



# ADAPTIVE FUZZY BACKSTEPPING CONTROLLER FOR POSITION CONTROL OF A PERMANENT MAGNET SYNCHRONOUS MOTOR DRIVE SYSTEM

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## ABSTRACT

In this paper, an adaptive fuzzy backstepping controller has been developed for the position tracking control of a permanent magnet synchronous motor (PMSM) drive. Fuzzy logic systems are used to approximate unknown nonlinearities appearing in the control law and an adaptive backstepping technique is employed to construct controller. To illustrate the performance of the proposed controller compared with the conventional backstepping that is studied by computer simulations. Simulation results verify that the proposed control structure is very simple and the position tracking error can converge to a small scope under parameter uncertainties and load torque disturbance.

**Keywords:** *Adaptive fuzzy control; Permanent magnet synchronous motor; Stability; Backstepping*

## 1. INTRODUCTION

Permanent magnet synchronous motors (PMSMs) have aroused great interest, since it has superior features such as compact size, high inertia ratio and low cost. However, it is a challenging problem to control PMSMs to get the perfect dynamic performance because it is very sensitive to external load disturbances and parameter variations in industrial applications. On the other hand, its dynamic model is highly nonlinear. Some control techniques such as nonlinear control <sup>[1]</sup>, sliding mode control <sup>[2]</sup> and adaptive intelligent control <sup>[3]</sup> have been developed to overcome these problems for speed and position control of PMSMs.

Much research has been witnessed in recent years to apply backstepping methodology to design controllers for nonlinear systems. This method provides a powerful control designing tool, for nonlinear systems in the strict feed back forms. The conventional backstepping is successfully applied to the control of PMSMs recently <sup>[4]</sup>. The most appealing point of the backstepping scheme is to use the virtual control variable to make the original high order system to be simple enough. thus the final control outputs can be derived step by step through the Lyapunov functions. Adaptive forms of this controller have been also developed. However,

a major disadvantage with backstepping approaches is that some tedious analysis is needed to determine a "regression matrix". In <sup>[5]</sup>, adaptive backstepping was used to compensate the nonlinearities in the speed control for a PMSM, but it is worth notice that the regression matrix almost covers one full paper. Another disadvantage is called the problem of "explosion terms" caused by the virtual variable.

Fuzzy logic controllers (FLCs) have been used in many areas. FLCs can be easily used in the control systems for which an exact mathematical model of the system cannot be obtained <sup>[6]</sup>. Unlike the existing backstepping design technique, the fuzzy control approach is suitable for dealing with the system's unknown nonlinearities. So the fuzzy control approach combining with backstepping technique can deal with the decoupling control problem of a class of nonlinear systems with unknown uncertainties <sup>[7-8]</sup>. In [9], the concept of fuzzy approximate disturbance decoupling based on backstepping technique is introduced for a class of MIMO nonlinear systems, and the proposed algorithm has less adaptive parameters. In [8], an adaptive fuzzy control method is developed to suppress chaos in the permanent magnet synchronous motor drive system via backstepping technology. Fuzzy logic systems are used to approximate unknown nonlinearities.

In this paper, we attempt to combine the direct adaptive fuzzy control approach with the backstepping technique to provide an effective position tracking control for the PMSM drive system. During the controller design process, fuzzy logic systems are employed to approximate the nonlinearities, and the adaptive technique and backstepping are used to construct fuzzy controllers. The designed fuzzy controller guarantees the uniform ultimate boundedness of the closed-loop adaptive systems; this means no regression matrices and the problem of “explosion of terms” are taken into account. So the major drawbacks of the classical backstepping are cured. To verify the performance of the proposed controller, a comparison between the classical backstepping and the proposed controller is implemented by computer simulations. The results

show that the proposed controller is reliable, effective and insensitive to parameter variations and external disturbance for the position control of the PMSM. Moreover, the proposed controller guarantees that the position tracking error converges to a small scope.

The paper is organized as follows. Section 2 introduced the mathematical model of PMSM. In Section 3, an adaptive fuzzy backstepping controller is designed. In Section 4, stability of the proposed control method is analyzed. In Section 5, the classical backstepping controller is designed and stability is analyzed. In Section 6, comparison between the two controllers is studied by computer simulations to illustrate the feasibility of the proposed control scheme. In Section 7, we conclude the work of this paper.

