

SHIP DYNAMIC POSITIONING SYSTEM BASED ON BACKSTEPPING CONTROL

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ABSTRACT

Ship dynamic positioning refers to ship which singly keeps its fixed position on the sea by its own installation propeller without anchor. Backstepping design method using in the uncertainty system is a kind of systematic controller synthesis method and is a regression design method which combine the selection of the lyapunov function with the design of the controller. This paper begin with the minimum order differential equation of the system, then introduce the concept of virtual control and design the virtual control meeting the requirements step by step, in the end design the real controller according with the required index. At last establish the modeling and simulate in the Matlab, realize the Backstepping control in combination with S-function. The simulation results suggest that we can use Backstepping method to realize the control of the ship dynamic positioning system.

Keywords: *Ship, Dynamic Positioning System, Backstepping Control, Matlab*

1. INTRODUCTION

As the development and utilization of the offshore petroleum resources, the continuous development of marine transport and marine science and technology, Ship and ocean engineering technology started to rise rapidly. Because the ship and the platform often need to locate its position in particular site, and the traditional way of anchor method restricted by the factors of operating water depth, working time and accuracy requirement. In order to solve this problem of the ship position and its course keeping, a ship control technology called dynamic positioning [1-3] arises at the historic moment and made a considerable progress at the development of correlation technique of electronic computer, sensor, and control theory.

The technology of dynamic positioning has been developed more than fifty years. With the advance of science and technology, new and more complex control method continually applied into the dynamic positioning system. More importantly, the development of nonlinear theory let the research of dynamic positioning control from linear system to nonlinear system. At most of the control application, various of uncertain factors inevitably existed in controlled object and its environment, so nonlinear control theory of the dynamic positioning system become the research front.

At the current time, researchers have put forward some control methods, such as fuzzy control, sliding mode control, model predictive control, robust control, and neural networks etc [4-8], using in dynamic positioning control system. The above-mentioned nonlinear control methods can well realize the ship dynamic positioning system.

This paper is organized as follows. Section 2 establishes a mathematical modeling of ship DP system. In section 3, we introduce a nonlinear control method, backstepping control algorithm [9-12]. In section 4 we firstly make use of the PID control method in the ship dynamic positioning system, and then apply the backstepping control algorithm into the ship DP system. Section 5 gives a conclusion to the whole paper.

2. THE MODEL OF SHIP DYNAMIC POSITIONING

2.1 Brief Introduction

Dynamic positioning, mainly consisting of measuring system, control system and thruster, is a ship control system separately using its installed propeller without anchoring to keep its fixed-position on the sea. Its simplified block diagram is as shown in Figure 1.

2.2 Model of Environmental Forces

The mathematical model of environmental forces can be described as follow:

$$\dot{b} = -T_b^{-1}b + E_b w_b \quad (1)$$

Where b being a three-dimensional vector, represents the environmental disturbances; T_b is a three-dimensional time-constant diagonal matrix; E_b is a three-dimensional diagonal matrix which denotes the amplitude of the environmental forces; w_b is a zero-mean white noise vector.

In addition, T_b Can also be zero, so the above expression can be simplified as:

$$\dot{b} = E_b w_b \quad (2)$$

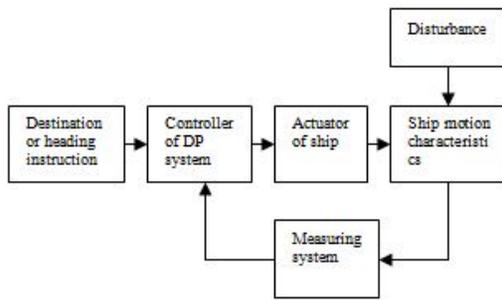


Figure 1: Simplified Block Diagram of DP System

2.3 Ship Low-Frequency Mathematical Model

Combined with the system noise, this simplified three degrees of freedom body-fixed coupled equations of the low-frequency motions in surge, sway and yaw can be described as follow:

