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SIMULATION OF PERMANENT MAGNET SYNCHRONOUS MOTOR CONTROL SYSTEM WITH EXTENTED KALMAN FILTER

[1]TAO XU, [2]ZHENGBIN ZHANG

[1]Assoc. Prof., Department of Automation, Shenyang Aerospace University, P. R. China
[2]Postgraduate Student, Department of Automation, Shenyang Aerospace University, P. R. China
E-mail: [1]xutao@sau.edu.cn, [2]binbincool.ok@163.com

ABSTRACT

With the development of Permanent Magnet Synchronous Motor (PMSM), it is applied to more and more areas. The control system of PMSM becomes the most important issue in applications. Because the sensorless control system possesses some good performances, it attracts many researchers' interests. This paper presents the application of Extended Kalman Filter (EKF) in PMSM speed control system. A PMSM vector control system model is established with Matlab/Simulink. The mathematical model of two-phase stationary coordinate system (α - β coordinate system) is adopted for linearization of the nonlinear mathematical model of PMSM. EKF algorithm estimates the rotation angle θ and rotation speed ω of the rotor of PMSM in the process of system control. The simulation results show that the control system works smoothly in the cases of the super-speed, slow-speed and with load. EKF estimations of the rotation angle θ and speed ω track actual angle and speed from sensors accurately.

Keywords: *Permanent Magnet Synchronous Motor (PMSM), Modeling and Simulation, Extended Kalman Filter (EKF), Vector Control*

1. INTRODUCTION

With the development of the power electronics, micro-electronics technology, new motor theory and the permanent magnet materials, PMSM has been promoted rapidly and application areas has been extended in recent years. Because of the permanent magnet excitation of PMSM, the motor's structure is relatively simple and the processing and assembly costs are reduced dramatically. Meanwhile, eliminates the need for error-prone collector rings and brushes and improve the reliability of the operation of the PMSM.

Therefore, PMSM possesses many good performances such as simple structure, small size, high efficiency, high power factor and low moment of inertia compared with traditional electrically synchronous motors. With the improvement of permanent magnet material, the vector control system of PMSM achieves a wide range of speed and position control with high precision and high dynamic performance in cases of small or medium power load, high reliability and wide speed range of the servo system.

This paper focuses on EKF algorithm and establishes simulation model of vector control system for PMSM with Matlab/Simulink. After programming the EKF algorithm by S function, sensorless vector control system of PMSM is studied based on the simulation model.

This paper is organized as follows. Section II presents the related works of PMSM in recent years. Section III presents the structure of PMSM vector control system. In Section IV, we introduce the mathematical model of PMSM. In Section V, we describe the state estimation with EKF. Section VI shows the simulation analysis of PMSM control system. Finally, we present meaningful conclusion.

2. RELATED WORK

Many researchers are interested in PMSM and the applications of it are promoted gradually [1]. Generally, PMSM control system is divided into speed sensor control and speed sensorless control. The former acquires rotation angle and rotation speed through mechanical sensors to modulate the motor speed in real time in order to achieve the desired speed value. The latter adopts the mathematical model of PMSM to estimate the angle and speed of the rotor in real time with some methods, including model reference adaption, high-

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frequency signal injection, state observer, Kalman optimal estimation, intelligent control and et cetera [2]-[6]. Among them, model reference adaption is simple. But it depends on the accuracy of parameters of the PMSM model. When the motor works in case of low rotation speed, the high-frequency signal injection method has certain advantages. But it will produce torque ripple, additional losses and requirement of saliency. Intelligent control algorithm possesses the characteristics of self-learning and adaption. But it is more complex than the proceeding methods and is not easy to modulate structure and design parameters [7]. Compared with other methods, the EKF algorithm can effectively suppress the affect of system error and measurement error on state estimation. Meanwhile, it possesses such good performances as fast convergence speed and high accuracy for state estimation. EKF is a better speed observer in those sensorless control methods [8].

