention in recent years, due to the fact that piezoelectric materials have simple mechanical properties, small volume, light weight, large useful bandwidth, efficient conversion between electrical and mechanical energy, good ability to perform vibration control and ease of integration into metallic and composite structures [1-3].

Different control techniques have been investigated in the control of smart structure. Abreu conducted experimental work for the vibration control of flexible beam by using piezoelectric sensors and actuators with Linear Quadratic Gaussian (LQG) controller [4]. There are many classical strategies that can be used when the mathematical model is available, for instance pole allocation and optimal control. However, if the model has uncertainties these methods are not indicated. There are many robust techniques in structural control literature. Li investigated two control strategies for robust vibration control of
We aim here to deal with the active vibration reduction problem in flexible structure with uncertainties through designing reasonable $\mu$-controller. In this paper, the vibration control of a flexible beam is investigated by using $\mu$ synthesis and experimental modal test method, and taking into account the random disturbance uncertainty, modal parameter uncertainty and measurement noise.

The paper is organized as follows. In section 2, a dynamic model of a flexible beam bonded with piezoelectric actuators and strain gauge sensors is constructed by using finite element method. In section 3, the $\mu$ controller is proposed. In section 4, experimental identification of the flexible cantilever beam is performed to obtain its modal parameters. And the experimental validation test is performed based on the dSPACE DS1103 platform. The conclusions are given in section 5.

2. DYNAMIC MODELING OF SMART STRUCTURE

The modeling of smart structure with piezoelectric actuators and sensors has been a subject of intense research for a long time and is only briefly described here.

The flexible structure is modeled by using a two-node beam element. The beam element is shown in Fig. 1, which has two nodes with four degrees of freedom at each node; namely $u_1$, $u_5$, the longitudinal displacement, and $u_2$, $u_6$, the transverse displacement, and $u_3$, $u_7$, the slope, and $u_4$, $u_8$, the curvature. $L_e$ is the length of element.

The nodal displacement vector $\mathbf{u}$ with respect to reference frame $A-\mathbf{r} \mathbf{y}$ is expressed as

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \end{bmatrix}^T$$  \hspace{1cm} (1)

The transverse and longitudinal displacement fields of two-node beam element are constructed using the quintic hermite and linear interpolation polynomials, respectively. $V$, parameter uncertain systems [5]. Mayhan combined intelligent control and smart materials to produce an adaptive and robust controller to dampen the fundamental vibration mode of the system in the presence of modeling uncertainties [6]. Zhang et al. studied the active vibration control problem for the high-speed flexible mechanisms all of whose members were considered as flexible by using complex mode method and robust $H_\infty$ control scheme [7-8]. Kawabe utilized neural networks (NN) theory for active control in a longitudinal cantilevered-beam system by simulation and experiment. It is found that fairly satisfactory active damping effect using the NN controller is obtained [9]. But the random disturbance and measurement noise of the actual system were not considered by these currently proposed vibration control strategies. The issue of robustness against external disturbances was not addressed, and therefore the proposed vibration controllers cannot be effectively applied to the smart structure under the random uncertain disturbances.

Very few attempts have been made toward the application of robust control methodology to control vibrations in lightweight flexible structures. Robust vibration control methodology has received much attention due to their wide applications, and led to a rapid development of various control strategies such as the LQG control and the fuzzy control, etc. In recent years, a great deal of attention has been paid to the $H_\infty$ control because it not only provides a unified and general control framework for all control structures, but also yields a controller with guaranteed margins. However, $H_\infty$ control models all uncertainties as a single complex full block, which results in a rather conservative design. Under such circumstances, the $\mu$ synthesis technique, which involves the use of $H_\infty$ optimization for synthesis and structured singular value for analysis, has been developed. During the controller design, an issue to be considered is the process of the mathematical model between the disturbance force and the manipulator, which is always assumed to be known. In engineering practice, however, it is difficult to obtain this model even though system identification or theoretical approach. Hence, how to design a $\mu$-controller without requiring a mathematical model between the disturbance force and structure is of great interest in the application of $\mu$-synthesis technique.

