OPTIMIZATION METHOD OF PILOT ALLOCATION FOR COOPERATIVE AMPLIFY-AND-FORWARD OFDM SYSTEM

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ABSTRACT

Pilot-assisted channel estimation has been studied only in common cooperative AF (Amplify-and-Forward) OFDM which has no feedback mechanism. In this paper, for cooperative AF-OFDM system, we propose a feedback technique in which the pilot allocation mechanism is adapted to the cooperative mobile channel. Then we show how uniform pilots may no longer be optimal compared with our pilots. After deducing the instantaneous system SNR is the function of the pilot allocation, we optimize the pilot locations using optimization method at the destination node. We examine the feedback overhead that is associated with our method and make comparisons with the conventional uniform pilot allocation. To improve system capacity, we suggest reduce the feedback information using K-means clustering algorithm. Simulation results confirmed that the proposed optimization method has an ideal performance gains and the algorithm of feedback reduction can reduce the feedback information under the premise of ideal performance.

Keywords: AF-OFDM system, Feedback mechanism, Optimization of pilot allocation, Reduce feedback.

1. INTRODUCTION

These years, in wireless communication, antenna diversity is widely used to fight fading. Although this technology will bring significant diversity gain, it cannot be applied to some mobile termination because of the limit of the termination’s size and power. Cooperative communication [1-2], which can form virtual antenna array between every node with single antenna can resolve the above problem effectively.

In wireless link, the method of using feedback is gradually becoming the focus of research. Aiming at the MIMO-OFDM system, [3] proposes a feedback framework which combines the beam formation vector quantization and the intelligent vector interpolation. This kind of feedback framework has a better performance than the existed technology under the condition of the same feedback data rate. Based on the study of traditional OFDM channel estimation [4], the authors of [5] estimate the channel of cooperative AF-OFDM system. The CRB (Cramer-Rao lower Bound) of the estimation and the minimum unbiased estimation which can reach the CRB are deduced in [5]. Since the authors of [5] estimate the channel using training sequence instead of pilots, we have to consider how to use pilots to accomplish the channel estimation of cooperative AF-OFDM system. On this basis, channel estimation for amplify-and-forward relaying: cascaded against disintegrated estimators is investigated in [6]. The authors of [6] investigate the performance of amplify-and-forward relaying with two different pilot-symbol-assisted channel estimation methods. However, Amin and Gedik [6] do not apply OFDM to their cooperative AF system, the channel cannot fight frequency selective fading.

In this paper, we propose an optimization method of pilot allocation for cooperative AF-OFDM system. For finding the optimal pilot allocation which can reduce symbol-error-rate (SER), we firstly deduce the instantaneous system SNR is the function of pilot allocation. Then feed the optimized pilot allocation back to the source using the feedback link at the destination. Finally, we propose a clustering algorithm (K-means algorithm) to reduce feedback overhead. Simulation results show considerable improvements for the cooperative AF-OFDM system and confirm that optimized pilot allocation has more ideal system performance compared with uniform pilots.

2. COOPERATIVE AF-OFDM SYSTEM MODEL

As shown in Fig.1, we consider a cooperative AF-OFDM system with one source $S$, one relay $R$ and one destination $D$. The dotted line expresses the
feedback link from destination to source. Each node is equipped with a single antenna and operates in half-duplex mode. Each channel will be flat Rayleigh fading channel because of using OFDM.

The node $S$ transmits OFDM symbol vector $X = [X(0), X(1), \ldots, X(N-1)]^T$. The time invariant channel impulse response (CIR) between nodes $i$ and $j$ is modeled as $h_{ij} = \sum_{l=0}^{L-1} h_{ij}(l) \delta(t - IT_s)$, where $h_{ij}(l)$ is the $l$th channel gain, $\delta(x)$ is the unit impulse function. The delays are $[0, T_s, 2T_s, \ldots, (L-1)T_s]$. $L$ is the channel length and it is the same for any pair of nodes. For brevity, we define $h_{ij}(n) = [h_{ij}(0), h_{ij}(1), \ldots, h_{ij}(L-1)]^T$. The channel frequency response (CFR) on the $n$th subcarrier can be defined as $H_{ij}(n) = \sum_{l=0}^{L-1} h_{ij}(l) e^{-j2\pi n l / N}$, and the frequency domain channel coefficient matrix is $H_{ij} = [H_{ij}(0), H_{ij}(1), \ldots, H_{ij}(N-1)]^T$, where $N$ is the total number of subcarriers.