$$\dot{\eta} = R^T(\psi)v \quad (3)$$

$$M\dot{v} + Dv = \tau + R^T(\psi)b + E_v w_v \quad (4)$$

Where $\eta = [x, y, \psi]^T$ denotes the ship's position (x, y) and heading ψ coordinated in the earth-fixed frame, $v = [u, v, r]^T$ indicates the surge, sway and yaw velocities of the ship coordinated in the body-fixed frame; $R^T(\psi)$ represents the rotation matrix represents the mass matrix; D represents the damping matrix; τ represents the forces and moments acting on ship coordinated in the body-fixed frame; w_v is a zero-mean white noise vector; E_v is a three-dimensional diagonal matrix which denotes the amplitude of the environmental forces.

2.4 Ship High-Frequency Mathematical Model

Ship high-frequency motion is actually a response to first order wave forces; it can be described as a second order harmonic oscillator adding damping term on position and angle:

$$h(s) = \frac{K_{wi}s}{s^2 + 2\xi_i w_{\sigma i} s + w_{\sigma i}^2} \quad (5)$$

Where K_{wi} ($i=1, \dots, 3$) relates to Wave intensity, the value of damping factor ξ_i ($i=1, \dots, 3$) should be between 0.02~0.05, $w_{\sigma i}$ ($i=1, \dots, 3$), related to the significant wave height, is the dominant frequency in wave spectrum; and the transfer function $h(s)$ represents the relation between wave disturbances forces $w_{waves} = [X_w, Y_w, N_w]^T$ and its frequency w_i as:

$$w_{waves} = \frac{K_{wi}s}{s^2 + 2\xi_i w_{\sigma i} s + w_{\sigma i}^2} w_i, i = 1, 2, 3 \quad (6)$$

2.5 Measurement Model

Ship measuring system provides the value of its position and heading accompanied by the measurement noise, so the measurement model of the DP system can be described as:

$$y = \eta + \eta_w + w_y \quad (7)$$

Where w_y is zero mean white noise vector; and η_w is the value of ship position and heading arouse by the environmental disturbances.

2.6 A State Space Description of Nonlinear Mathematical Model of Ship Motion

According to the above description, we can get a state space description of nonlinear mathematical model of ship motion:

$$\dot{x} = f(x) + Bu + Ew \quad (8)$$

$$y = Hx + v \quad (9)$$

Where state vector $x = [\xi^T, \eta^T, b^T, v^T]^T$; $u = \tau$ is a control vector; $w = [w_h^T, w_b^T, w_y^T]^T$ represents system white noise vector; v represents measuring system white noise vector.

$$f(x) = \begin{bmatrix} A_w \xi \\ R(\psi)v \\ -T_b^{-1}b \\ -M^{-1}Dv + M^{-1}R^T(\psi)b \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M^{-1} \end{bmatrix}$$

$$H = [C_h, I, 0, 0], E = [E_h, 0, E_b, M^{-1}E_v]^T$$

If we linearize the nonlinear equations, we can get a state space description of linear mathematical model of ship motion:

$$\dot{x} = Ax + Bu + Ew \quad (10)$$

$$y = Hx + v \quad (11)$$



3. DESIGN OF BACKSTEPPING CONTROLLER

Lyapunov theory [13-14] has already been one of the most important tools in researching the stability of nonlinear control system. As we known, one Non-negative differentiable function $V(t)$ is a monotonically decreasing function when its derivative satisfies $\dot{V}(t) \leq 0$. If $\dot{V}(t) \leq -cV(t)$, we can obtain $V(t_0) \leq V(t) \leq e^{-c(t-t_0)}V(t_0)$, it means $V(t)$ exponential decay to zero. Lyapunov second method is actually further expand from above idea using in stability analysis. That is about estimating one system's stability performance through a positive definite function and its derivative symbol.