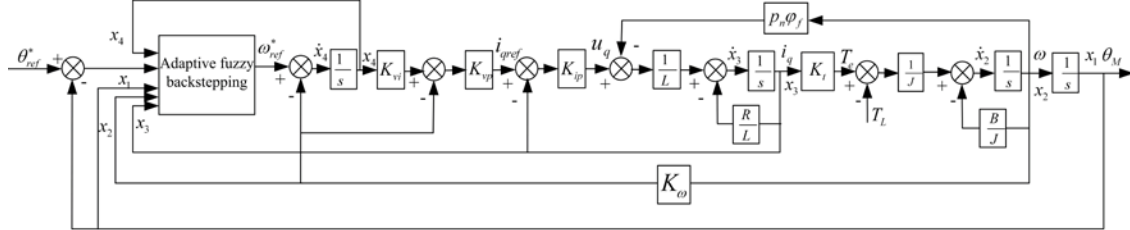


Figure 1: Position Control Of PMSM Drive Using The Proposed Adaptive Fuzzy Backstepping

## 2. PMSM MODEL

With the assumption that the PMSM is unsaturated and eddy currents and hysteresis losses are negligible, the stator  $d, q$ -axes voltage equations of the PMSM in the synchronous rotating reference frame are given by

$$\begin{cases} U_d = Ri_d + p_n L_d \dot{i}_d + p_n \phi_f - \omega \Psi_q \\ U_q = Ri_q + p_n L_q \dot{i}_q + \omega \Psi_d \end{cases} \quad (1)$$

$L_d$  and  $L_q$  are called  $d$ - and  $q$ -axis synchronous inductances, respectively,  $\omega$  is motor electrical speed,  $\Psi_d$  and  $\Psi_q$  are the flux linkages in the  $dq$  frame,  $\phi_f$  is rotor flux linkage. Note that  $L_d$  and  $L_q$  are equal and are taken as  $L$  for the surface PMSM.

Using the method of field oriented control of the PMSM, the  $d$ -axis current is controlled to be zero to maximize the output torque<sup>[10]</sup>. The motor torque is given by

$$T_e = \frac{3}{2} p_n \phi_f i_q = K_t i_q \quad (2)$$

Where  $K_t$  is the torque constant and  $p_n$  is the number of poles in the motor.

In general, the mechanical equation of the PMSM can be represented as:

$$T_e = J\dot{\omega} + T_L + B\omega \quad (3)$$

Where  $J$  is the total inertia and  $B$  is the frictional coefficient.

Substituting Eq. (2) into Eq. (3), the mechanical dynamic of the PMSM drive system can be represented as

$$\ddot{\theta}_M = -\frac{B}{J}\dot{\theta}_M + \frac{K_t}{J}i_q - \frac{T_L}{J} \quad (4)$$

Where  $\theta_M$  is rotor angular,  $\dot{\theta}_M = \omega$ .

Using the vector control technique, the simplified block diagram of the PMSM drive



system including current control and speed control can be represented and shown in Figure 1.

For the purpose of further analysis in Figure 1, the dynamic model of the PMSM drive system can be described by the following differential equations:

$$\dot{x}_1 = x_2 \tag{5}$$

$$\dot{x}_2 = \frac{K_t}{J} x_3 - \frac{B}{J} x_2 - \frac{T_L}{J} \tag{6}$$

$$\dot{x}_3 = -\frac{p_n \phi_f + K_{ip} K_{vp} K_\omega}{L} x_2 - \frac{R + K_{ip}}{L} x_3 + \frac{K_{ip} K_{vp} K_{vi}}{L} x_4 \tag{7}$$

$$\dot{x}_4 = -K_\omega x_2 + \omega_{ref}^* \tag{8}$$

Where  $x_1 = \theta_M, x_2 = \omega, x_3 = i_q, x_4 = \int(\omega_{ref}^* - \omega)dt$ ;  $K_{ip}$  is the current controller gain;  $K_{vp}$  is the speed controller proportional parameter;  $K_{vi}$  is the speed controller integral parameter;  $K_\omega$  is the speed

feedback coefficient.  $\omega_{ref}^*$  is the reference speed.

$$H_2 = -(p_n \phi_f + K_{ip} K_{vp} K_\omega) / L ;$$

$$H_3 = -(R + K_{ip}) / L ; H_4 = K_{ip} K_{vp} K_{vi} / L .$$

The control objective is to design an adaptive fuzzy controller so that the state variable  $x_i$  ( $i=1, 4$ )

follows the given reference signal  $\theta_{ref}^*$  and all the closed-loop signals are bounded. To this end, we adopt the singleton fuzzifier, product inference, and the central defuzzifier to deduce the following fuzzy rules <sup>[11]</sup>:

