Journal of Theoretical and Applied Information Technology
30th April 2013. Vol. 50 No.3
© 2005 - 2013 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

A SIMPLE COMPUTER CONTROL METHOD OF INVERTED PENDULUM

SUN QIANLAI, SUN ZHIYI

School of Electronic Information Engineering, Taiyuan University of Science and Technology, China

E-mail: sql_sun@163.com, sunzhiyi@263.net

ABSTRACT

In order to guarantee real time, a simple control method is presented to stabilize the fast-moving system-inverted pendulum. The method was obtained by mathematical derivation based on the dynamical model of the inverted pendulum system, but the control strategy is independent of the model. It is applicable for controlling other multi input single output plants besides inverted pendulum. A controller was designed to stabilize an inverted pendulum based on that method. The structure of the controller is simplified, and the parameters which need to be adjusted are reduced. The simulation result shows that the simple control method is valid to stabilize the inverted pendulum.

Keywords: *Computer control, Inverted pendulum, Mathematical derivation*

1. INTRODUCTION

The inverted pendulum system is a typical nonlinear, unstable, non-minimum phase, fasting-moving and under-actuated systems. It is a very important problem to stabilize inverted pendulum for research of control theory. As a typical controlled plant, inverted pendulum is not only a challenge for design of controller, but also a very good experiment platform for research and application of many advanced control methods. Moreover, the inverted pendulum has very important practical values for the control of robot, aircraft, aerospace craft and etc. Thus, the control method of inverted pendulum not only has theory value but also has universal use in war industry, spaceflight, robot and general industry areas. The control problem of inverted pendulum has been studied widely by many scholars [1-4] from the first success of development of single inverted pendulum system.

According to the published literature, there are many theories and successful examples to stabilize the inverted pendulum. At present, common methods to stabilize inverted pendulum include linear control [5-7], fuzzy control [8] [9], Neural Network control [10] [11], nonlinear control [12], inversion system control [13] [14] and the combination of two or more different methods [15]. But those control methods mentioned above are becoming more and more complex, and the design of corresponding controllers is more and more difficult especially for intelligent control methods, and so is the parameter tuning of controller. Therefore the particle swarm optimization [16] and Genetic Algorithm [17] are adapted to simply the design and parameter tuning of controller. Then the control methods are further complicated. Thus, much more time is taken to calculate the output of controller. Those more and more complicated control methods are not suitable for inverted pendulum with hard real-time requirement, not to mention their controlling effect.

In order to guarantee real time, a very simple but valid method is presented to stabilize inverted pendulum. The design of controller is simplified and the parameters need to be adjusted is reduced. Simulation results show that the controlling effect is satisfied. It is more suitable for real-time control of inverted pendulum.

Section 2 presents the model of inverted pendulum. In section 3, the control law is designed. In section 4, the simulation results are given. Section 5 gives a conclusion to the whole paper expecting the next work.

2. MODEL OF INVERTED PENDULUM

A typical inverted pendulum is mainly composed of pendulum and a moving platform driven by one motor. The controlled variables include the deviating angle from plumb line of the pendulum and placement of moving platform. Neglecting friction and air resistance, the typical inverted pendulum system can be

© 2005 - 2013 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

abstracted as Figure 1. For inverted pendulum, the physical parameters include the length and mass of pendulum represented by l and m respectively, and the mass of moving platform represented by M. The placement of platform referenced to the starting point is represented by x, and the deviating angle from plumb line of pendulum by a.

Then model of inverted pendulum will be built according to the energy characteristics of mechanical systems based on Lagrange Equations. First, assume that the pendulum is uniform rigid body. Let ds represent an arbitrarily short part of pendulum, which position referenced to the starting point can be specified by

$$r(x,\alpha) = \begin{bmatrix} x + s\sin\alpha \\ s\cos\alpha \end{bmatrix} \tag{1}$$

Thus, the kinetic energy of ds will be

$$dT = \frac{1}{2} dm \|\dot{r}\|^2$$
$$= \frac{1}{2}\rho ds(\dot{x}^2 + 2s\cos\alpha\dot{\alpha}\dot{x} + s^2\dot{\alpha}^2)$$

Here $dm = ds \cdot \rho$, and ρ represents the ratio between the mass and length of pendulum similar to the density of objects.