Figure 1: Planar beam element showing nodal degrees of freedom and coordinate systems
$W$ denotes longitudinal and transverse elastic displacement of arbitrary point, respectively. Subscript T denotes matrix transpose. They can be written by the following form

$$
\begin{bmatrix}
V(\bar{x},t) \\
W(\bar{x},t)
\end{bmatrix} = \begin{bmatrix} N_1(\bar{x}) \\
N_2(\bar{x}) \end{bmatrix} u
$$

(2)

where $N(\bar{x}) = [N_1(\bar{x}) N_2(\bar{x})]^T$ is shape function, $u$ is the nodal displacement vector.

The system dynamic equations can be obtained by using finite element method

$$
M\ddot{U} + C\dot{U} + KU = P
$$

(3)

where $M, C, K$ are the systematic mass, damping, stiffness, respectively. $U, \dot{U}, \ddot{U}$ are the generalized displacement, velocity, and acceleration vectors of the system, respectively. $P$ is the systematic generalized disturbance force vector corresponding to vector $U$.

The piezoelectric patches work as actuators are perfectly bonded on the upper and lower surfaces of the beam at the same location. For the modeling of PZT actuators, literature [10, 11] provides a detailed derivation of coupling of PZT actuators and a host beam. These bending moments induced by the actuators is given by

$$
T_a = T_b = -d_{31}E_p b_p(t_i + t_v)V_{in}
$$

(4)

where $d_{31}$, $E_p$, $b_p$, $t_v$ is piezoelectric constant, elastic module, width, and thickness of PZT patch, respectively. $t_i$ is the thickness of the beam element, $V_{in}$ is the vector of input voltage to the piezoelectric actuators. The moments are assembled as a part of the external moments exerted on node.

Strain is the amount of deformation of a structure due to an applied force. Strain gauge is the most common sensor for measuring strain. A strain gauge’s electrical resistance varies in proportion to the amount of strain placed on it. The deformations mainly include compression or tension and bending deformation for the flexible beam. The strain $\varepsilon$ in $\bar{x}$ direction is given by

$$
\varepsilon = \varepsilon_L + \varepsilon_B
$$

(5)

where $\varepsilon_L$ is compression or tension strain and $\varepsilon_B$ are longitudinal strain due to bending deformation, respectively. They can be given by

$$
\varepsilon_L = N_1^*u = N_2^*B,U,
$$

$$
\varepsilon_B = -hN_2^*u = -hN_2^*B,U
$$

(6)

where $h$ is the distance between the neutral axis of the beam and the outer surface of the beam, $N_1^*$, $N_2^*$ are the first-order and the second-order differential of the shape function $N_1$ and $N_2$, respectively, $B$ denotes the transformation matrix. Thus, the exogenous perturbation and the control inputs have no direct effect on the measured outputs.

There are $a$ piezoelectric actuators and $s$ strain gauge sensors on the flexible structure. Combining Eq. (3)~ Eq.(6), the dynamic equations of the structure equipped with piezoelectric actuators and strain gauge transducer can be expressed as

$$
M\ddot{U} + C\dot{U} + KU = P + D,V\dot{u}
$$

(7)

where $D_s$ is the systematic control matrix related to configuration of actuators, $D_c$ is the systematic output matrices determined by configuration of sensors, a $a$-by-$1$ vector, and $\dot{y}$ is the strain from the sensors, a $s$-by-$1$ vector.

It was shown that the dynamic response of the flexible structure is composed mainly of the lower modes. In order to control the lower modes, the physical-coordinate equations must be first transformed into modal coordinates. Here, we choose the first $c$ order modes as control modes. Applying the modal theory, the normalized modal transformation is introduced by

$$
U = \psi_c \eta_c
$$

(8)

where $\psi_c$ is the controlled normalized modal matrix, $\eta_c$ is controlled modal coordinate vectors. Substituting Eq. (8) into Eq.(7), the system dynamic equations are rewritten as

$$
\ddot{\eta}_c + C_c \eta_c + K_c \eta_c = N_c + D_s,V\dot{u}
$$

(9)

$$
\dot{y} = D_c,\dot{\eta}_c
$$

where $N_c, D_s, D_c, C_c, K_c$ is $c \times c$ diagram matrix, which is determined by system natural frequency and damping ratio.