IF  $x_1$  is  $A_1^i$  and ...and  $x_n$  is  $A_n^i$  THEN  $y$  is  $B^i$  ( $i=1,2,...,N$ )

Where  $x=[x_1, \dots, x_n]T \in R_n$ , and  $y \in R$  are the input and output of the fuzzy system, respectively,

$A_i^j$  and  $B^i$  are fuzzy sets in  $R$ . Since the strategy of singleton fuzzification, center-average defuzzification and product inference is used, the output of the fuzzy system can be formulated as

$$y(x) = \frac{\sum_{j=1}^N W_j \prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^N [\prod_{i=1}^n \mu_{A_i^j}(x_i)]} \tag{9}$$

Where  $W_j$  is the point at which fuzzy membership function  $\mu_{B^j}(W_j)$  achieves its maximum value. Let

$$p_j(x) = \frac{\prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^N [\prod_{i=1}^n \mu_{A_i^j}(x_i)]}$$

$S(x)=[p_1(x), p_2(x), \dots, p_N(x)]T$  and  $W=[W_1, W_2, \dots, W_N]T$ , then the fuzzy logic system above can be rewritten as

$$y(x) = W^T S(x) \tag{10}$$

If all memberships are taken as Gaussian functions, then the following lemma holds.

Lemma 1: Let  $f(x)$  be a continuous function defined on a compact set  $\Omega$ . Then for any scalar  $\varepsilon > 0$ , there exists a fuzzy logic system in the form (10) such that

$$\sup_{x \in \Omega} |f(x) - y(x)| \leq \varepsilon \tag{11}$$

### 3. ADAPTIVE FUZZY CONTROLLER DESIGN WITH BACKSTEPPING

In this section, we will develop a control algorithm for the PMSM system. The system (1) leads a system structure, namely, the system with

$(x_1, x_2, \dots, x_4)$  as state variables and  $\omega_{ref}^*$  as control input. The backstepping design procedure contains four steps. At each design step, a virtual control function  $\alpha_i$  ( $i=1,2,3$ ) will be constructed by using an appropriate Lyapunov function. At the last step, the real controller is constructed to control the system.

Step1: For the reference signal  $\theta_{ref}^*$ , define the

tracking error variable as  $e_1 = x_1 - \theta_{ref}^*$ . From the first differential equation of (1), the error dynamic system is given by  $\dot{e} = \dot{x}_2 - \dot{\theta}_{ref}^*$ .

Choose Lyapunov function candidate as  $V_1 = e_1^2 / 2$ , then the time derivative of  $V_1$  is computed by

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1(x_2 - \dot{\theta}_{ref}^*) \tag{12}$$

Construct the virtual control law  $\alpha_1$  as



$$\alpha_1 = -k_1 e_1 + \dot{\theta}_{ref}^* \tag{13}$$

With  $k_1 > 0$  being a design parameter. By using (12) and (13) can be rewritten as the following form:

$$\dot{V}_1 = -k_1 e_1^2 + e_1 e_2 \tag{14}$$

With  $e_2 = x_2 - \alpha_1$ .

Step 2: Differentiating  $e_2$  gives

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = \frac{K_t}{J} x_3 - \frac{B}{J} x_2 - \frac{T_L}{J} - \dot{\alpha}_1 \tag{15}$$

Now, choose the Lyapunov function candidate

$V_2 = V_1 + \frac{J}{2} e_2^2$ . Obviously, the time derivative of  $V_2$  is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + J e_2 \dot{e}_2 = -k_1 e_1^2 + e_1 e_2 \\ &+ e_2 (K_t x_3 - B x_2 - T_L - J \dot{\alpha}_1) \end{aligned} \tag{16}$$

Remark 1: in this paper, due to the motor torque TL is unknown but its upper bound is  $d > 0$  in practice system, namely,  $0 \leq T_L \leq d$ . Obviously,

$e_2 T_L \leq \frac{1}{2\epsilon_2^2} e_2^2 + \frac{1}{2} \xi_2^2 d^2$ , where  $\xi_2$  is an arbitrary small positive constant. Since  $J$  and  $B$  are unknown, they cannot be used to construct the control signal.