Backstepping control method is actually a comprehensive method focusing on the uncertain system. It is a regression design method combining the selection of the Lyapunov function and the design of controller. Its basic idea is to resolve a complicated system into some subsystem according to the system orders. Then separately design Lyapunov function and intermediate virtual control quantity for every subsystem through backstepping control method, until complete the design of controller. Backstepping control method can be suitable for linear and nonlinear system, especially is effective in nonlinear system taking parametric strict feedback forms.

Suppose the state space description of the controlled object's nonlinear mathematical model can be described as:

$$\dot{x}_1 = x_2 \tag{12}$$

$$\dot{x}_2 = f(x,t) + b(x,t)u \tag{13}$$

$$y = x_1 \tag{14}$$

Where $f(x,t), b(x,t)$ are nonlinear expressions satisfying $b(x,t) \neq 0$

The basic design procedure of the backstepping control method can be described as:

Step 1:

Definition of the position error

$$e_1 = y - y_d = x_1 - r \tag{15}$$

Where e_1 is the value of error of the system; y_d is the desired position; that is the input command signal r ; then we can obtain:

$$\dot{e}_1 = \dot{x}_1 - \dot{r} \tag{16}$$

Definition of the controlled variable

$$\alpha_1 = -c_1 e_1 + \dot{r} \tag{17}$$

Where $c_1 > 0$

Then a new error is defined as:

$$e_2 = x_2 - \alpha_1 \tag{18}$$

Select the Lyapunov function candidate as:

$$V_1 = \frac{1}{2} e_1^2 \tag{19}$$

We can obtain:

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1(x_2 - \dot{r}) = e_1(e_2 + \alpha_1 - \dot{r}) \tag{20}$$

Substituting Eq. (17) into Eq. (20), we can obtain:

$$\dot{V}_1 = -c_1 e_1^2 + e_1 e_2 \tag{21}$$

If $e_2 = 0$, we can obtain $\dot{V}_1 \leq 0$, so we design the next step.

Step 2:

Select the Lyapunov function candidate as:

$$V_2 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \tag{22}$$

Because of

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = f(x,t) + b(x,t)u + c_1 \dot{e}_1 - \ddot{r} \tag{23}$$

We can obtain:

$$\dot{V}_2 = -c_1 e_1^2 + e_1 e_2 + e_2 [f(x,t) + b(x,t)u + c_1 \dot{e}_1 - \ddot{r}] \tag{24}$$

If we want $\dot{V}_2 \leq 0$, the expression of the controller can be given as:

$$u = \frac{1}{b(x,t)} [-f(x,t) - c_2 e_2 - e_1 - c_1 \dot{e}_1 + \ddot{r}] \tag{25}$$

Where $c_2 > 0$, we can obtain:

$$\dot{V}_2 = -c_1 e_1^2 - c_2 e_2^2 \leq 0 \tag{26}$$

Through the design of the controller, we get a system satisfying Lyapunov stability theory condition, and e_1 and e_2 are exponential asymptotically stable to ensure that this designed system is globally exponential asymptotically stable.

4. CONTROL SYSTEM SIMULATIONS

In order to test the DP control method, in this paper we example one DP supply ship to complete the simulation. The ship size is shown in table 1.

Table 1: Parameters of the Example Ship

Ship length	76.2m	Draft	6.25m
Ship width	18.8m	Displacement	4200t
Ship height	82.5m	Power	3533kW