For control synthesis, the system must be written as a system of first-order ordinary differential equations (ODEs). We can define the controlled state variables by the following form

$$
X_c = [\eta_1, \eta_2, \eta_3, \ldots, \eta_c]^T
$$

(10)

Due to the controlled mode number $c$, the number of controlled state variables is $2c$. Taking into account measurement noise $v$, the state-space model for the system can be written as
Robust control theory is exploited to develop a more simple and reliable controller for the flexible structure vibration control applications. The robust controller requires both nominal model and uncertainty model. So in order to make actual control become robust, it is necessary to provide an accurate and detail modeling of the uncertainties that is relevant to the structure model. Because the plant to be controlled is a flexible structure modal model is best one described its dynamic behavior.

In general, modal parameters of a flexible structure model are: resonance frequencies, damping ratios, modal vibration shape. Each of these modal parameters contains uncertainty. In addition, the truncated modes in the model reduction and signal noise in actuators and sensors also increase the uncertainty in the assumed model. The mathematical models for uncertainties established in robust theory are additive uncertainty. Since the natural frequencies and mode shapes of the system are functions of the time, Equations (11) are continuous time-varying state space representation of the mechanism system.

Equations (11) are continuous time-varying state space representation of the mechanism system. Designing a controller with time-varying state space matrices is not of the scope of this study. The resulting closed-loop system is the weighted function of the signal. In order to find a controller to suppress \( N_c \) in desired frequency bandwidth \( y \) should be weighted by a performance weighting function matrix. \( z \) represents the evaluation signal and \( W_{per} \) is the weighted function of the signal. \( V_n \) is the control voltage. In addition, the control voltage should be weighted by the performance weighting function matrix so as to prevent the saturation of controller output. In order to limit the voltage range, the weighted function \( W_{act} \) is used to penalize the signal. \( y \) is the measurement output of the system, the output strains from strain gages. Measurement noise represents electrical noise, sampling errors, drifting calibration, and other effects that impair measurement precision and accuracy.

In the design of the \( \mu \) controller, we can choose the two different evaluation signals in accordance with the actual physical meaning of the
controlled plant, the first type signal is the controlled mode displacement, and the other is the output strain signal from the sensor. And taking into account the range of input voltage of actuator, the two different evaluation signals \( z_1 \) and \( z_2 \) are represented by the following form

\[
\begin{align*}
\dot{z}_i &= \begin{bmatrix} \eta_c \\ V_{in} \end{bmatrix} = \begin{bmatrix} C_i \\ \theta_{N_x} \end{bmatrix} X_c + \begin{bmatrix} 0_c \\ I_{N_x} \end{bmatrix} V_{in} \\
Y_i &= C_i X_c \end{align*}
\]

where \( C_i \) is a \( N_x \times N_a \) zero matrix, respectively. \( \theta_c, \theta_{N_x} \) is \( c \times c \), \( N_x \times N_a \) zero matrix. The external disturbance signal \( w \) includes the modal force signal and measurement noise signal, and it can denotes

\[
w = \begin{bmatrix} N_x \\ N_a \end{bmatrix}.
\]

Combined equation (11) and equation (12), the state-space equation of the generalized controlled plant can be written as follow

\[
\begin{align*}
\dot{X}_c &= A_c X_c + B V_{in} + N_c \\
Y_i &= C_c X_c \\
\dot{z}_i &= \begin{bmatrix} \eta_c \\ V_{in} \end{bmatrix} = \begin{bmatrix} C_i \\ \theta_{N_x} \end{bmatrix} X_c + \begin{bmatrix} 0_c \\ I_{N_x} \end{bmatrix} V_{in} \\
Y_i &= C_i X_c
\end{align*}
\]

The determination of weighting function matrices \( W_d(s), W_a(s), W_{act}(s), W_{per}(s) \) is a crucial step in the controller design. According to the amplitude and frequency characteristics of modal force and measurement noise signal, the weighting function \( W_d(s) \) and \( W_a(s) \) are set to respectively

\[
\begin{align*}
W_d &= N_{max} f_d(s) \\
W_a &= M_a
\end{align*}
\]

where \( N_{max} \) is the maximum amplitude of the mode force signal, \( f_d(s) \) denotes its characters in the frequency domain, \( M_a \) is the amplitude of the measurement noise signal.