![AF-OFDM system with one source, one relay and one destination](image)

**Fig. 1** AF-OFDM system with one source, one relay and one destination

During the first time slot, the source node $S$ transmits information to destination $D$ and relay $R$. For the second time slot, relay $R$ amplifies and forwards the signal received from source $S$, and retransmits the amplified signal to destination $D$. The input-output relations at relay and destination for $n$th subcarrier are given by

$$Y_{R1}(n) = \sqrt{E_{SR}} H_{SR}(n) X(n) + N_{R1}(n)$$

$$Y_{D1}(n) = \sqrt{E_{SD}} H_{SD}(n) X(n) + N_{D1}(n)$$

$$Y_{D2}(n) = \sqrt{E_{SD}} [F_2 h_{SD} X(n)] + N_{D2}(n)$$

$$= \sqrt{E_{SR}} E_{RD}/[E_{SR} + N_0] H_{RD}(n) X(n) + \hat{N}_{D2}(n)$$

where $H_{SR}(n)$ is the product of $H_{SR}(n)$ and $H_{RD}(n)$. Subscript $l \in \{1, 2\}$ indicates the time slots. $Y_i$ is the received symbol at terminal $i$. $N_i$ is the Gaussian white noise with zero mean and variance $N_0 I_N$, where $I_N$ denote an $N \times N$ identity matrix. $E_i$ is the average received signal energy over one symbol period at terminal $j$, which includes the path loss and shadowing effects for simplicity. $i, j \in \{S, R, D\}$.

The relay normalizes $Y_{R1}$ by a scaling factor of $1/\sqrt{E_{SR} + N_0}$ and retransmits it to the destination.

$$\hat{N}_{D2}(n)$$

is the sum of $\sqrt{E_{RD}}/\sqrt{E_{SR} + N_0} N_{R1}$ and $N_{D2}$.

For simplicity, we denote $\rho = \sqrt{E_{SR}E_{RD}/(E_{SR} + N_0)}$.

The input-output relations from (3) can be rewritten as

$$Y_{D2} = \rho H_{SD} X + \hat{N}_{D2} = \rho F_2 h_{SD} X + \hat{N}_{D2}$$

we denote $H_{SD} = F_2 h_{SD}$, where $F_L$ is a matrix that consists of the first $L$ columns of the $N \times N$ Fourier transform matrix $F$, and $H_{sd} = [H_{SD}(0), H_{SD}(1), \ldots, H_{SD}(N-1)]^T$. The complex AWGN vector $N_{D2} = [N_{D2}(0), N_{D2}(1), \ldots, N_{D2}(N-1)]^T$ distributed as $CN(0, N_0 I_N)$. The received symbol vector at the destination is denoted as $Y_{D2} = [Y_{D2}(0), Y_{D2}(1), \ldots, Y_{D2}(N-1)]^T$.

### 3. Optimum Pilot Allocation for Cooperative AF-OFDM System

The proposed system only requires feedback the current best pilot allocation and the feedback link need not include the entire CSI. With Least Square (LS) channel estimation, we proceed to derive the instantaneous SNR as a function of the pilot allocation.