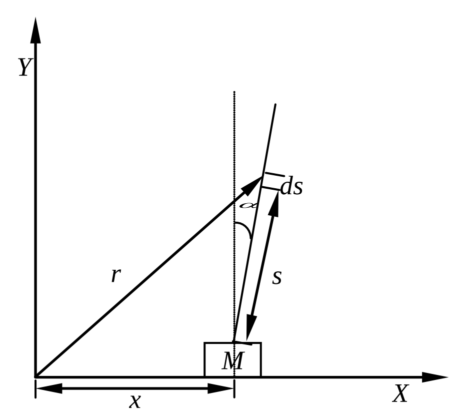

Figure 1. Sketch Map of Typical Inverted Pendulum

Likewise the potential energy of ds is

$$dV = \rho \cdot ds \cdot g \cdot \cos\alpha \, .$$

Then the kinetic energy and potential energy of the whole pendulum are respectively

$$T = \frac{1}{2}M\dot{x}^2 + \int_0^l dT = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}ml\dot{\alpha}\dot{x}\cos\alpha + \frac{1}{6}ml^2\dot{\alpha}^2$$

$$V = \int_0^l dV = \frac{1}{2}mgl\cos a$$

So there is

$$L(q,\dot{q}) = T - V$$
$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml\,\dot{\alpha}\,\dot{x}\cos\alpha + \frac{1}{6}ml^2\dot{\alpha}^2 - \frac{1}{2}mgl\cos\alpha \tag{2}$$

Here L is the Lagrangian and q is the generalized coordinate of system. Let $q = (x,\alpha)^T$ and substitute it in Lagrange Equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \gamma_i \quad i = 1,\ldots,m \, ,$$

Here γ_i is the external force in generalized coordinate. Then there are

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = (M+m)\ddot{x} + \frac{1}{2}ml(\ddot{\alpha}\cos\alpha - \dot{\alpha}^2\sin\alpha)$$
$$\frac{\partial L}{\partial x} = 0$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\alpha}} = \frac{1}{2}ml(\ddot{x}\cos\alpha - \dot{x}\dot{\alpha}\sin\alpha) + \frac{1}{3}ml^2\ddot{\alpha}$$
$$\frac{\partial L}{\partial \alpha} = -\frac{1}{2}ml\dot{\alpha}\dot{x} + \frac{1}{2}mgl\sin\alpha$$

Thus, the kinetic equation of inverted pendulum is derived as

$$(M+m)\ddot{x}\cos\alpha + \frac{1}{2}ml\ddot{\alpha}\cos\alpha - ml\dot{\alpha}^2\sin\alpha = \tau \tag{3}$$
$$m\ddot{x}\cos\alpha + \frac{1}{3}ml^2\ddot{\alpha} - \frac{1}{2}mgl\sin\alpha = 0 \tag{4}$$

Here τ represents the force of motor.

Considering the frictional force, the model of inverted pendulum can be modified as

$$(M+m)\ddot{x}\cos\alpha + \frac{1}{2}ml\ddot{\alpha}\cos\alpha - \frac{1}{2}ml\dot{\alpha}^2\sin\alpha + c_M\dot{x} = \tau \tag{5}$$
$$\frac{1}{2}m\ddot{x}\cos\alpha + \frac{1}{3}ml^2\ddot{\alpha} + c_m\dot{\alpha} - \frac{1}{2}mgl\sin\alpha = 0 \tag{6}$$

Here c_M is sliding friction coefficient of the moving platform in proportion to its speed; c_m is the friction coefficient of pendulum proportionate to its angle acceleration.

In order to make the expression much terser, let

$$\tilde{l} = l/2$$
$$M_0 = M + m$$
$$J_0 = \frac{4}{3}m\tilde{l}^2 \, .$$

And substitute them in equations (5) and (6), then (7) and (8) are obtained as follow:

$$M_0\ddot{x} + m\tilde{l}\,\ddot{\alpha}\cos\alpha - m\tilde{l}\,\dot{\alpha}\sin\alpha + c_M\dot{x} = \tau \tag{7}$$
$$m\tilde{l}\,\ddot{x}\cos\alpha + J_0\ddot{\alpha} + c_m\dot{\alpha} - mgl\sin\alpha = 0 \tag{8}$$