The weighting function matrix \( W_{act}(s) \) is selected as to prevent the saturation of controller output due to the limits of DS1103 D/A converter output. Suppose the maximum allowable value of the actuator \( V_{max} \), the frequency character of the signal \( f_{act} \), the weighting function \( W_{act}(s) \) is

\[
W_{act} = \frac{1}{V_{max}} f_{act}(s)
\]

The introduction of \( W_{per}(s) \) is to reduce the influence of disturbance on sensor outputs. In general, it is chosen with large amplitude so as to suppress the low-frequency vibration. There are two type evaluation signals. One is modal displacement and \( W_{per1} \) is its weighting function; the other is output strain and \( W_{per2} \) denotes the weighting function corresponding to the signal. They are given by

\[
\begin{align*}
W_{per1} &= P_1 f_{per1}(s) \\
W_{per2} &= P_2 f_{per2}(s)
\end{align*}
\]

where \( P_1 \) and \( P_2 \) is the weighting coefficient, \( f_{per1}(s) \) and \( f_{per2}(s) \) is chosen as a one-order diagonal matrix, and they can be given by

\[
\begin{align*}
f_{per1}(s) &= \text{diag} \left( \frac{1}{s/\omega_a + 1}, \ldots, \frac{1}{s/\omega_a + 1} \right) \\
f_{per2}(s) &= \text{diag} \left( \frac{1}{s/\omega_a + 1}, \ldots, \frac{1}{s/\omega_a + 1} \right)
\end{align*}
\]

where \( \omega_a \) is a slightly larger value than the controlled natural frequency value. That is, the output signal whose frequency is less than \( \omega_a \) can be attenuated under the action of the controller.

In general, the design of a \( \mu \) -control system includes the synthesis of the controller and the selection of weighting functions. The consideration of the physical system is crucial. The so-called physical system includes both the system to be controlled and the actuator/sensor configuration. Different selections of actuator/sensor locations lead to very different sensor/actuator transfer functions and accordingly affect the design of \( \mu \) controllers. As is known, the actuator/sensor configuration determines the controllability and observability of the system, which has direct impact on the modes to be controlled and the control energy required. When piezoelectric actuators are used, they should be usually located at regions where the strain deformations of the dominant modes are large enough. Ideally, sensors should be collocated (or at least very close to the actuators) to ensure the plant model described by the transfer function as a minimum phase system in the controlled frequency band. The number of sensors and actuators to be used depends on the number of modes to be controlled.

Combining equation (13)–(17), the transfer function of the generalized controlled plant \( P \) can be written by the following form
\[
\begin{bmatrix}
    z \\
    y_c
\end{bmatrix} = P \begin{bmatrix}
    w \\
    V_{in}
\end{bmatrix} = \begin{bmatrix}
    P_1 & P_2 \\
    P_{21} & P_{22}
\end{bmatrix} \begin{bmatrix}
    w \\
    V_{in}
\end{bmatrix} \tag{18}
\]

Suppose the controller is given by
\[
V_{in} = Ky_c \tag{19}
\]

Substituting equation (19) into equation (18), the closed-loop transfer function from \(w\) to \(z\) can be obtained
\[
F_c(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{20}
\]

The controller \(K\) can be obtained by solving two algebraic Riccati equations which the closed-loop system is stable and the norm of the closed-loop transfer function can reach to the minimum value.