3. STRUCTURE OF PMSM VECTOR CONTROL SYSTEM

Vector control system structure of PMSM is shown in figure 1, including Proportional Integral (PI) modulator, Space Vector Pulse Width Module (SVPWM), EKF estimation module, Clark-Park transform module, three-phase inverter and PMSM module.

PMSM adopts Field Oriented Control (FOC) controlling method with dual closed-loop control scheme of speed and current regulated by PI modulator. System estimates rotation angle θ and rotation speed ω from EKF in real time. Here, ω is the feedback variable of speed control loop and θ is the parameter of the Park transform and inverse transform to implement relevant calculation.

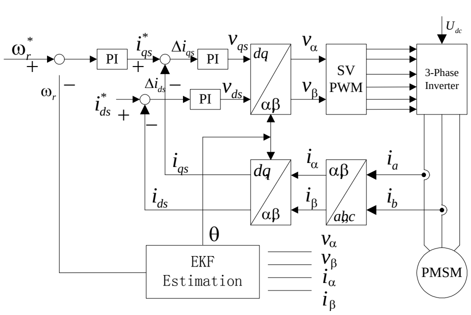

Fig. 1 Sensorless Vector Control System Structure of PMSM

4. MATHEMATICAL MODEL OF PMSM

The mathematical model of PMSMS can be divided into three kinds of representation by the axis systems, including the three-phase stationary coordinate system (*A-B-C* shafting coordinate system), the stator phase stationary coordinate system (α - β coordinate system) and the rotor two-phase rotating coordinate system (*d-q* coordinate system). The mathematical model utilized for EKF estimation is the α - β coordinate system in this paper. Therefore, this coordinate system is represented as follow.

The voltage equations of PMSM in the α - β coordinate system are equations (1) and (2).

$$U_\alpha = R_s i_\alpha + L_s \frac{di_\alpha}{dt} - \omega_r N_p \lambda_r \sin\theta_r \quad (1)$$

$$U_\beta = R_s i_\beta + L_s \frac{di_\beta}{dt} + \omega_r N_p \lambda_r \cos\theta_r \quad (2)$$

From equations (1) and (2), we have

$$\frac{di_\alpha}{dt} = -\frac{R_s}{L_s} i_\alpha + \omega_r \frac{N_p \lambda_r}{L_s} \sin\theta_r + \frac{U_\alpha}{L_s} \quad (3)$$

$$\frac{di_\beta}{dt} = -\frac{R_s}{L_s} i_\beta - \omega_r \frac{N_p \lambda_r}{L_s} \cos\theta_r + \frac{U_\beta}{L_s} \quad (4)$$

In digital systems, the sampling cycle is very short. And each sampling period can be considered as constant. So, we have the next equations (5) and (6).

$$\frac{d\omega_r}{dt} = 0 \quad (5)$$

$$\frac{d\theta_r}{dt} = \omega_r \quad (6)$$

Here, U_α and U_β present the axis voltages of α and β respectively, i_α and i_β present the axis currents of α and β respectively. R_s is the equivalent phase resistance of the PMSM's stator and L_s is the equivalent phase inductance of the PMSM's stator. N_p is the number of motor poles and λ_r is the rotor flux. ω_r is the mechanical angular velocity of the rotor and θ_r is the angular position of the rotor.

From the equations (3)-(6), we have the state equation and output equation of PMSM in equation (7) and (8).

$$\frac{dx}{dt} = Ax + Bu \quad (7)$$

$$y = Hx \quad (8)$$

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Here, $x=\begin{bmatrix} i_\alpha \\ i_\beta \\ w_r \\ \theta_r \end{bmatrix}$, $u=\begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix}$, $y=\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$,

$$A=\begin{bmatrix} -\frac{R_s}{L_s} & 0 & \frac{N_p\lambda_r}{L_s}\sin\theta_r & 0 \\ 0 & -\frac{R_s}{L_s} & -\frac{N_p\lambda_r}{L_s}\cos\theta_r & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & N_p & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, H=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Suppose sampling period is T, we have the discrete representation from equation (7) and (8). Therefore, discrete state equations of PMSM are equations (9) and (10) as follow.