Assume that the pendulum is stabilized nearby the balance position in the course of actual control, and the angle of pendulum equals zero. Considering the essence of inverted pendulum system, equations as follow are tenable: $\cos\alpha = 1$, $\sin\alpha = \alpha$ and $\dot{\alpha} = 0$. Substitute

© 2005 - 2013 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

them in (7) and (8), the linearized model of inverted pendulum system is obtained as (9) and (10).

$$M_0\ddot{x} + m\tilde{l}\ddot{\alpha} + c_M\dot{x} = \tau \tag{9}$$

$$m\tilde{l}\ddot{x} + J_0\ddot{\alpha} + c_m\dot{\alpha} - mg\tilde{l}\alpha = 0 \tag{10}$$

3. DESIGN OF CONTROL LAW

In normal conditions, the pendulum and the moving platform are coupled with ball bearing. Therefore the rolling friction coefficient is relatively small when the pendulum is rotating. Then c_m can be neglected. Equation (10) can be expressed as (11).

$$m\tilde{l}\ddot{x} + J_0\ddot{\alpha} - mg\tilde{l}\alpha = 0 \tag{11}$$

Here $\ddot{x}$ is the acceleration of the moving platform.

Via appropriately transforming, the acceleration can be transformed into force from motor to moving platform. Thus, above equation can be expressed as (12) considering $\ddot{x}$ as the input variable of inverted pendulum system.

$$\ddot{x} = \frac{-J_0\ddot{\alpha} + mg\tilde{l}\alpha}{m\tilde{l}} \tag{12}$$

Then the system can be considered as a simple linear system. If let α_d , $\dot{\alpha}_d$ and $\ddot{\alpha}_d$ are angle, angular velocity, angular acceleration of the pendulum respectively, the feedback control law can be designed according to (12). The expression of control law is (13).

$$\ddot{x} = \frac{-J_0\ddot{\alpha}_d + k_1(\alpha - \alpha_d) + k_2(\dot{\alpha} - \dot{\alpha}_d) + mg\tilde{l}\alpha}{m\tilde{l}} \tag{13}$$

Where k_1 and k_2 are the feedback gains of angle and angular velocity respectively.

Substituted above expression in (12), (14) can be obtained.

$$-J(\ddot{\alpha} - \ddot{\alpha}_d) + k_1(\alpha - \alpha_d) + k_2(\dot{\alpha} - \dot{\alpha}_d) = 0 \tag{14}$$

Let the error between the reference angle and the actual angle is e=α+αd. and substitute it in (14), which can be expressed as follow

$$\frac{d}{dt}\left(\frac{de}{dt}\right) + k_d\frac{de}{dt} + k_p e = 0\,.$$

Then its characteristic equation is $\lambda^2 + k_d\lambda + k_p = 0$. Known from the control theory, that system is asymptotically stable. When the time is long enough, α, the angle of pendulum converges α_d. In the real time control, the control target is to stabilize the pendulum on the vertical position, so that let the given angle is zero. Then (14) is changed as (15).

$$\ddot{x} = a\ddot{\alpha} + k_1\alpha + k_2\dot{\alpha} \tag{15}$$

Here $a = -J/m\tilde{l}$.

According to the control target of inverted pendulum, the influence from displacement of moving platform is secondary. But generally speaking, the movable scope of moving platform is limited, and then let its linear displacement equal zero. Transform (15) to (16), the control law is obtained.

$$\ddot{x} = a\ddot{\alpha} + k_1\alpha + k_2\dot{\alpha} + k_3x + k_4\dot{x} \tag{16}$$

Here k_1, k_2, k_3 and k_4 are feedback gains of corresponding variables respectively.

Obviously, equation (16) is intricate and unachievable. Because there are certain relation between the acceleration of the platform and the output force of the motor, let the output force of the motor as the input of the inverted pendulum. Substitute (10) in (16), (17) is obtained.

$$\tau = M_0 g\ddot{\alpha} - \frac{(M_0J_0 - m^2\tilde{l}^2)\alpha}{m\tilde{l}} + c_M\dot{x} \tag{17}$$

According to the derivative process of (16), the control law of the inverted pendulum shown as (18) is obtained.

$$u_c = k_1\alpha + k_2\dot{\alpha} + k_3x + k_4\dot{x} - k\ddot{\alpha} \tag{18}$$

here $u_c = \tau / a_0$; α_0 is the relation coefficient between the voltage and the output force of the servo motor; k_1, k_2, k_3, k_4 and k are feedback gains of corresponding variables respectively. And $k_1 = \frac{(M_0J_0 - m^2\tilde{l}^2)}{a_0 m\tilde{l}}$, $k_4 = c_M$, $k = \frac{M_0 g}{a_0}$.