Thus, let  $\hat{J}$  be the estimation of  $J$  and  $\hat{B}$  be the estimation of  $B$ . The virtual control  $\alpha_2$  is constructed as

$$\alpha_2 = \frac{1}{K_t} (-k_2 e_2 - \frac{1}{2\xi_2^2} e_2 + \hat{B} x_2 + \hat{J} \dot{\alpha}_1 - e_1) \tag{17}$$

Then the time derivative of  $V_2$  can be expressed as

$$\begin{aligned} \dot{V}_2 &\leq -k_1 e_1^2 - k_2 e_2^2 + K_t e_2 e_3 + e_2 (\hat{B} - B) x_2 \\ &+ e_2 (\hat{J} - J) \dot{\alpha}_1 + \frac{1}{2} \xi_2^2 d^2 \end{aligned} \tag{18}$$

With  $k_2 > 0$  being a design parameter,  $e_3 = x_3 - \alpha_2$ .

Step 3: Choose Lyapunov function candidate as  $V_3 = V_2 + \frac{1}{2} e_3^2$ , then the time derivative of  $V_3$  is computed by

$$\dot{V}_3 = \dot{V}_2 + e_3 \dot{e}_3 \tag{19}$$

Construct the virtual control law  $\alpha_3$  as

$$\alpha_3 = \frac{1}{H_4} (-k_3 e_3 - H_2 x_2 - H_3 x_3 + \dot{\alpha}_2 - K_t e_2) \tag{20}$$

With  $k > 0$  being a design parameter. By using (19) and (20) can be rewritten of the following form.

$$\dot{V}_3 \leq -\sum_{i=1}^3 k_i e_i^2 + H_4 e_3 e_4 + e_2 (\hat{B} - B) x_2 + e_2 (\hat{J} - J) \dot{\alpha}_1 + \frac{1}{2} \xi_2^2 d^2 \tag{21}$$

With  $k_3 > 0$  being a design parameter,  $e_4 = x_4 - \alpha_3$ .

Step 4: At this step, we will construct the control law  $\omega_{ref}^*$ . In the end, choose the following Lyapunov function candidate as  $V_4 = V_3 + \frac{1}{2} e_4^2$ . Then the derivative of  $V_4$  is given by

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + e_4 \dot{e}_4 \leq -\sum_{i=1}^3 k_i e_i^2 + e_2 (\hat{B} - B) x_2 + e_2 (\hat{J} - J) \dot{\alpha}_1 \\ &+ \frac{1}{2} \xi_2^2 d^2 + e_4 (\omega_{ref}^* + f_4) \end{aligned} \tag{22}$$

Where  $f_4 = -K_\omega x_2 - \dot{\alpha}_3 + H_4 e_3$ .

Notice that  $f_4$  containing the derivative of  $\alpha_3$ . This will make the classical adaptive backstepping design become very complex and troubled, and the

designed control law  $\omega_{ref}^*$  will have the complex structure. To avoid the trouble in design procedure and simplify the control signal structure, we will employ the fuzzy logic system to approximate the function  $f_4$ . According to Lemma 1, for any given  $\xi_4 > 0$  there exists a fuzzy logic system  $W_4^T S(x)$  so that



$$f_4 = W_4^T S(x) + \sigma_4 \tag{23}$$

With  $\sigma_4$  being the approximation error and satisfying  $|\sigma_4| \leq \xi_4$ . Consequently, a simple method computing produces the following inequality:

$$e_4 f_4 = e_4 (W_4^T S_3 + \sigma_4) \leq e_4 \left( \frac{\|W_4\| \|S_4\| \|W_4^T l_4\|}{l_4 \|W_4\|} + \xi_4 \right) \leq \frac{1}{2l_4^2} e_4^2 \|W_4\|^2 S_4^T S_4 + \frac{1}{2} l_4^2 + \frac{1}{2} e_4^2 + \frac{1}{2} \xi_4^2 \tag{24}$$

It follows immediately from substituting (24) into (22) that

$$\dot{V}_4 \leq -\sum_{i=1}^4 k_i e_i^2 + e_2 (\hat{B} - B) x_2 + e_2 (\hat{J} - J) \dot{\alpha}_1 + \frac{1}{2l_4^2} e_4^2 (\|W_4\|^2 - \hat{\theta}) S_4^T S_4 + \frac{1}{2} l_4^2 + \frac{1}{2} \xi_4^2 + \frac{1}{2} \xi_2^2 d^2 \tag{25}$$

The control input  $\omega_{ref}^*$  is designed as

$$\omega_{ref}^* = -k_4 e_4 - \frac{1}{2} e_4 - \frac{1}{2l_4^2} e_4 \hat{\theta} S_4^T S_4 \tag{26}$$