This supply ship's model parameters are identified from the sea trial. Its mass matrix M and damping matrix D are given as follows:

$$M = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{bmatrix}$$

At the same time, if we linearize the rotation matrix $R^T(\psi)$, we can obtain:

$$R^T(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

If we Integrate the Eq. (2) and Eq. (6) into τ representing the forces and moments acting on ship, we can get the disturbed forces and moments expressing by a new symbol $\tau = u + \tau_{di}$, where

$$\tau_{di} = \frac{K_{wi}s}{s^2 + 2\xi_i w_{\sigma i} s + w_{\sigma i}^2} w_i + b_i, i = 1,2,3$$

$$b_i = \frac{1}{s} w_{bi}, i = 1,2,3 \quad (27)$$

At this time, the ship mathematical model can be simplified by:

$$\dot{\eta} = I v$$

$$\dot{v} = -M^{-1} D v + M^{-1} \tau$$

$$y = \eta \quad (28)$$

4.1 PID Control Method

Consider the motion of surge and sway, ignoring the disturbances of wind and wave, ship high frequency motion, and the ship coupling of three directions. We can get a state equation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a x_2 + b u$$

$$y = x_1 \quad (29)$$

Through calculation we can get the value of the state equation of surge motion model is $a = -0.0318$ and $b = 0.8870$, the value of the state equation of sway motion model is $a = -0.0641$ and $b = 0.5415$. This ship DP system is demonstrated by simulations using the Matlab / Simulink toolbox.

Suppose the initial position is (0m, 0m), the desired position is (10m, 10m). Select the module of Constant as our input and set its value as 10, parameters of PID valued as $p=10$; $i=0.01$; $d=3$. The module of $Pid1$ and $Pid2$ are the state equation of ship's surge and sway motion. The outputs of the simulation delegating the ship's position are as shown in Figure 2 and Figure 3. Ship's motion trail is as shown in Figure 4. The simulation time is 10s.

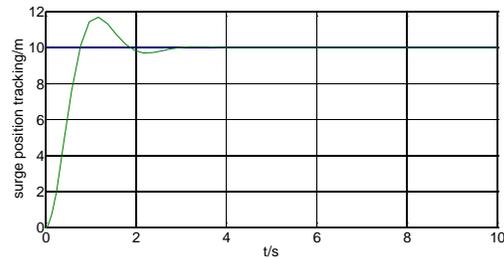


Figure 2: the Position of the Ship's Surge Motion

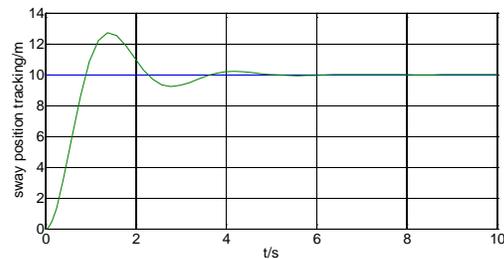


Figure 3: the Position of the Ship's Sway Motion

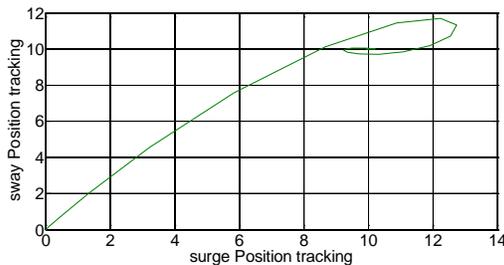


Figure 4: Ship's Motion Trail

From those figures, we can see the DP system's effective control result through PID control theory; its oscillation amplitude is between 0.9-1.2 standard units.

4.2 Backstepping Control Algorithm

Now we use the backstepping control algorithm describing in this paper. Also suppose the initial position is (0m, 0m), the desired position is (10m, 10m). Select the module of Constant as our input

and set its value as 10. The module of back based s-function is the backstepping controller; the module of plant is the state equation of ship's surge motion; the module of plant1 is the state equation of ship's sway motion. Parameters of backstepping controller are $c_1=10$ and $c_2=10$. At the same time we add the Band-Limited White Noise denoting the

environmental disturbances in this simulation. The outputs of the simulation delegating the ship's position are as shown in Figure 5 and Figure 6. The outputs of the backstepping controller are shown in Figure 7 and Figure 8. Ship's motion trail is as shown in Figure 9. The simulation time is 10s.