Here k_1, and k_4 can be computed from the parameters of the inverted pendulum.

Known from (18), the control law of an inverted pendulum is similar to the PD control law. That control law is easy to be understood and realized. If adopting this method, the structure of controller is very simple. Some of the gains can be calculated from physical parameters of the inverted pendulum system. Then there are few parameters need to be adjusted.

The control method is easy to expand to control a double or triple inverted pendulum. As an example, (19) can be used to stabilize a double inverted pendulum system.

Journal of Theoretical and Applied Information Technology
30th April 2013. Vol. 50 No.3
© 2005 - 2013 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$u_c = k_1\alpha_1 + k_2\dot{\alpha}_1 + k_3\alpha_2 + k_4\dot{\alpha}_2 + k_5 x + k_6\dot{x} - k\ddot{\alpha} \quad (19)$$

If neglect the effect of the acceleration of the angle to the control performance in (14), namely, let $k=0$, that control law is just as same as the human-imitating control law [18]. That article gives only a control law without theoretical basis.

4. SIMULATION

The controller was designed according to the control law shown in (18) for inverted pendulum. The structure of the control system for inverted pendulum is shown in Figure 2. Two sensors were used to measure the placement of the moving platform and deviating angle of pendulum. The physical parameters of the inverted pendulum system are shown in Table1.

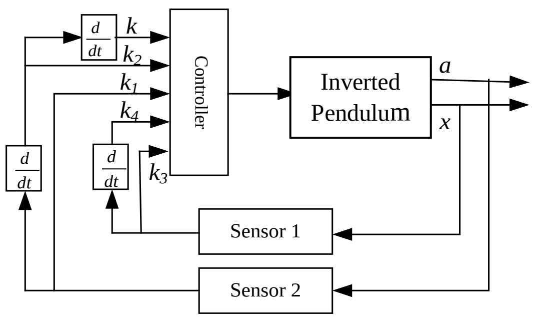

Figure2. Structure of Control System for Inverted Pendulum

Table 1. Physical parameters of inverted pendulum

Parameter	***Value***	***Unit***
l	0.31	m
m	0.192	Kg
a_0	13.16	N/V
c_M	43.94	N/ms^{-1}
M	0.968	Kg
c_m	3.49	$10^{-3}N/ms^{-1}$
J_m	1.64	$10^{-3}Kgm^2$

In order to verify the validity of the presented method in this article, the simulation experiment is made based on the simple control law with MATLAB. In the simulation experiment, parameters of the controller are shown as Table 2. Then, when sampling time interval is adjusted to 20 millisecond seconds, the simulation results are shown in Figure 3 and 4. Figure 3 is the displacement of the moving platform and Figure 4 is the deviating angle from plumb line of pendulum.

Table 2. Feedback gains of controller

Parameter	***Value***
k_1	50
k_2	5
k_3	20
k_4	1
k	5.6

The results of simulation show that the simple control method is feasible and valid to stabilize the inverted pendulum. It is illustrated that the simple control method has a good performance. With this control method, the inverted pendulum system has not only good dynamic feature but also good steady-state performance.