#### 3.1 LS Channel Estimation for Cooperative AF-OFDM System

In the proposed cooperative AF-OFDM system, pilots locations in each OFDM symbol may conveniently be represented by a $N \times N$ diagonal pilot allocation matrix $P$ which can be defined as

$$p_{ii} = \begin{cases} 1 & \text{if } i \text{th subcarrier is pilot} \\ 0 & \text{if } i \text{th subcarrier is data} \end{cases}, \quad p_{ij} = 0 \forall i \neq j$$

The set of all possible pilot allocation matrices may denoted as $I_p = \{P_1, P_2, \ldots, P_{|I_1|}\}$, where $|I_1| = \binom{N}{N_p}$ is the cardinality of the set, $N_p$ is the number of pilot subcarrier. Each matrix is constrained by $tr\{ P \} = N_p, \forall P \in I_p$.

Using the pilot allocation matrix, the received vector may be decoupled into pilot and data bearing
subcarriers as \( Y_{D2} = Y_{D2}^{\text{pilot}} + Y_{D2}^{\text{data}} \), where
\[ Y_{D2}^{\text{pilot}} = \rho X_{p} \mathbf{P}_{L} \mathbf{h}_{\text{SRD}} + \mathbf{P}_{N_{D2}} \]
Substituting from (4), we have
\[ Y_{D2}^{\text{pilot}} = \rho X_{p} \mathbf{P}_{L} \mathbf{h}_{\text{SRD}} + \mathbf{P}_{N_{D2}} \tag{6} \]
where \( x_{p} \) is the common pilot data symbol. Pre-multiplying both sides of (6) by \( x_{p}^{*} \), we get an observation vector \( \hat{Y}^{\text{pilot}}_{D2} = x_{p}^{*} Y_{D2}^{\text{pilot}} \), therefore
\[ \hat{Y}^{\text{pilot}}_{D2} = \rho \mathbf{x}_{p}^{*} \mathbf{P}_{L} \mathbf{h}_{\text{SRD}} + \mathbf{P}(x_{p}^{*} \mathbf{N}_{D2}) \tag{7} \]
where \( \mathbf{N}_{D2} \) is statistically equivalent to \( \mathbf{N}_{D2} \). From \( \hat{Y}^{\text{pilot}}_{D2} \), the least squares (LS) estimate [7] of the CIR may be obtained via Moore-Penrose matrix inverse as
\[ \mathbf{h}_{\text{SRD}} = (\rho \mathbf{P}_{L}^{H})^{-1} \hat{X}^{\text{pilot}}_{D2} \]
\[ = (\rho \mathbf{F}_{L}^{H} \mathbf{P}_{L})^{-1} \mathbf{F}_{L}^{H} \mathbf{P} \hat{Y}^{\text{pilot}}_{D2} \tag{8} \]
Using (8) and invariance property of the LS estimation [7], the CFR estimation may be given as
\[ \hat{H}_{\text{SRD}} = \mathbf{F}_{L} \mathbf{h}_{\text{SRD}} = \mathbf{F}_{L} (\rho \mathbf{F}_{L}^{H} \mathbf{P}_{L})^{-1} \mathbf{F}_{L}^{H} \mathbf{P} \hat{Y}^{\text{pilot}}_{D2} \tag{9} \]
Substituting (7) to (9), and simplifying using the fact \( \mathbf{P} \mathbf{P}^{H} = \mathbf{I} \), we get
\[ \hat{H}_{\text{SRD}} = \mathbf{H}_{\text{SRD}} + \mathbf{F}_{L} (\rho \mathbf{F}_{L}^{H} \mathbf{P}_{L})^{-1} \mathbf{F}_{L}^{H} \mathbf{P} \mathbf{N}_{D2} \tag{10} \]
where \( \mathbf{F}_{L} (\rho \mathbf{F}_{L}^{H} \mathbf{P}_{L})^{-1} \mathbf{F}_{L}^{H} \mathbf{P} = \mathbf{J} \) is denoted as a \( N \times N \) projection matrix and the estimated error vector is \( \mathbf{E} = \mathbf{JN}_{D2}^{*} \). For each subcarrier, \( e(n) \) is the \( n \)-th element of \( \mathbf{E} \), which is a complex, circularly symmetric Gaussian random variable with zero mean and variance \( \alpha \sigma^{2} \).