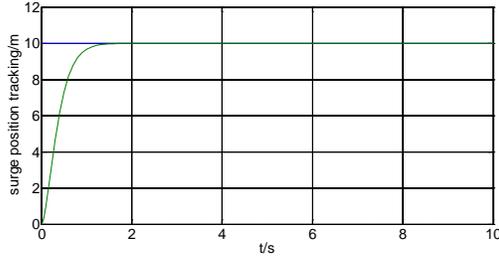


Figure 5: the Position of the Ship's Surge Motion

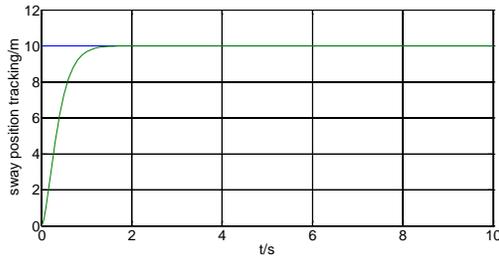


Figure 6: the Position of the Ship's Sway Motion

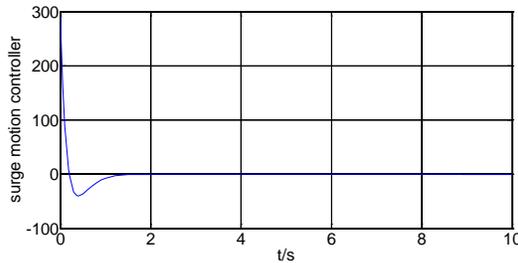


Figure 7: the Output of Controller of the Surge Motion

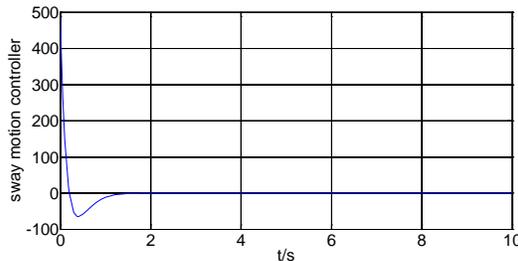


Figure 8: the Output of Controller of the Sway Motion

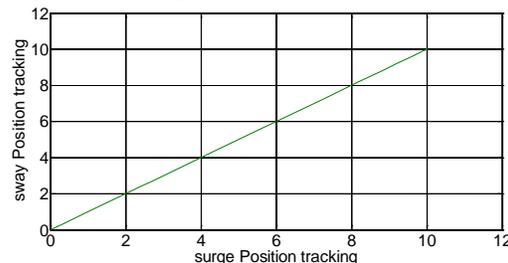


Figure 9: Ship's Motion Trail

From those figures, we can see the DP system's effective control result through backstepping control theory. This DP system doesn't appear obvious overshoot and oscillation, and enters into the steady state in one second. Parameters needing to adjust in the simulation only have two of c_1 and c_2 , and the adjusting is very easy. Comparing different parameters with its simulation result we can conclude that the values of the two parameters are linear with its control effect. Even though we add the environmental disturbances, the DP system can rapidly reach into the steady state, and the outputs of the controller u remain stable after the ship arrive the desired position.

4.3 Backstepping Algorithm of the Coupled System

From the above design of the backstepping controller, we can obtain one system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= fx_2 + bu \end{aligned}$$