Then, define  $\theta = \|W_2\|^2$ , at the present stage, to estimate the unknown parameters  $B, J$  and  $\theta$ , define the adaptive variables as follows:  $\tilde{B} = \hat{B} - B, \tilde{J} = \hat{J} - J$  and  $\tilde{\theta} = \hat{\theta} - \theta$ . In order to determine the corresponding adaptation laws, choose the following Lyapunov function candidate:

$$V = V_4 + \frac{1}{2r_1} \tilde{B}^2 + \frac{1}{2r_2} \tilde{J}^2 + \frac{1}{2r_3} \tilde{\theta}^2 \tag{27}$$

Where  $r_i$  ( $i=1, 2, 3$ ) are positive constant. The derivative of  $V$  is given by

$$\dot{V} \leq -\sum_{i=1}^4 k_i e_i^2 + \frac{1}{2} \xi_2^2 d + \frac{1}{2} l_4^2 + \frac{1}{2} \xi_4^2 + \frac{1}{r_1} \tilde{B} (e_2 r_1 x_2 + \dot{\hat{B}}) + \frac{1}{r_2} \tilde{J} (e_2 r_2 \dot{\alpha}_1 + \dot{\hat{J}}) + \frac{1}{r_3} \tilde{\theta} \left( -\frac{r_3}{2l_4^2} e_4^2 S_4^T S_4 + \dot{\hat{\theta}} \right) \tag{28}$$

According to (28), the corresponding adaptive laws are chosen as follows:

$$\dot{\hat{B}} = -e_2 r_1 x_2 - m_1 \hat{B}$$

$$\dot{\hat{J}} = -r_2 e_2 \dot{\alpha}_1 - m_2 \hat{J}$$

$$\dot{\hat{\theta}} = \frac{r_3}{2l_4^2} e_4^2 S_4^T S_4 - m_3 \hat{\theta} \tag{29}$$

Where  $m_i$  ( $i=1, 2, 3$ ) and  $l_4$  are positive constant.

#### 4. STABILITY ANALYSIS<sup>[12]</sup>

In this section, substitute (29) into (28) gives

$$\dot{V} \leq -\sum_{i=1}^4 k_i e_i^2 + \frac{1}{2} \xi_2^2 d + \frac{1}{2} l_4^2 + \frac{1}{2} \xi_4^2 - \frac{m_1}{r_1} \tilde{B} \hat{B} - \frac{m_2}{r_2} \tilde{J} \hat{J} - \frac{m_3}{r_3} \tilde{\theta} \hat{\theta} \tag{30}$$

We can change the term  $\tilde{B} \hat{B}$  as  $-\tilde{B} \hat{B} \leq -\tilde{B} (\tilde{B} + B) \leq -\frac{1}{2} \tilde{B}^2 + \frac{1}{2} B^2$

Similarly, we have

$$-\tilde{J} \hat{J} \leq -\frac{1}{2} \tilde{J}^2 + \frac{1}{2} J^2$$

$$-\tilde{\theta} \hat{\theta} \leq -\frac{1}{2} \tilde{\theta}^2 + \frac{1}{2} \theta^2 \tag{31}$$

Consequently, substitute (31) into (30) yields

$$\dot{V} \leq -\sum k_i e_i^2 + \frac{1}{2} \xi_2^2 d + \frac{1}{2} l_4^2 + \frac{1}{2} \xi_4^2 - \frac{m_1}{2r_1} \tilde{B}^2 - \frac{m_2}{2r_2} \tilde{J}^2 + \frac{m_3}{2r_3} \tilde{\theta}^2 + \frac{m_1}{2r_1} B^2 + \frac{m_2}{2r_2} J^2 + \frac{m_3}{2r_3} \theta^2 \leq -a_0 V + b_0 \tag{32}$$

Where

$$a_0 = \min\{2k_1, 2k_2, 2k_3, 2k_4, m_1, m_2, m_3\}$$

$$b_0 = \frac{1}{2} \xi_2^2 d + \frac{1}{2} l_4^2 + \frac{1}{2} \xi_4^2 + \frac{m_1}{2r_1} B^2 + \frac{m_2}{2r_2} J^2 + \frac{m_3}{2r_3} \theta^2$$

Furthermore, (32) implies that

$$V(t) \leq (V(t_0) - b_0 / a_0) e^{-a_0(t-t_0)} + b_0 / a_0 \leq V(t_0) + b_0 / a_0, \quad \forall t \geq t_0 \tag{33}$$



As a result, all  $e_i$  ( $i=1,2,3,4$ ),  $\tilde{B}$ ,  $\tilde{J}$  and  $\tilde{\theta}$  belong to the compact set

$$\Omega = \left\{ (e_i, \tilde{B}, \tilde{J}, \tilde{\theta}) \mid V \leq V(t_0) + b_0 / a_0, \forall t \geq t_0 \right\} \quad (34)$$

So from (34) we have

$$\lim_{t \rightarrow \infty} e_1^2 \leq 2b_0 / a_0$$

Namely, all the signals in the position closed-loop system are bounded.