\[
\begin{bmatrix}
    \Delta_{F1} & \Delta_{F2} \\
    \Delta_A & \Delta_B
\end{bmatrix}
\]

\[
V_{in} \quad K \quad y_c
\]

\[z_1 \text{ or } z_2 \]

\[\begin{bmatrix}
\Delta_{F1} \\
\Delta_{F2} \\
\Delta_A \\
\Delta_B
\end{bmatrix}
\]

\[A_p = \begin{bmatrix}
\Delta_{F1} & \Delta_{F2} \\
\Delta_A & \Delta_B
\end{bmatrix} \tag{23}
\]

\[
\Delta A_1(\Delta \omega, \Delta \zeta) = A_1(\Delta \omega, \Delta \zeta) - A_1 \quad \Delta B_1(\Delta g) = B_1(\Delta g) - B_1 \tag{21}
\]

where \(A_1(\Delta \omega, \Delta \zeta)\) and \(B_1(\Delta g)\) is the actual coefficient matrix, \(A_1\) and \(B_1\) is the nominal value, \(\Delta A_1(\Delta \omega, \Delta \zeta)\) and \(\Delta B_1(\Delta g)\) represent the bounded uncertainty of these parameters. In order to design the controller by using \(\mu\) -synthesis method, \(\Delta A_1\) and \(\Delta B_1\) can be described as

\[
\Delta A_1(\Delta \omega, \Delta \zeta) = D_1\Delta \omega E_1 \quad \Delta B_1(\Delta g) = D_2\Delta g E_2
\]

where \(D_1, E_1, D_2\) and \(E_2\) are uncertainty structured constant matrix, respectively. \(A_1(\Delta \omega, \Delta \zeta)\) and \(B_1(\Delta g)\) is unknown uncertainty parameters which is a bounded perturbation.

According to the performance index introduced by above part, the \(\mu\) -synthesis augmented model is shown as Fig.3. The dotted lines indicate the hypothetical perturbation \([\Delta A_1 \Delta A_2]\). \([\Delta A_1 \Delta B]\) is the actual perturbations. Thus, the diagonal block \(\Delta_1\) of uncertainty matrix is obtained to construct a standard \(\mu\) -synthesis framework.

\[
\begin{bmatrix}
\Delta_{F1} & \Delta_{F2} \\
\Delta_A & \Delta_B
\end{bmatrix}
\]

\[
A_p = \begin{bmatrix}
\Delta_{F1} & \Delta_{F2} \\
\Delta_A & \Delta_B
\end{bmatrix} \tag{23}
\]

obviously, this is the standard \(\mu\) -control problem, and the design can be based on the Matlab \(\mu\) -toolbox, in which the \(D-K\) iteration is adopted to perform the synthesis procedure. \(D-K\) iteration is a two-step minimization process: the first step is a minimization of the \(H_\infty\) norm over all stabilizing controllers \(K\) while the scaling matrix \(D\) is held fixed and second step is a minimization over a set of scaling \(D\) while the controller \(K\) is held fixed.

4. EXPERIMENTAL INVESTIGATION

In this section, we shall experimentally evaluate the effectiveness of the proposed control method in the vibration control of a flexible beam.

4.1 Experimental setup

The length of the cantilever beam is 300 mm, its width and thickness are 20 mm, 1.5 mm, respectively. Its material is steel with elastic module, density and Poisson’s ratio 2100 GPa, 7800 kg/m³, 0.3, respectively. The beam is divided into 7 beam elements and 28 generalized coordinates shown as in the Fig. 4. The lengths of the elements are 50 mm, 40 mm, 40 mm, 40 mm, 40 mm, 40 mm, 50 mm, respectively. The symbols \(N, E, A, S\) denote node, actuator, sensor, respectively, in the Fig. 4. The actuators are made of PZT-5H piezoelectric ceramic, which its thickness is 0.5 mm, length is 40 mm and width is 20 mm, piezoelectric constant \(d_{31}\), elastic
module and density is $200\cdot10^{-12}$ m/V, 1200 GPa, 7650 kg/m$^3$, respectively. The sensors are resistance strain gage. The configuration of the actuators and sensors are shown in Fig.4. The three pairs actuator bonded on the link are located at elements $E_1$, $E_4$ and $E_6$. Two sensors $S_1$, $S_2$ are located at the midpoint of elements $E_3$ and $E_5$. Three actuators $A_1$, $A_2$, $A_3$ are located at elements $E_2$, $E_4$ and $E_6$, respectively.

![Figure 4](image-url)  
Figure 4: Configuration of the elements and nodes, actuator and sensor. $N$, $E$, $A$, $S$ denote node, actuator, sensor, respectively.