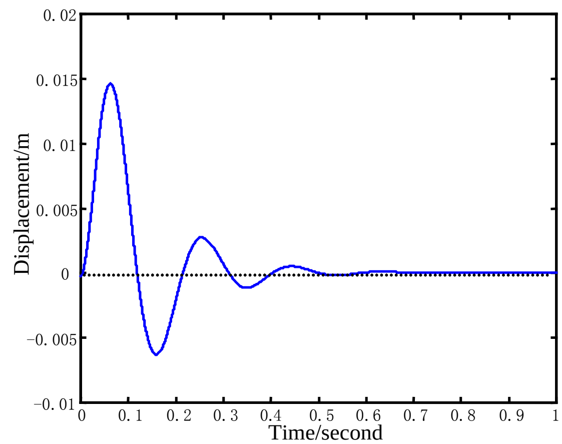

Figure 3. Displacement of the moving platform

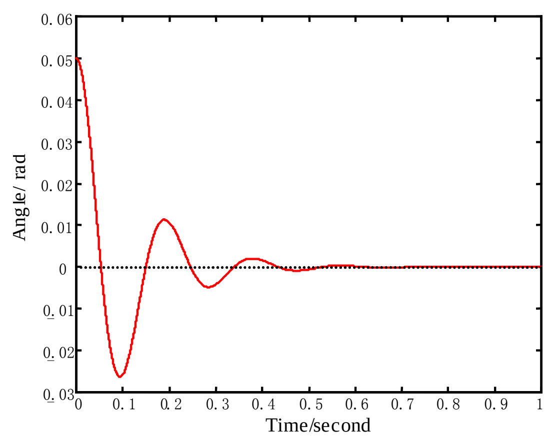

Figure 4. Deviating angle from plumb line of pendulum

5. CONCLUSION

As the control methods of inverted pendulum are coming more and more complex, a simple control strategy is presented to stabilize the inverted pendulum system. Compared with the other control methods, it is much simpler. The design of

© 2005 - 2013 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

controller and the modifying of parameters are thus greatly simplified. The simulation result shows that the control method is feasible and valid. Utilizing the method, the control performance is satisfied. And the real time of control is guaranteed greatly.

The control law in this article is not dependent on the model even though it is derived from the dynamical model of an inverted pendulum system. It is easy to transform the control law to stabilize a double or triple inverted pendulum system. And the method can be adopted to control other multi input single output systems.

The disadvantage of the control method presented in this article is that the dynamic performance is barely satisfactory. So the next work is to improve the response speed of control system.

ACKNOWLEDGMENTS

This work is supported by The Key Technology R&D Program of Shanxi (Granted No. 20120322005-01), which is gratefully appreciated.

REFERENCES

[1] AstrÄom K J, Furuta K, "Swinging Up A Pendulum by Energy Control", *Automatica*, Vol. 36, No.2, pp. 287-295, 2000.

[2] Mason P, Broucke M, Piccoli B. "Time Optimal Swing-up of The Planar Pendulum", *IEEE Transactions on Automatic Control*, Vol. 53, No.8, 1876-1886, 2008.

[3] Srinivasan B, Huguenin P, Bonvin D. "Global Stabilization of An Inverted Pendulum-Control Strategy and Experimental verification", *Automatica*, Vol.45, No.1, pp. 265-269, 2009.

[4] Mori S., Nishihara H., Furuta K, "Control of Unstable Mechanical System Control of Pendulum" , *International Journal of Control*, Taylor and Francis Group, vol. 23, no. 5, pp.673-692,1976.

[5] Sturgeon R., Loscutof M R., "Application of Modal Control and Dynamic Observer to Control an Inverted Pendulum", *Proceeding of JACC*, pp.857-865, 1972.

[6] Anderson M.J, Grantham. W.J., "Lvyapunov Optimal Feedback Control of A nonlinear Inverted Pendulum", *Trans. ASME, The American Society of Mechan*, Vol.111, No.5, pp.554-558, 1989.

[7] Wang J J., "Simulation Studies of Inverted Pendulum Based on PID Controllers", *Simulation Modeling Practice and Theory*, Vol.19, No. 1, pp. 440-449, 2011.

[8] Li Hongxing, Miao Zhihong, Wang Jiayin. "Variable Universe Adaptive Fuzzy Control on The Quadruple Inverted Pendulum", *Science in China (Series E)*, Vol. 32, No. 1, pp.65–75, 2002.

[9] Chen Fuguo, Deng Guannan, Tan Yanhua, "The Design of Improved Variable Universe Fuzzy Controller of Triple Inverted Pendulum", *Journal of Control Theory and Applications*, Vol. 27, N0. 2, pp.233–237, 2010.

[10] Nelson J., Kraft.L G, "Real-time Control of An Inverted Pendulum System Using Complementary Neural-network and Optimal Techniques", *Proceeding of American Control Conference*, pp.2553-2554, 1994.

[11] Sahba M, "Computer-aided Design of Feedback Controller for Nonlinear Systems with Applications to Control of a Double Inverted Pendulum", *IEE Proceeding D*, pp. 350-358, 1983.

[12] Wang Jiajun, Liu Dongliang, