



NUMERICAL ANALYSIS AND RESEARCH ON COMPUTING VISUALIZATION OF STRENGTH REDUCTION FINITE ELEMENT METHOD BASED ON DIFFERENT INSTABILITY CRITERIA

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ABSTRACT

In the analysis of strength reduction finite element method, there are a variety of computational judgments of instability. Therefore, it is necessary to approach the applicability and discrepancy of different instability criteria. Based on the thought of real-time visual analysis, this article gives the research and development of corresponding visual programs, and finishes the real-time visual display technique of view about the relationship between three instability criteria and the strength reduction factor in computation of strength reduction finite element method. Then this article makes the numerical computation and analysis of a rather complicated slope. Research indicates that with the combination of finite element computation of strength reduction and the visual technique of computer, the sliding surface and stability of a slope can be analyzed intuitively. For slopes with local sliding surface or complicated shapes, safety factors are somewhat different because of employing the different instability criteria in the computation of strength reduction finite element.

Keywords: *Slope, Safety Factor, Instability Criterion, Numeric Computation, Visual Analysis, Strength Reduction Finite Element Method*

1. INTRODUCTION

Limit equilibrium method, limit analysis method, slip line method and so on, are traditional computational methods to study stability of slope, regardless of stress-strain relationship and deformation compatibility condition of the soil body. What's more, the above methods are difficult to be applied to stability analysis of slopes with complicated geometrical condition, loading condition or all kinds of it being reinforced. Shear strength reduction finite element method can overcome above weaknesses, and is becoming one of those computing approaches to analyze stability of the slope.

Nowadays, though the strength reduction finite element method is successfully used to analyze stability of slope, there are some instability criteria such as the iteration times of non-convergence, the connectivity of equivalent plastic shear strain, the abruptness of displacement in characteristic point. And these instability criteria are deeply influenced

by subjective judgment. Therefore, in order to approach the discrepancy of these criteria and to further analyze the evolutionary trend of different instability criteria in computation, it is necessary to carry out research on computation of strength reduction finite element method in terms of visualization. It not only can improve the numeracy of strength reduction finite element method, but also improve the instability judgment of the slope, with extremely important significance.

Section 2 presents a review of related work. In section 3, the computation principle of strength reduction finite element method is discussed. Section 4 presents the menu and function of developed ContourPlot software. Section 5 analyzes a complicated slope in computation by ContourPlot software. Section 6 draws a conclusion that the visual analytical software can be used in judgment of the stability of slope in computation.

2. RESEARCH REVIEW

The concept of strength reduction finite element method was first proposed by Zienkiewicz and so on [1]. Wong provided to analyze what caused the inaccuracy of the slope stability by means of the finite element method [2]. Ugai, Matsui and San[3], Griffiths and Lane[4], Dawson and Roth[5], Manzari and Nour[6], they all made further research and analysis of the strength reduction finite element method, and pushed the development and application of this method. Duncan pointed out that the slope safety factor could be defined as, when the slope exactly reached the critical destructive status, the extent to reduce the shearing strength of soil [7]. That is to say, the definition of safety factor is the ratio between the actual shearing strength of soil and the shearing strength after the reduction at the time of being critical destructive when the slope exactly reaches the critical destructive status. This research indicates that strength reduction finite element method and limit equilibrium method are accordant when it is used to define the concept safety factor. The stress of computation of the strength reduction finite element method is just more reasonable. Besides, the strength reduction method attracts widespread interest of scholars, because it does not need to preliminarily assume the shape and the position of sliding surface, and it can reflect the gradually destructive process of the earth slope to some extent.

Though strength reduction finite element method has successfully been applied to evaluate the stability of slope, the instability criteria in computation are not united. There are some kinds of instability criteria, for example, the iteration times of non-convergence, the connectivity of equivalent plastic shear strain, the abruptness of displacement in characteristic point, and so on. Deeply influenced by subjective factors, these criteria always cause different results. Liu Jinlong and et al approached the characteristics of instability criterion in strength reduction finite element method [8]. Fang Shaoshi and et al discussed factors which influenced the accuracy of computation in the analysis of strength reduction finite element method, based on the case of a slope [9].