3.2 Optimization Method for Pilot Allocation

In this section we discuss how, by utilizing a feedback link, the source node may adopt a more intelligent pilot allocation strategy. At the destination, the combined signal at the MRC detector input is
\[ Y = a_{1} Y_{SR}(n) + a_{2} Y_{RD}(n) \tag{11} \]
where \( a_{1} \) and \( a_{2} \) are determined constants which can maximize SNR in \( Y \). For simplicity, we assume that the relay has the knowledge of \( \hat{H}_{SR}(n) \) and the destination can get \( \hat{H}_{RD}(n) \) and \( \hat{H}_{SR}(n) \) using LS estimation. From [8], the coefficients at the \( n \)-th pilot subcarrier can be obtained as follows
\[ a_{1} = \sqrt{P_{s} / P_{N_{SR}}} \left[ N_{0} + 2 \alpha \sigma_{1} \right] \]
\[ a_{2} = \sqrt{P_{s} / P_{N_{RD}}} \left[ N_{0} + 2 \alpha \sigma_{2} \right] \]
where
\[ \mu = P_{s} \left( P_{N_{SR}} / N_{0} + 2 \alpha \sigma_{1} \right) + P_{P_{RD}} \left( P_{N_{RD}} / N_{0} + 2 \alpha \sigma_{2} \right) \]
\( P_{1}, P_{2} \) are transmitted power from source and relay, respectively. \( P_{av} \) is the average transmitted power from source. \( \hat{H}_{SR}(n) \) is the \( n \)-th element of \( \hat{H}_{SRD} \).

The corresponding SNR of the MRC outputs is
\[ \gamma(n) = P_{N_{SR}} / N_{0} + 2 \alpha \sigma_{1} \]
\[ = P_{N_{RD}} / N_{0} + 2 \alpha \sigma_{2} \]
\[ = P_{av} \]
\[ \mu = P_{s} \left( P_{N_{SR}} / N_{0} + 2 \alpha \sigma_{1} \right) + P_{P_{RD}} \left( P_{N_{RD}} / N_{0} + 2 \alpha \sigma_{2} \right) \]
\[ = P_{av} \]

For M-QAM modulation, the minimum achievable conditional SER can be expressed as
\[ p_{e}(n) = 4 K Q \left( \frac{2 \gamma(n)}{P_{w}} - 4 K^{2} Q^{2} \right) \tag{12} \]
where \( K \) is the Gaussian Q-function. Using the definition in (6), each subcarrier may be identified by \( 1 - P_{mm} \). The average SER on the data-bearing subcarriers is a function of \( P \) which may given as
\[ \overline{p_{e}(P)} = \frac{4}{N - N_{p}} \sum_{i=1}^{N_{p}} \left[ K Q \left( \frac{2 \gamma(n)}{P_{w}} - 4 K^{2} Q^{2} \right) \right] \tag{13} \]
Thus we have the following optimization problem in \( P[9] \)
\[ \overline{p_{e}(P)} = \frac{4}{N - N_{p}} \sum_{i=1}^{N_{p}} \left[ K Q \left( \frac{2 \gamma(n)}{P_{w}} - 4 K^{2} Q^{2} \right) \right] \tag{14} \]

For high computation complexity of (14), we have to find a suboptimal solution. From [10], \( Q(n) \leq (1/2) e^{-n^{2}/2} \). Therefore, the upper bound of (14) is
\[ \overline{p_{e}(P)} \leq \frac{2 K}{N - N_{p}} \sum_{i=1}^{N_{p}} \left[ 1 - P_{mm} \left( e^{-\gamma(n)/P_{w}} - 4 K^{2} Q^{2} \right) \right] \leq \frac{2 K}{N - N_{p}} \sum_{i=1}^{N_{p}} \left[ 1 - P_{mm} \left( \gamma(n) - 2 K \gamma(n) \right) \right] \tag{15} \]
where we used the expression \( e^{-n^{2}} < x, 0 \leq a \leq 1 \) and \( x >> 0 \). Therefore, the optimization problem can be simplified as a sum-power problem which may be expressed as
\[ p_{max} = \arg \min_{P_{mm}} \overline{p_{e}(P)} \tag{16} \]
In addition, the error rate of (16) is prominently dominated by the fading
subcarriers at high SNRs. Therefore, the optimization problem can be described as max-min problem