Its backstepping controller can be given as

$$u = \frac{1}{b} [-fx_2 - c_2e_2 - e_1 - c_1\dot{e}_1 + \ddot{r}]$$

So about a coupled system:

$$\dot{x}_1 = f_1x_1 + f_2x_2 + b_1u_1 + b_2u_2 \quad (30)$$

$$\dot{x}_2 = f_3x_2 + f_4x_1 + b_3u_1 + b_4u_2 \quad (31)$$

Define a new variable:

$$u_a = -f_1x_1 - f_2x_2 - c_2e_2 - e_1 - c_1\dot{e}_1 + \ddot{r}$$

$$u_b = -f_3x_2 - f_4x_1 - c_2e_2 - e_1 - c_1\dot{e}_1 + \ddot{r}$$

From the Eq. (30) and Eq. (31), we can obtain:

$$b_1u_1 + b_2u_2 = u_a ; b_3u_1 + b_4u_2 = u_b \quad (32)$$

From the Eq. (32), we can solve:

$$u_1 = \frac{1}{(b_1b_4 - b_2b_3)} (b_4u_a - b_2u_b) \quad (33)$$

$$u_2 = \frac{-1}{(b_1b_4 - b_2b_3)} (b_3u_a - b_1u_b) \quad (34)$$

That is the backstepping controller of the coupled system (the Eq. (30) and Eq. (31)).

About the above simplified mathematical model of ship motion: $\dot{\eta} = Iv$; $\dot{v} = -M^{-1}Dv + M^{-1}\tau$; $y = \eta$. Where $\tau = [X, Y, N]^T$ represent the forces and moments acting on ship. $\tau = u + \tau_{di}$,

$$\tau_{di} = \frac{K_{wi}s}{s^2 + 2\xi_i w_{\sigma_i} s + w_{\sigma_i}^2} w_i + \frac{1}{s} w_{bi}, i = 1, 2, 3$$

In order to guarantee the ship's maximum physical limit, we must add a saturation element in simulation to revise those equations, that is:

$$b_i = \frac{1}{s} w_{bi} \leq d_{i,max} \approx 0.1$$

Generally, according to the customs, we assume $\xi_i = 0.1$, $w_{\sigma_i} = 0.8976$, $K_{wi} = 2\xi_i w_{\sigma_i} \sigma \approx 0.25 (\sigma \approx \sqrt{2})$.

So we can obtain:

$$\tau_{di} = \frac{0.25s}{s^2 + 0.18s + 0.79} w_i + \frac{1}{s} w_{bi}, i = 1, 2, 3$$

Substituting the value of the mass matrix M and the damping matrix D into Eq. (28), we can obtain:

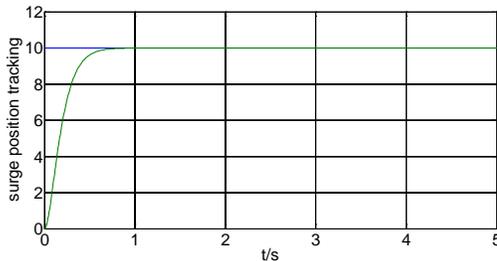


Figure 10: the Position of the Ship's Surge Motion

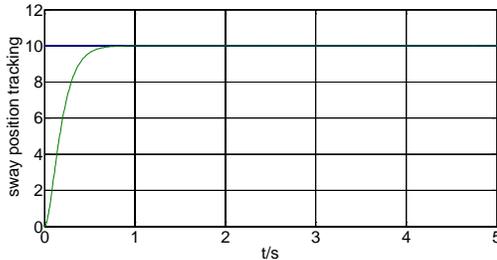


Figure 11: the Position of the Ship's Sway Motion

$$\begin{aligned} \dot{x} &= u \\ \dot{u} &= -0.0318u + 0.8870X \\ \dot{y} &= v \\ \dot{v} &= -0.0641v + 0.0039r + 0.5415Y + 0.3152N \\ \dot{\psi} &= r \\ \dot{r} &= -0.2467r + 0.0013v + 0.3152Y + 8.0082N \end{aligned}$$

Using the regulation of Eq. (33) and Eq. (34), we write control program in s-function. Suppose the initial position is (0m, 0m, 0°), the desired position is (10m, 10m, 10°). Select the module of Constant as our input and set its value as 10. The module of a based s-function is the backstepping controller; the module of b is the state equation of ship's motion; Parameters of backstepping controller are c1=10 and c2=10. At the same time we add the Band-Limited White Noise denoting the environmental disturbances in this simulation. The outputs of the simulation delegating the ship's position are as shown in Figure 10, Figure 11 and Figure 12. The outputs of the backstepping controller are shown in Figure 13, Figure 14, and Figure 15. Ship's motion trail is as shown in Figure 16. The simulation time is 10s.