4.2. Experimental modal test

Experimental modal analysis is a case of system identification where a priori model form consisting of modal parameters is assumed. Because it is hard to obtain damping ratio of structure by finite element method (FEM), the experimental modal test is a good method to get accurate natural frequency and damping ratio, which provide a basis for adjustment of the control model of the flexible structure. The setup of the experimental modal test is shown as Fig.5. Kistler 8690CS50-type piezoelectric accelerometer is used as acceleration sensor. The ZonicBook/618E is the dynamic signal acquisition system. The eZ-Analyst software is real-time vibration analysis software equipped with a dynamic signal acquisition system which provides a real-time analysis capability in the frequency domain and time domain. ME’scopeVES software is used to be a post-processing test which is capable of analyzing mechanical and structural static and dynamic characteristics to obtain modal parameters.

Impulse hammer method is applied to perform experimental modal test. To excite the bending vibration, the cantilever beam was hit with a hammer at the specified excitation points. Using a hammer to produce a wide band of excitation, it can excite each mode in a wider frequency range. The locations of hammer and accelerometers are located at midpoints of element $E_1$ and elements $E_i = E_j$, respectively. A miniaturized accelerometer PCB is sequentially placed at different locations. The tap position of the hammer is fixed, and the measurement points are 7 different positions. In order to eliminate measurement noise, the multiple measurements value of measurement point is averaged, the times of measurement is set to 5. The eZ-Analyst software is used to collect data of excitation point and all measurement points, to obtain the frequency response function of the measuring point, and then export the data format which ME’scopeVES can support. The ME’scopeVES software is used to identify the modal parameters of the flexible beam. The synthesis of the controller is based on a nominal model constructed by low-frequency modes. In the present case, the first four-order mode of the link is identified. The first four-order nature frequencies by using experimental identification and FEM are shown in Table 1, respectively, and the corresponding damping ratios are identified and
tabulated in Table 1. As can be seen from the table, relative error of calculated and experimental values of natural frequency is close to 2-6 \%, which indicates that the finite element model is not fully consistent in the actual system. That is to say, the model used to design controller is uncertain.

<table>
<thead>
<tr>
<th>Mode order</th>
<th>Natural frequency (Hz)</th>
<th>Relative error</th>
<th>Damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculation value</td>
<td>Experimental value</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10.5</td>
<td>10.8</td>
<td>1.52</td>
</tr>
<tr>
<td>2</td>
<td>73.8</td>
<td>72.6</td>
<td>2.37</td>
</tr>
<tr>
<td>3</td>
<td>180.9</td>
<td>189.6</td>
<td>4.56</td>
</tr>
<tr>
<td>4</td>
<td>366.7</td>
<td>387.5</td>
<td>5.37</td>
</tr>
</tbody>
</table>

4.3. Experimental results

Schematic diagram of vibration control experiment is shown as Fig. 7. The principal of the vibration control is described as follows. When the exogenous disturbances are exerted on the beam, the vibration response will be generated. The output of the strain gage is given as input to the dynamic strain gauge which filters out the noise contents. The conditioned sensor signal is given as analog input card through the electric bridge connector box. The vibration signal measured by a sensor is transformed into a voltage signal by a dynamic strain transducer, and through a low-pass filter and an A/D converter, the analog voltage signal is converted into a digital signal to the dSPACE controller board. And the control voltage applied to actuator can be obtained through the designed controller. As the control voltage from D/A port is relatively low, the voltage exerted on the piezoelectric patch must be amplified by the voltage amplifier to implement the vibration control. The control signal calculated by the dSPACE is converted into an analog signal by a D/A converter, and then is magnified 15 times by a voltage amplifier. Signals are then amplified and fed to a digital control system. The control algorithms are implemented using dSPACE DS1103 system with necessary Matlab/Simulink software installed in an industrial computer. The control algorithm is implemented using Simulink software and Real Time Workshop (RTW) is used to generate C code from the developed Simulink model. The C code is then converted to target specific code by real time interface (RTI) and target language compiler (TLC) supported by DS1103 controller board. Then we can design a vibration control experiment in real time by using ControlDesk software provided by dSPACE. The control objective is to minimize the output strain of two sensor outputs within the control bandwidth under the excitation of the disturbance force induced by external force.