$$x_k = f(x_{k-1},u_{k-1}) = (I + A\cdot T)\cdot x_{k-1} + B\cdot T\cdot u_{k-1} \qquad (9)$$

$$y_k = h(x_k) = H\cdot x_k \qquad (10)$$

5. STATE ESTIMATOIN WITH EKF

Kalman filter is the state estimation for linear system. EKF extends the application of Kalman filter into nonlinear systems. Because control system of PMSM is a non-linear system, this paper adopts EKF to estimate state for PMSM. The general expressions of EKF are equations (11) and (12) [6].

$$\frac{d\hat{x}}{dt} = A(\hat{x})\hat{x} + Bu + K(y-\hat{y}) \qquad (11)$$

$$\hat{y} = H\hat{x} \qquad (12)$$

Structure scheme of EKF equations is shown in figure 2 as follow.

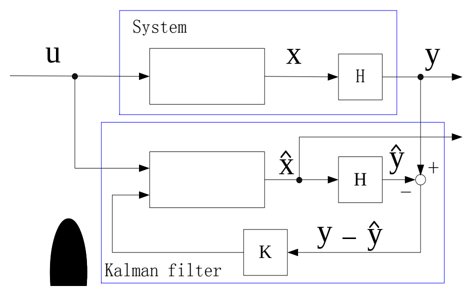

Fig. 2 Structure Scheme of EKF

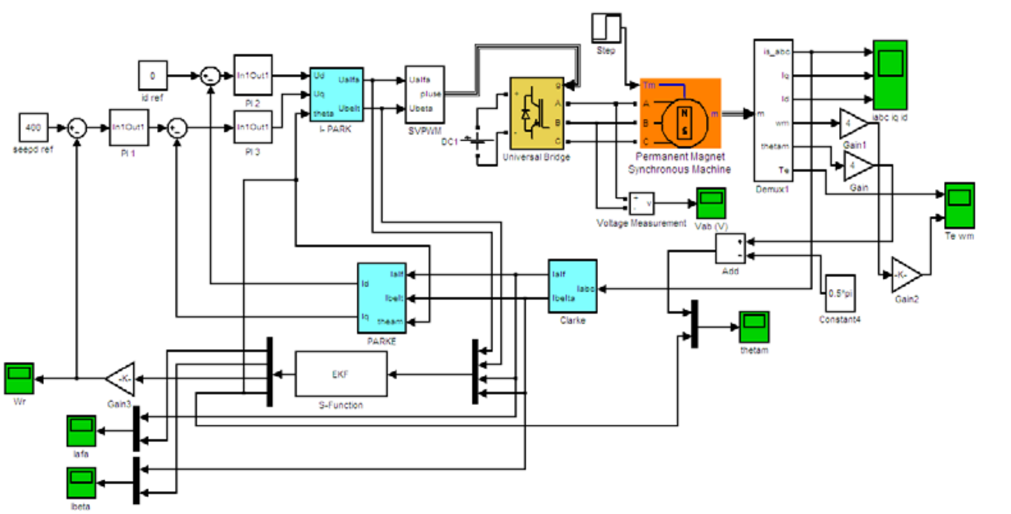

Fig. 3 System Simulation Diagram

After the system noise and observation noise are involved into the control system, non-linear discrete expressions of PMSM are equations (12) and (13).

$$x_k = f(x_{k-1},u_{k-1}) + W_k = (I + A\cdot T)\cdot x_{k-1} + B\cdot T\cdot u_{k-1} + W_k \qquad (12)$$

$$y_k = h(x_k) + V_k = H\cdot x_k + V_k \qquad (13)$$

Here, W_k and V_k are zero-mean Gaussian white noise sequences. Covariance matrixes of W_k and

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V_k are $Q = \mathrm{cov}(\mathrm{W}) = \mathrm{E}\{\mathrm{WW}^{\mathrm{T}}\}$ and $R = \mathrm{cov}(\mathrm{V}) = \mathrm{E}\{\mathrm{VV}^{\mathrm{T}}\}$ respectively.

When lineation process for the nonlinear discrete state equation (12) of PMSM, suppose the optimal estimation $\hat{x}$ of state variable x is known ahead of time at a moment. From equation (12), extend $f(\square)$ into Taylor series at $\hat{x}$ and omit those items more than second-order [9]. The nonlinear function acquired is equation (14).

$$x_k \approx f(\hat{x}_{k-1}) + \frac{\partial f}{\partial \hat{x}_{k-1}}(\mathrm{x}_{k-1} - \hat{x}_{k-1}) + W_{k-1} \quad (14)$$

Now, define that

$$\Phi_k = \frac{\partial f}{\partial \hat{x}_{k-1}} = \left.\frac{\partial f(\hat{x}_{k-1}, k-1)}{\partial \hat{x}_{k-1}}\right|_{x_{k-1}=\hat{x}_{k-1}}$$

$$= \begin{bmatrix} 1-T\frac{R_s}{L_s} & 0 & T\frac{\lambda_r N_p}{L_s}\sin\theta_r & T\frac{\lambda_r N_p}{L_s}\omega_r\cos\theta_r \\ 0 & 1-T\frac{R_s}{L_s} & -T\frac{\lambda_r N_p}{L_s}\cos\theta_r & T\frac{\lambda_r N_p}{L_s}\omega_r\sin\theta_r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & N_p T & 1 \end{bmatrix}_k$$

Estimate the state variables x_k at different time with recursive EKF algorithm.

$$\tilde{x}_k = f(\hat{x}_{k-1}, \mathrm{u}_{k-1}) = (I + A\cdot T)\cdot\hat{x}_{k-1} + B\cdot T\cdot u_{k-1} \quad (15)$$

$$\tilde{P}_k = \Phi_k \hat{P}_{k-1}\Phi_k^T + Q_k \quad (16)$$

$$K_k = \tilde{P}_k H_k^T (H_k \tilde{P}_k H_k^T + R_k)^{-1} \quad (17)$$

$$\hat{x}_k = \tilde{x}_k + K_k(y_k - H_k\tilde{x}_k) \quad (18)$$

$$\hat{P}_k = \tilde{P}_k - K_k H_k \tilde{P}_k \quad (19)$$

In the equations above, $\tilde{x}_k$ presents the state variable at time k and it is predicted by optimal state estimation $\hat{x}_{k-1}$. Also, it is called state prediction value at time k. K_k is the gain matrix of Kalman filter and $\tilde{P}_k$ is the covariance matrix of state prediction value $\tilde{x}_k$. $\hat{P}_k$ is the covariance matrix of optimal state estimation $\hat{x}_k$.

6. SIMULATION ANALYSIS OF PMSM CONTROL SYSTEM

Speed sensorless control system simulation model of PMSM is established with Matlab/Simulink. The simulation model diagram is shown in figure 3.

In the simulation model, PMSM model is designed with SimpowerSystems Toolbox and its parameters are configured be the reference literature [7]. $R_s = 2.875\Omega$, $L_s = 8.5\mathrm{mH}$, $\lambda_r = 0.175\mathrm{Wb}$, $J = 0.0008\mathrm{kg}\cdot\mathrm{m}^2$, $N_p = 4$ =4。

In the S function for EKF estimation, configure the initial state $x = [0;0;0;-\pi/2]$, the initial covariance matrix of state error P=diag(0.5,0.5,100,10), the initial covariance matrix of system noise Q=diag(5,5,200,1) and the initial covariance matrix of observation noise R=diag(0.5,0.5). In the process of EKF estimation, the calculation of the Kalman filter gain K is related with Q, R and P. If the covariance matrix Q changes, R will influence the dynamic character and stability of the EKF. Meanwhile, that the Q and R are appropriate or not greatly influences the accuracy of the EKF estimation and convergence of the algorithm. Generally, Q and R are unknown ahead of time. The determine principle is just considered based on the statistical characteristics of the system and measurement noise. In the actual system, they are generally determined by experience and simulation result. This paper chooses appropriate values by the previous experience from literature [10]. After the repetition of simulation and adjustment of the trial-and-error, those initial values are configured as before.