To further compare the rationality and applicability based on three instability criteria: the iteration times of non-convergence, the connectivity of equivalent plastic shear strain and the abruptness of displacement in characteristic

point, this article makes a research on the implement method of visual analysis of instability criterion in the computation of strength reduction finite element method. Besides, this article further analyzes the evolution law of different instability criteria in computation, and inspects and verifies the applicability and rationality of research, based on analysis of examples in the computation of stability of the slope in complicated conditions.

3. COMPUTATIONAL PRINCIPLE OF THE STRENGTH REDUCTION FINITE ELEMENT METHOD

Mohr-Coulomb criterion is one of those frequently used yield criteria in soil and rock mechanics. The yield criterion consists of two parameters: cohesion c , and internal frictional angle φ . In computation of strength reduction elasto-plastic finite element method, Mohr-Coulomb criterion is also adopted as the failure criterion. To compute safety factor of slope with strength reduction finite element method, it is essential to reduce the values of cohesion and internal frictional angle of the soil and rock of slope, and when one or several of three conditions such as the iteration times of non-convergence, the connectivity of equivalent plastic shear strain and the abruptness of displacement in characteristic point happen, the reduction factor in the previous step is the safety factor of slope.

In the computation of strength reduction finite element method, at first, initial reduction factor is chosen, according to which, reduction of strength of every element in slope is in progress. That is to say, soil parameter value c , and φ are divided by a reduction factor F , a new group of value c_F and φ_F gotten, as new parameter. Besides, the corresponding yield criterion of strength is updated, and computation of elasto-plastic finite element method is in progress by the new yield criterion of strength.

To every step of reduction strength:

$$c_F = \frac{c}{F} \quad (1)$$

$$\tan \varphi_F = \frac{\tan \varphi}{F} \quad (2)$$

In computation of elasto-plastic finite element method, if the computation of strength reduction finite element method does not reach the set

instability criterion, the slope is still in stable condition. Then relevant reduction factor is increased, until computation of elasto-plastic finite element method reaches a instability criterion set by computation. At this time, slope soil reaches critical damage state, and reduction factor in the previous step is safety factor of slope and corresponding sliding surface is gotten.

It is worth noting that in computation of strength reduction elasto-plastic finite element method, one or several of three conditions like the iteration times of non-convergence, the connectivity of equivalent plastic shear strain and the abruptness of displacement in characteristic point adopted may cause some discrepancies among safety factors of slope. Nowadays, there is a united cognition in analysis of slope stability about how to choose instability criterion in the computation of strength reduction finite element method, but every kind of instability criterion somehow has shortcomings. Therefore, in computation of strength reduction finite element method, how to evaluate instability criterion and its certain safety factor is reasonably one of the key issues to be solved.

4. VISUAL IMPLEMENTING FUNCTION OF CONTOURPLOT SOFTWARE IN COMPUTATION

Visual Studio software is a development environment which can be used to initialize Windows application program on Windows platform. Visual FORTRAN compiler encapsulates nearly complete function Win32API and OpenGL, and it provides fine and stable programmatic interface for visual program.

In order to dynamically draw relevant visual graph about analytic process and outcome of slope using finite element method, also to be able to clearly show the relationship between computation process of strength reduction finite element method and potential sliding surface of slope this article makes research and development of a visual program, ContourPlot software through the integrated development environment Visual Studio 2008, and Visual FORTRAN compiler. Main column of the menu of ContourPlot software is divided into 8 submenus, File, Edit, Database, Plot, View, State, Window, Help, and other submenus. As illustrated in Fig.1. Functions of menu mainly used by this software are File function, Database function, and Plot function.

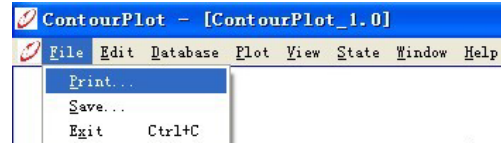


Fig.1 The File Menu Of Contourplot Software

It is function view of menu operation that ContourPlot uses. In Fig.1, Print option under File menu can start the printer to print every kind of view of computational analysis; save option can save view as a picture; exit option can log off the window. In Fig.2, Datinput option can put in data to acquire data again, and to draw relevant view. In Fig.3, RePlot option is to draw view once more; Other 4 options, Contour option, Contours option, GaussPoints option and γ -Fs option can draw relevant view separately.



Fig.2 The Database Menu Of Contourplot Software

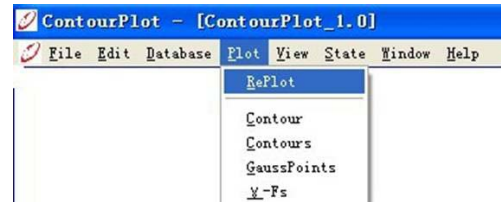


Fig.3 The Plot Menu Of Contourplot Software

Open Datinput option under Database menu, and Fig.4 can be seen. Fill in relevant filename in Filename column in window interface of data, and click OK button and then visual view of formerly calculating data can be achieved. Plot menu can draw every kind of visual view in computation, and make further analysis of auxiliary data. There is a modeling example below:

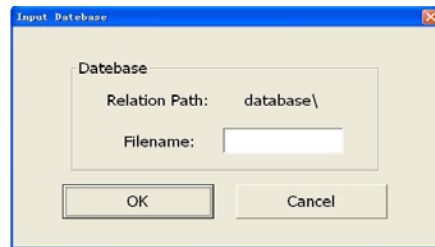


Fig.4 The Window Interface Of Datinput

The silt slope shown in Fig.5 is 20 m high. It has a natural unit weight of 19.08 kN/m³, a cohesion of 32 kPa, an internal frictional angle of 17 ° and a gradient of 45 °. In the mesh of two-dimensional finite element, there are 526 elements and 582

nodes. The left boundary and right boundary are horizontal restraint, the bottom is fixed, and the up-part is free boundary.

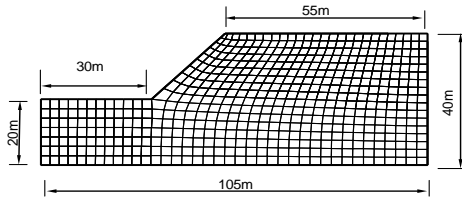


Fig.5 The Two-Dimensional Mesh

The visual function picture in ContourPlot software can reveal flood and isoclines of increment displacement in the computational process of strength reduction finite element. Fig. 6 is flood of increment displacement in computational process. Fig. 7 is isoclines of increment displacement. To contrast Fig. 6 with Fig. 7, the position of sliding surface of slope can be identified more clearly, and we can make out that potential sliding surface is located in the area where isoclines are relatively concentrated. In Fig. 6 and Fig. 7, separate functions of 4 windows are: the picture left above is a mesh of finite element; the picture right above is a display of increment displacement in the computational process; the picture left down is a display of horizontal increment displacement; the picture right down is a display of vertical increment displacement. In Fig.7, the pictures right above show the position of the greatest gradient of isoclines of increment displacement, according to which we can identify the potential sliding surface.

The visual function picture in ContourPlot software can reveal the graph of relation between three instability criteria and the strength reduction factor in the computational process of strength reduction finite element. In Fig.8, separate functions of 4 windows are: the picture left above is a mesh picture, which can choose Gauss point and the nodes, and it can be clicked for operation by the mouse in the software which is convenient to analyze the relation between three instability criteria and the strength reduction factor in the computational process element by element or node by node; the picture right above is a real-time calculating and displaying picture of computation based on the criterion of the iteration times of non-convergence, which can show us the safety factor of slope by curvilinear endpoint; the picture left down is a real-time calculating and displaying picture of computation based on the criterion of the abruptness of displacement in characteristic point, which indicates the situation of change about the

characteristic point; the picture right down is a real-time calculating and displaying picture of computation based on the criterion of connectivity of equivalent plastic shear strain, in which we can decide the critical point if the plastic area of slope is connectivity.

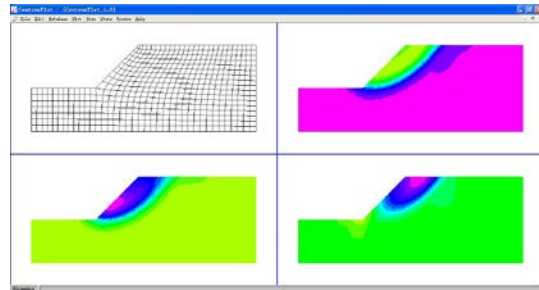


Fig.6 Flood Of Displacement In Computation

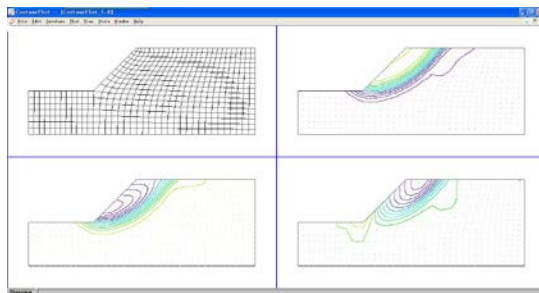


Fig.7 Contour Of Increment Displacement

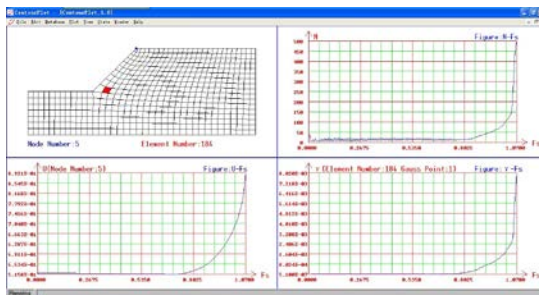


Fig.8 The Curves Of Different Instability Criterion

The goal of real-time display of this computational figure is to provide convenience for the judgment of the rationality of instability criterion by the visual window interface. From the numerical computation, the criterion of the iteration times of non-convergence, the criterion of connectivity of equivalent plastic shear strain and the criterion of the abruptness of displacement in characteristic point, we can get different strength reduction factors F_s . We can compare their conditions of change intuitively by real-time display of this computational view. This windows interface can be easily displayed by dynamic visual graphs. Besides, it has advantages like real-time display, and being optional of different observing elements and nodes.

Through the computation of strength reduction finite element, we get curved lines based on three instability criteria: the iteration times of non-convergence, the variation tendency of equivalent plastic shear strain of slope, and the abruptness of displacement in characteristic point in slope. From the feature of curved lines in pictures, we can decide F_s , the safety factors of slope, is 1.07, 1.05 and 0.97 individually. The outcomes from the third and second criteria are a little bit smaller than the outcome from the first criterion. Though the first criterion can obtain an outcome of safety factor of the slope, this outcome lags behind the outcome from the second criterion.

The analysis shows that, under the condition of simple slope, outcomes of safety factors from three instability criteria have no big differences, which are reasonable.

5. NUMERIC APPLICATION OF THE CONTOURPLOT SOFTWARE IN COMPUTATION

The Longdong Landslide is located at the north shore of the Yangtze River, 40 km away from the east of county of Yunyang near the reservoir area of the Three Gorges. It belongs to the landform with low mountains, and the landform is complicated. The sliding bed is the rock bed, while soil of the slope is mainly clay. The soil of sliding belt is mainly silt clay, which has thin particle, and it is a little bit wet and moldable with relatively smooth surface, and 30 cm in thickness, as illustrated in Fig. 9.

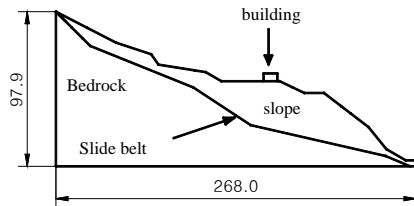


Fig.9 Sliding Slope Profile (Unit: M)

According to the codes and provisions of survey, the transfer coefficient method should be adopted in computation of stability of the landslide. Based on the hypothesis that the landslide slides along the whole sliding belt, the safety factor calculated by the transfer coefficient method is 1.05, which is in under stable condition. The Longdong slope has several features, for example, the upper part of the slide zone and gradient in this slope is steep while the lower part is flat. The slope

body is narrow in height, long in direction of horizon and irregular in shape. For these features, this article holds the point that there may be an instable condition that the local sliding is ahead of the entire sliding along sliding belt. And according to this condition, safety factors from three instability criteria may have relatively large differences.

From the slope engineering in practice, a numerical analysis modeling of finite element is set up in Fig.10. There are 2581 elements and 2705 nodes. Besides, the soil zone of slide belt with the thickness of 30 cm is divided into 3 layer elements. The bedrock is simplified as fixed restraint and the right boundary is horizontal restraint.

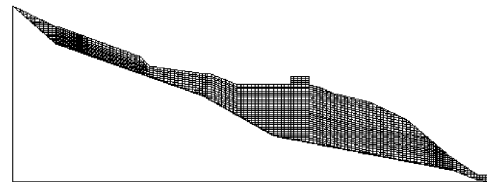


Fig.10 Finite Element Mesh

The material parameters are as shown in table.1, γ_{sat} and γ respectively represents saturated unit weight and natural unit weight; E for modulus of elasticity; ν for Poisson's ratio; c' for effective cohesion; ϕ' for effective frictional angle.

Table 1 The Parameter Of Materials

Material	γ_{sat} /(kN/m ³)	γ /(kN/m ³)	E /MPa	ν	c' /kPa	ϕ' /(°)
Slope	20.6	20	4.888	0.3	37	17.8
Slide belt	20.3	20	3.276	0.3	18	8.20

In order to investigate issues of the local sliding surface within the slope, we can analyze the flood of increment displacement (Fig.11) and isoclines of increment displacement (Fig.12), through the safety factor $F_s=0.9050$ which is calculated by strength reduction finite element.

Through analysis, we can know that the position where the gradient of isoclines of increment displacement is biggish does not mean that the position where the displacement increment is biggish. Distribution of the slope displacement registers as the displacement increment of upside soil body of slope is biggish while the gradient of isopleths of displacement increment is not biggish. The reason of this phenomenon is that partly sliding surface of the upside slope mostly forms a local sliding zone along the sliding belt which is 30 cm

thickness. The slide zone is only 30 cm thickness; therefore, though the gradient of isoclines of increment displacement in slide zone is biggish, it cannot be intuitively displayed in the reprocessed picture. That is why displacement of upside slope performs as obvious local slip sliding of rigid body. Therefore, the upside slope slides in advance, and it is a typical slope existing in the local sliding surface.

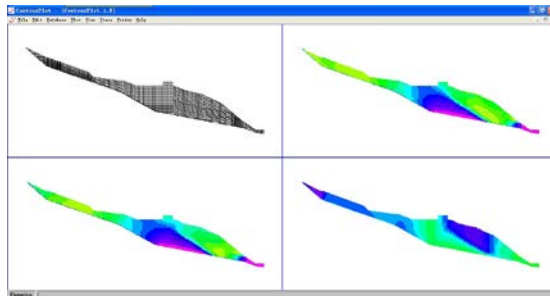


Fig.11 Flood Of Increment Displacement

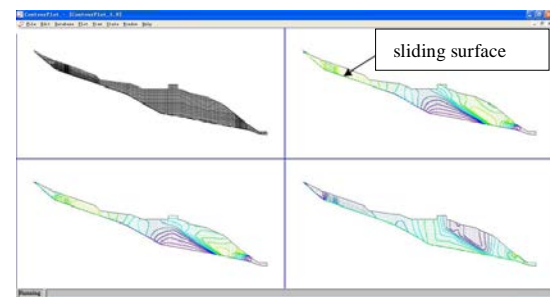


Fig.12 Contour Of Increment Displacement

For this slope, this article chooses the element and nodes in mesh of finite element to research the criterion of the iteration times of non-convergence, the criterion of connectivity of equivalent plastic shear strain, and the criterion of the abruptness of displacement in characteristic point. This article analyzes and decides the applicability and discrepancy of these three instability criteria in the computation of strength reduction finite element under the complicated conditions of slope. Thus we can decide instability of slope and definite sliding surface more effectively.

In the upside slope, we choose element 4 and node 6 as reference points. From computation and visual analysis, we get changing relationship between three instability criteria and reduction factors, as illustrated in Fig. 13. In Fig. 13, we can clearly observe the variation trend of F_s , the reduction factor and the reference values of three instability criteria in computation of strength reduction finite element. They are respectively the iteration times, the equivalent plastic shear strain of Gauss point in element 4, and the abruptness of

displacement in node 6. From activity in the dynamic visual graphs, we can decide safety factors F_s of slope, and they are respectively 0.905, 0.846 and 0.819.

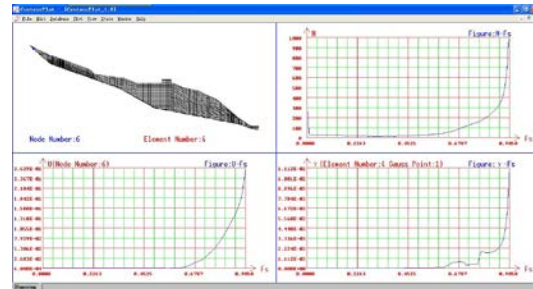


Fig.13 Curves Of Different Instability Criterion In Element 4 And Node 6

In the middle slope, we choose element 1452 and node 1513 as reference points. From computation and visual analysis, we get changing relationship between three instability criteria and reduction factors, as illustrated in Fig. 14. In Fig. 14, we can clearly observe the variation trend of F_s , the reduction factor and reference values of three instability criteria in computation of strength reduction finite element. They are respectively the iteration times, the equivalent plastic shear strain of Gauss point in element 1452, and the abruptness of displacement in node 1513. From activity in dynamic visual graphs, we can decide safety factors F_s of slope, and they are 0.905, 0.815 and 0.769 respectively.

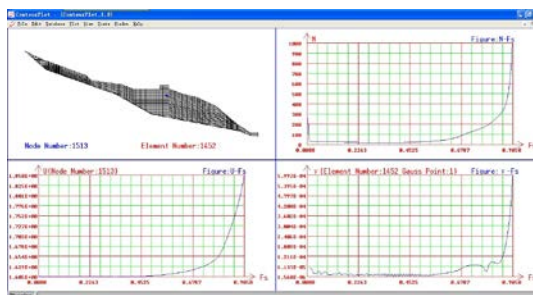


Fig.14 Curves Of Different Instability Criterion In Element 1452 And Node 1513

On the basis of the above analysis, we can know that under complicated slope conditions, safety factors of slope are respectively decided by the criterion of the iteration times of non-convergence, the criterion of connectivity of equivalent plastic shear strain, and the criterion of the abruptness of displacement in characteristic point have relatively obvious differences. In Fig. 14, the position where the maximum curvature of displacement- F_s ($U-F_s$) curved line occurs a little bit earlier than the one of shear strain- F_s ($\gamma-F_s$) curved line. While the position where the maximum



curvature of γ -Fs curved line occurs a little bit earlier than one of number of iteration-Fs (N-Fs) curved lines. Therefore, in analysis of stability of slope with local sliding surface, different instability criteria adopted in computation of strength reduction finite element may cause discrepancies among certain safety factors of slope.

Therefore, for slope with complicated shape, in computation of strength reduction finite element, different instability criteria adopted may cause certain discrepancies of safety factors of slope.

6. CONCLUSIONS

This article develops the visualized analytical software, which can be used in computation of strength reduction finite element method. Based on computations of strength reduction finite element method by different instability criteria and visual analysis, conclusions gained are as shown below:

(1) In strength reduction finite element computation, instability criteria of certain slope are inconsistent, for example, the criterion of the iteration times of non-convergence, the criterion of connectivity of equivalent plastic shear strain, and the criterion of the abruptness of displacement in characteristic point. They have their own advantages and disadvantages. In order to decide which criterion is more proper, the combination of strength reduction finite element method and visualization of computer can make it applicable to analyze sliding surface and stability of slope more intuitively.

(2) In strength reduction finite element computation, outcome from the criterion of the abruptness of displacement and the criterion of equivalent plastic shear strain are a little bit smaller; however, when adopting the criterion of the iteration times of non-convergence as the instability criterion of the slope, the outcome is a little bit larger. It could be indicated that, for the simple slope, outcomes calculated by these three instability criteria are roughly the same.

(3) For the slope with complicated shape, safety factors from different instability criteria in strength reduction finite element computation have certain discrepancies because of the local sliding surface.

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