The objective in the experimental studies is to control the first three vibration modes. In order to generate the disturbance force exerted on the beam, we use the actuator $A_3$ as excitation source. The actuators $A_1, A_4$ are applied to vibration control. The gain of the dynamic strain gauge is set to 1000, the bridge voltage 6 V, cutoff frequency of filter is set to 100 Hz. The sampling frequency of analog input and output port is set to 1000 Hz. The covariance of measurement noise is estimated as $10^{-4}$. The sampling period of the controller is 1 ms. The real-time control time is set to 1 s. The schematic diagram of the feedback control system is depicted in Fig. 7. In order to experimentally investigate the control performance, the two excitation sources are exerted to the cantilever beam, respectively. The first excitation source called excitation 1 is the free-vibration one by
taping instantly the free end of the beam. The second excitation source named as excitation 2 is the forced one by applying a voltage to the piezoelectric ceramic patch $A_1$. The excitation voltage is generated by the signal generator which generates a white noise signal, amplified by voltage amplifier. And the covariance of the excitation signal is set to 120 V.

In the following section, the control effect of the controller is analyzed. In order to meet better the situation of the actual system, the measurement noise is considered in the experimental validation, and the signal is treated as white noise whose amplitude is set to $2 \times 10^{-4}$. Since the frequency band to control is between 0 and 100 Hz including the first two modes of the structure, the cut-off frequency of low-pass filter of dynamic strain gauge is set to 5 Hz. The weighting matrix of performance index function shown as equation (14), (15) and (16) are set as follows

$$W_d = 150, W_v = 1 \times 10^{-4} \times \text{diag}(1,1)$$

$$W_{act} = \frac{1}{150} \times \frac{s + 200}{s + 600} \times \text{diag}(1,1)$$

$$W_{per} = 500 \times \text{diag}
\left(\frac{75}{s + 75}, \frac{470}{s + 470}\right)$$

$$W_{per2} = 500 \times \text{diag}
\left(\frac{930}{s + 930}, \frac{930}{s + 930}\right)$$

where $\text{diag}(\cdot)$ denotes diagonal matrix. The minimum and maximum value of input voltage is set to $V_{\min}(k) = -150$ V and $V_{\max}(k) = 150$ V.

The effectiveness and control performance of the four controllers are analyzed from the time domain point of view. The controller synthesis and analysis techniques described above are applied to the smart beam. Two $H_\infty$ controllers and two $\mu$ controllers are designed according to above two performance index. In subsequent section, the $H_\infty$ controller with modal displacement for evaluation signal is called controller1. The $H_\infty$ controller with output strain for evaluation signal is called controller2. The $\mu$ controller with modal displacement for evaluation signal is called controller3, and the $\mu$ controller with output strain for evaluation signal is named as controller4. Figure 8 (a), (b) shows the output strain from two sensors without controller. Figure 9 and 10 (a), (b), (c), (d) represents the output strain from two sensors with controller1, controller2, controller3 and controller4, respectively. By comparing Fig.9 and 10 with Fig.8, we can see that the two output strains from sensors are reduced rapidly under the action of the four controllers. The results show that the four kinds of controllers can suppress vibration response of the flexible manipulator. By comparing the analysis of Figure 9, Figure 10(a), (b), the analysis results showed that two $H_\infty$ controllers have the same control effect. Similarly, the two $\mu$ controllers also have the same control performance. However, the results of comparative analysis of Fig.9-10 (a), (b), (c), (d) show that the suppression of the output strain with $\mu$ controller is faster than one with $H_\infty$, which it also indicates the control performance of $\mu$ controller is better than $H_\infty$ controller. Figure 11 and 12 show the two actuator input voltages correspond to the controller 1, 2, 3, 4, respectively. From these diagrams, we can observe that these input voltages are with the available range. Through comparative analysis of the above two input voltages, the input voltage of the controller3 and controller4 are higher than the controller1 and controller2. This indicates that the $\mu$ controller can consume more energy to obtain better control performance.

![Figure 8: Strain of from two sensors in case of without controller](image1)

![Figure 9: Strains from sensor $S_1$ with controller 1, 2, 3 and 4](image2)
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