\[ P_{\text{max-min}} = \arg \max_{p \in P} \left( \min_{i \in \mathcal{N}} \frac{2K}{N - N_p} \left[ \tilde{y}(n) - 2K\tilde{y}(n) \right] \right) \quad (17) \]

In conclusion, at timestamp \( t \), the task of destination is given as: Firstly, using the current pilot allocation matrix \( \mathbf{P}(t) \) according to (9), estimate the channel response \( \mathbf{H}(t) \) and perform data detection via (11). Then, update \( \mathbf{P}(t) \rightarrow \mathbf{P}(t+1) \) using (14), (16) or (17). Finally, feed the updated pilot allocation \( \mathbf{P}(t+1) \) back to source, which will be used in the next time slot.

### 4. A CLUSTERING ALGORITHM OF FEEDBACK REDUCTION

As we know the amount of overhead that is associated with the feedback information and the feedback reduces the capacity of the system. Therefore, we have to reduce the amount of feedback as much as possible. For each channel response \( \mathbf{H}_{\text{env}} \), there will be a \( \mathbf{P} \) is selected from \( \mathbf{I}_p = \{ \mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_{N_p} \} \) using the aforementioned optimization criteria. Hence, each \( \mathbf{P} \) transfers \( \log_2 [N_{\text{ps}}] \) bits of information to the source.

For simplicity, we assume that the feedback link is error free and instantaneous. To reduce feedback, we propose using subcarrier clustering algorithm (K-means algorithm), which will be explained as follows:

Assuming that the size of the pilot allocation codebook \( I_p \) is \( K = |I_p| \), then partitioning the original codebook into a total of \( K' \) subsets, where \( K' \leq K \) and construct a new lower order codebook such as \( I'_p = \{ \mathbf{P}_1^{(1)}, \mathbf{P}_2^{(2)}, \ldots, \mathbf{P}_{K'}^{(K')} \} \), \( |I'_p| = K' \).

One method of creating the new codebook is to use vector quantization. K-means algorithm is an iterative quantization method that converges to an optimum quantization while minimizing some user-defined distortion function. The distortion function is designed specifically. Therefore, we choose the following metric as the clustering criterion function

\[ D(I') = E \left( \min_{p \in I'} \left\Vert \mathbf{P} - \mathbf{P}^{(p)} \right\Vert^2 \right) \quad (18) \]

The objective of an optimal quantizer is to seek the codebook that minimizes the average distortion over all possible codebooks. We can summarize the clustering algorithm as follows:

a) Randomly generate \( M \) AF-OFDM CFRs, constructing the codebook \( \mathbf{H} = \{ \mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \ldots, \mathbf{H}^{(M)} \} \). For each channel within this codebook, determine the corresponding optimum (or suboptimum) pilot allocation matrix \( \mathbf{P} = \{ \mathbf{P}^{(1)}, \mathbf{P}^{(2)}, \ldots, \mathbf{P}^{(M)} \} \).

b) Of the \( M \) pilot allocation matrices of \( \mathbf{P} \), randomly choose \( K' \) to construct the initial pilot index codebook \( \mathbf{P}_0 = \{ \mathbf{P}_0^{(1)}, \mathbf{P}_0^{(2)}, \ldots, \mathbf{P}_0^{(K')} \} \) which means \( K' \) clustering, where each pilot allocation matrix is a centroid of the clustering. Then calculate the mean value of each clustering and set \( i = 1 \) as the iteration counter.