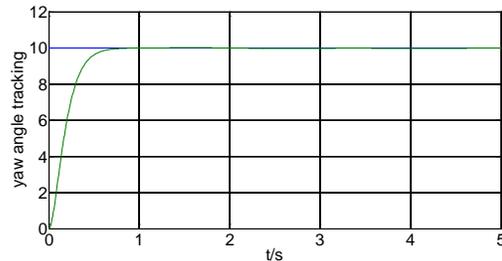


Figure 12: the Angle of the Ship's Yaw Motion

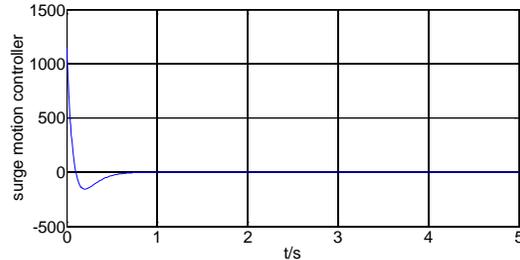


Figure 13: the Output of Controller of the Surge Motion

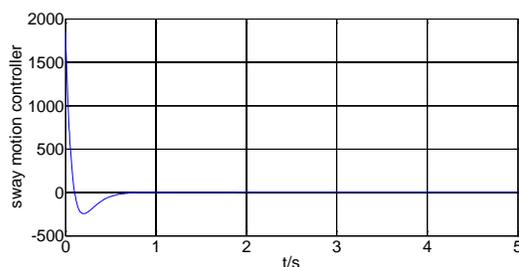


Figure 14: the Output of Controller of the Sway Motion

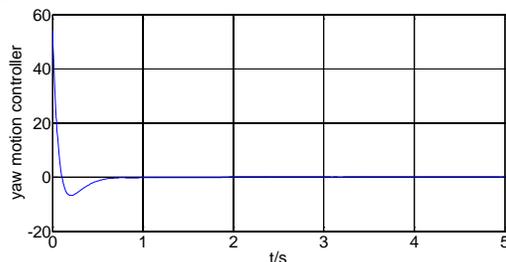


Figure 15: the Output of Controller of the Yaw Motion

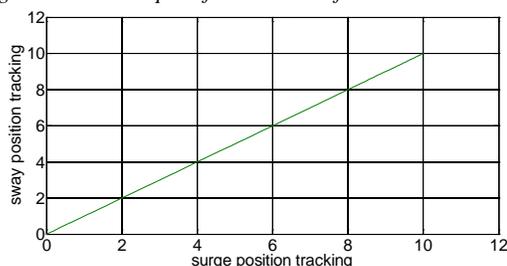


Figure 16: Ship's Motion Trail

From those figures, we can see the coupled DP system's effective control result through backstepping control theory. This DP system doesn't appear obvious overshoot and oscillation, and enters into the steady state in one second. Even though we add the environmental disturbances, the DP system can rapidly reach into the steady state, and the outputs of the controller u remain stable after the ship arrive the desired position.

5. CONCLUSIONS

In order to ensure that the DP system is globally exponential asymptotically stable, this paper based on nonlinear mathematical model of the ship DP system presents an overview of the backstepping controller based on Lyapunov theory. At first, we design a PID controller as a reference, then we design a nonlinear controller for DP system using backstepping control algorithm. From the simulation result we can see that backstepping control algorithm has a good control performance and preferably achieves the requirements of the dynamic positioning system. In order to obtain a more precise simulation result, we can add the

Kalman filter into the simulation program in future research.

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