In the process of system simulation, configure the reference speed as 400rad/s. After 0.2s, speed becomes 200rad/s. The simulation duration is 0.5s. Experiment simulation result of speed is shown in figure 4.

In the figure 4(a), the red curve presents the actual measurements from speed sensor. As can be seen from the figure above, system disturbances emerge in the process of system starting and state transition. But, the transient to stable state is very quick. In the figure 4(b), blue curve presents the estimation of speed with EKF in sensorless condition. Compared with actual speed measurements, estimations from EKF slightly lag in the process of system starting and speed transition.

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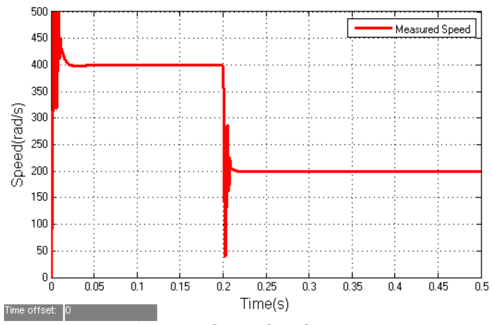
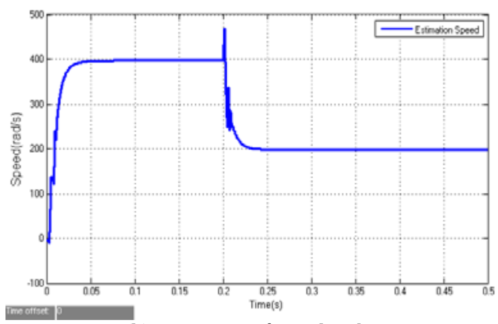
But, they still meet the performance requirements of the control system and capable of tracking the actual measurement speed values. After the system transition to steady-state, sensorless control is consistent with control by sensor accurately.



(a) Actual speed with sensor



(b) Estimation of speed with EFK

Fig. 4 Simulation Result Of Speed

Angle position estimations when sensorless and angle position measurements with sensor are shown in figure 5, in which red curve presents the angle measurements and blue curve presents the angle estimations from EKF. It clearly shows that the estimations are nearly identical with the actual measurements. Simulation result shows the feasibility of EKF estimations in the PMSM control system in case of sensorless condition.

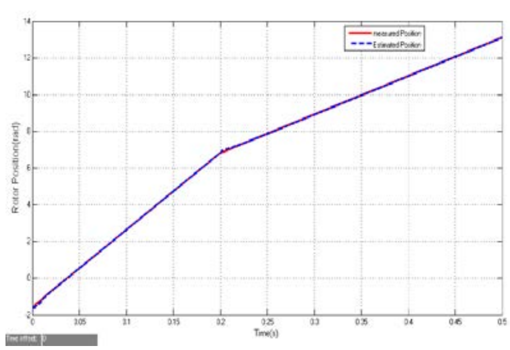


Fig. 5 Simulation Result of Angular Position

CONCLUSION

Different control models of PMSM are studied by simulation in this paper. We compare the sensorless control system with the sensor control system by Matlat/Simulink. Simulation results show that EKF algorithm accurately tracks rotation position and rotation speed of PMSM when the accurate PMSM model is available. Therefore, EKF state estimation meets the performance requirements of control system and possesses the similar performance of sensor control system.

Yet, the tracking error of the estimation model is relatively large in the beginning stage of the speed estimation and the speed change stage, especially in the beginning stage. To decrease the tracking error, more complicated model will be studied, such as the intelligent control system that may play the import role in the process of system controlling.

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