### 5. A COMPARISON WITH THE CLASSICAL BACKSTEPPING DESIGN

In this section, we will give a comparison between the adaptive fuzzy backstepping and the classical backstepping method. Thus the classical backstepping is used to control design for the position system of the PMSM drive. And the implementation of the control algorithm using the MATLAB/Simulink blocks is carried out by both control technique.

#### 5.1 Classical Backstepping Design

Step 1: For the position signal  $\theta_{ref}^*$ , define the tracking error variable as  $e_1 = x_1 - \theta_{ref}^*$ . From the first differential equation of (1), the error dynamic system is given by  $\dot{e}_1 = \dot{x}_2 - \dot{\theta}_{ref}^*$

Choose Lyapunov function candidate as  $V_1 = e_1^2 / 2$ , then the time derivative of  $V_1$  is computed by

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1 (x_2 - \dot{\theta}_{ref}^*) \quad (35)$$

Construct the virtual control law  $\alpha_1$  as

$$\alpha_1 = -k_1 e_1 + \dot{\theta}_{ref}^* \quad (36)$$

With  $k_1 > 0$  being a design parameter. By using (35) and (36) can be rewritten as the following form:

$$\dot{V}_1 = -k_1 e_1^2 + e_1 e_2 \quad (37)$$

With  $e_2 = x_2 - \alpha_1$ .

Step 2: Differentiating  $e_2$  gives

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = \frac{K_t}{J} x_3 - \frac{B}{J} x_2 - \frac{T_L}{J} - \dot{\alpha}_1$$

Now, choose the Lyapunov function candidate as

$$V_2 = V_1 + \frac{J}{2} e_2^2$$

Obviously, the time derivative of  $V_2$  is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + J e_2 \dot{e}_2 = -k_1 e_1^2 + \\ &e_2 (K_t x_3 - B x_2 - T_L - J \dot{\alpha}_1 + e_1) \end{aligned} \quad (39)$$

The virtual control  $\alpha_2$  is constructed as

$$\alpha_2 = \frac{1}{K_t} (-k_2 e_2 + B x_2 + J \dot{\alpha}_1 - e_1 + T_L) \quad (40)$$

Where  $k_2 > 0$  is a positive design parameter. Adding an subtracting  $\alpha_2$  in (39) shows that

$$\dot{V}_2 \leq -k_1 e_1^2 - k_2 e_2^2 + K_t e_2 e_3 \quad (41)$$

With  $e_3 = x_3 - \alpha_2$ .

Step 3: Differentiating  $e_3$  results in the following equation

$$\begin{aligned} \dot{e}_3 &= \dot{x}_3 - \dot{\alpha}_2 = H_2 x_2 + \\ &H_3 x_3 + H_4 x_4 - \dot{\alpha}_2 \end{aligned} \quad (42)$$

Choose the Lyapunov function as

$$V_3 = V_2 + \frac{1}{2} e_3^2$$

Thus differentiating  $V_3$  gives

$$\dot{V}_3 \leq -\sum_{i=1}^3 k_i e_i^2 + H_4 e_3 e_4 \quad (43)$$

Where

$$\alpha_3 = \frac{1}{H_4} (-k_3 e_3 - H_2 x_2 - H_3 x_3 + \dot{\alpha}_2 - K_t e_2)$$

$k_3 > 0$  is a positive design parameter and  $e_4 = x_4 - \alpha_3$ .

Step 4: Choose the Lyapunov function candidate as  $V_4 = V_3 + \frac{1}{2} e_4^2$ , then the time derivative of  $V_4$  is computed by

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + e_4 \dot{e}_4 \leq -\sum_{i=1}^3 k_i e_i^2 + \\ &e_4 (-K_\omega x_2 + \omega_{ref}^* - \dot{\alpha}_3 + H_4 e_3) \end{aligned} \quad (44)$$

And the control law  $\omega_{ref}^*$  is designed as

$$\omega_{ref}^* = -k_4 e_4 + K_\omega x_2 + \dot{\alpha}_3 - H_4 e_3 \quad (45)$$

Where  $k_4 > 0$  is a positive design parameter.