c) Clustering. For the remainder of the pilot allocation codebook, assigning these codebook to the most similar clustering according to the distance between pilot allocation codebook and each cluster centroid. Partitioning the vectors of \( \mathbf{P} \) into a total of \( K \) quantization regions using minimum distance criteria such that the \( k \)th region is defined as

\[ R_k = \left\{ P \in \mathbf{P} \mid \left\| P - \bar{P}_{i-k} \right\| \leq \left\| P - \bar{P}_{i-k} \right\| , \forall l \neq k \right\} \]

d) Construct a new codebook \( \bar{P}_{i-k} \), which means calculating a new centroid of each clustering. The \( k \)th matrix of the \( k \)th quantization region given as

\[ \bar{P}_{i-k} = \arg \min \mathbb{E} \left( \left\| P - \bar{P}_{i-k} \right\|^2 \right) = \frac{1}{|R_k|} \sum_{P \in R_k} P \] \quad \forall P \in R_k \]

The last step is an approximation by choosing the center of mass of the quantization region. If \( D(\mathbf{P}_i) - D(\mathbf{P}_{i-1}) \leq \varepsilon \) or \( i \geq I \), terminate the algorithm. Otherwise, set \( i = i+1 \), go to step (3).

### 5. NUMERICAL RESULTS

In this section we simulate the performance of the proposed optimization method and the algorithm of feedback reduction by Matlab software[11]. To compare the proposed optimization method with uniform pilot allocation, we simulated the SER over a multipath channel of length \( L=2 \) with a normalized Doppler frequency of \( f_d = 10^{-4} \). A total number of \( N = 16 \) subcarriers is used for modulation, of which \( N_p = 4 \) reserved for pilot symbols.
The SER performance of proposed optimization method at a wider range of SNR and lower SNR are shown in Fig.2 and Fig.3, respectively. Fig.2 shows the SER performance with QPSK modulation over a wide range of SNR for different optimization strategies. Clearly, as shown in Fig.2, with the increasing of SNR, the SER of the proposed optimization strategy is gradually decreasing. There is a significant improvement in using feedback to optimize pilot allocation, especially at high SNR, where fading is a more prominent factor. To achieve ideal gain, the required feedback payload is 

$$\log_2\left(\frac{16}{4}\right) = 11 \text{ bits.}$$

As Fig.3 shows, in the low SNR regime (<10dB), where AWGN is dominant, the SER of the proposed optimization method is decreasing.

In order to simulate the performance of the proposed algorithm of feedback reduction, we design different bit codebooks for $N=16$, $N_p=4$ and $L=2$ using K-means algorithm mentioned in section 4. We assume the initial channel instances is $2 \times 10^4$ and $I=5$ iteration. We use the optimum SER criteria in (14) to obtain the optimal pilot allocation for all quantization regions.

Note that the objective function is a function of SNR. Therefore, we design our codebook at high SNR (30dB). According to the distortions calculated from (18), the respective SER performance is shown in Fig.4. With the increase of the feedback bits, the SER is deducing. Note despite a 5-bit reduction in feedback, the performance is still ideal even reducing the feedback information. Fig.5 shows the performance with different codebook from the distortion in (18). It is shown in Fig.5 that the performance of 6-bit codebook is the optimal, while the performance of 1-bit is the worse. When $I>2$ the distortion with each codebook is unchanged that means clustering criterion function is becoming stable after two iterations.

**6. CONCLUSION**

In this paper, optimization method of pilot allocation for cooperative AF-OFDM system has been proposed. The proposed optimization method can improve the performance effectively. After deducing the instantaneous SNR was the function of pilot allocation, we proposed to feed the optimum pilot allocation which can maximize the instantaneous SNR back to the source.

Therefore, the source could transmit the OFDM symbol according to the updated pilot allocation.
Our simulations have shown the proposed optimization strategy had a lower SER compared with uniform pilot allocation. We have suggested a clustering algorithm (K-means algorithm) to reduce feedback information and computer simulation confirmed that the performance of the system was still ideal even reducing the feedback information.

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