### 5.2 Stability Analysis

Furthermore, using the equality (45), it can be easily verified that

$$\dot{V}_4 \leq -\sum_{i=1}^4 k_i e_i^2 \quad (46)$$

By comparing the control law (26) with (45), it is easy to see that the proposed adaptive fuzzy backstepping controller have simpler structure than the classical backstepping. In addition, the controller (45) requires the precise information on the nonlinear functions, when the nonlinear functions are unknown; the classical backstepping cannot be used to obtain the control law.

## 6. SIMULATION

The simulation is run for PMSM with the parameters:

$$\begin{aligned} J &= 0.002625 \text{Kg} \cdot \text{m}^2, R = 1.32 \Omega, \\ B &= 0.0001034 \text{N} \cdot \text{m}/(\text{rad}/\text{s}), L = 0.0335 \text{H}, p_n = 3, \\ K_t &= 1.34 \text{N} \cdot \text{m}/\text{A}, K_{vi} = 24, K_{vp} = 25, K_{ip} = 10, \\ d &= 25 \text{N} \cdot \text{m}. \end{aligned}$$

The fuzzy membership functions are

$$\begin{aligned} \mu_{A_1^1} &= \exp\left[\frac{-(x+5)^2}{2}\right], & \mu_{A_2^1} &= \exp\left[\frac{-(x+4)^2}{2}\right], \\ \mu_{A_3^1} &= \exp\left[\frac{-(x+3)^2}{2}\right], & \mu_{A_4^1} &= \exp\left[\frac{-(x+2)^2}{2}\right], \\ \mu_{A_5^1} &= \exp\left[\frac{-(x+1)^2}{2}\right], & \mu_{A_6^1} &= \exp\left[\frac{-(x+0)^2}{2}\right], \\ \mu_{A_7^1} &= \exp\left[\frac{-(x-1)^2}{2}\right], & \mu_{A_8^1} &= \exp\left[\frac{-(x-2)^2}{2}\right], \\ \mu_{A_9^1} &= \exp\left[\frac{-(x-3)^2}{2}\right], & \mu_{A_{10}^1} &= \exp\left[\frac{-(x-4)^2}{2}\right], \\ \mu_{A_{11}^1} &= \exp\left[\frac{-(x-5)^2}{2}\right]. \end{aligned}$$

The control parameters are chosen as follows:

$$\begin{aligned} k_1 &= 17, k_2 = 11, k_3 = 35, k_4 = 13, r_1 = r_2 = r_3 = 2, \\ m_1 &= m_2 = m_3 = 0.004, l_4 = 0.7. \end{aligned}$$

The classical backstepping are also used to control the PMSM drive system. The controller parameters are chosen as  $k_1 = 17, k_2 = 11, k_3 = 35, k_4 = 13$ .

To give the further comparison, the simulation is run under the same assumption that the system parameters and the nonlinear functions are unknown.

The reference signals are taken as  $x_{1d} = 2\sin(t) + 2\sin(0.5t)$  with  $T_L$  being

$$T_L = \begin{cases} 15, & 0 \leq t \leq 10 \\ 5, & t \geq 10 \end{cases}$$

The simulation results for both adaptive fuzzy control and classical backstepping control are shown in Figures 2-18. The reference signals for both control approaches are shown in Figure 2. Figures 3-6 and Figures 11-14 display the system state responses. Figures 8-10 and Figures 16-18 show the virtual control state signals. Figures 3-7 shows our control scheme can achieve the better control performances than the classical backstepping in Figures 11-15. From Figures 8-10 and Figures 16-18, it can be seen that under both control methods, the system follow the desired virtual control state signals well.

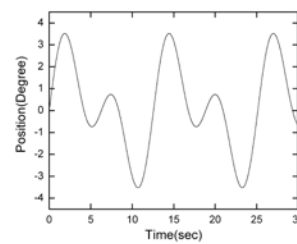


Figure 2: Reference Signal

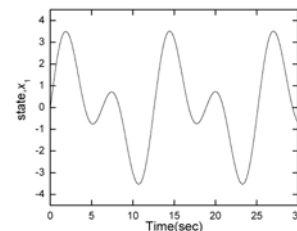


Figure 3:  $x_1$

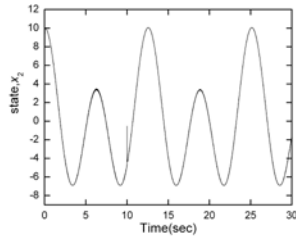


Figure 4:  $x_2$

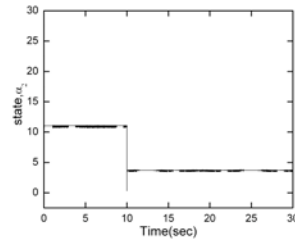


Figure 9:  $\alpha_2$

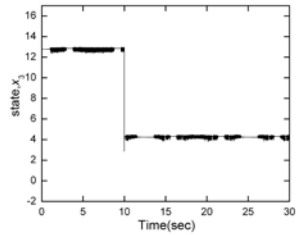


Figure 5:  $x_3$

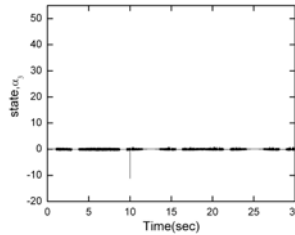


Figure 10:  $\alpha_3$

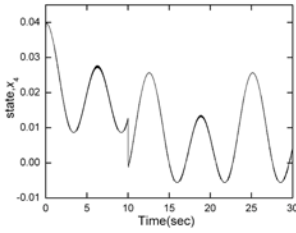


Figure 6:  $x_4$

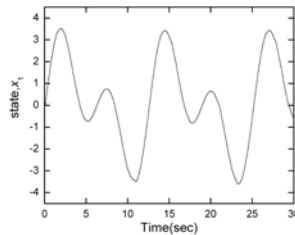


Figure 11:  $x_1$  For Classical Backstepping

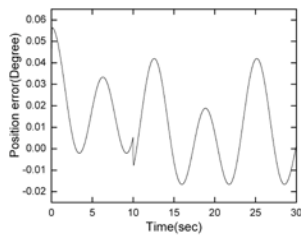


Figure 7:  $e_1$

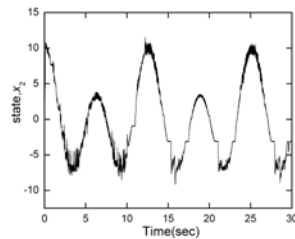


Figure 12:  $x_2$  For Classical Backstepping

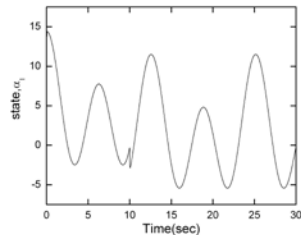


Figure 8:  $\alpha_1$

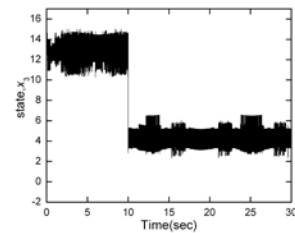


Figure 13:  $x_3$  For Classical Backstepping



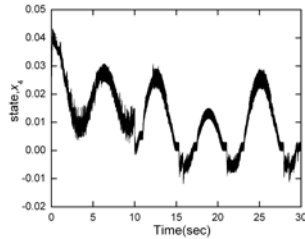


Figure 14:  $x_4$  For Classical Backstepping

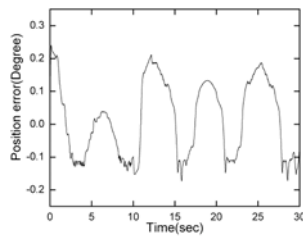


Figure 15:  $e_1$  For Classical Backstepping

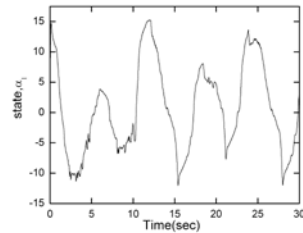


Figure 16:  $\alpha_1$  For Classical Backstepping

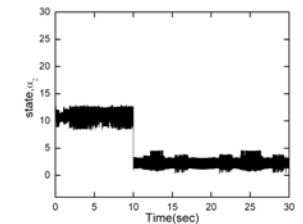


Fig.17.  $\alpha_2$  for classical backstepping

Figure 17:  $\alpha_2$  For Classical Backstepping

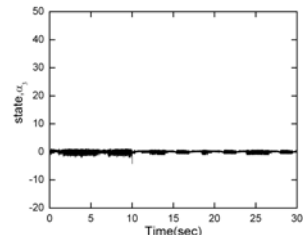


Figure 18:  $\alpha_3$  For Classical Backstepping

## 7. CONCLUSION

In this paper, an adaptive fuzzy backstepping is developed to control PMSM in unmeasured states. Fuzzy logic systems are used to approximate the uncertain nonlinear functions. The proposed controllers which overcome the problems of the classical backstepping guarantee that the position tracking error converges to a small scope and all the closed-loop signals are bounded. Simulation results show that the effectiveness of the proposed control method.

## ACKNOWLEDGEMENTS:

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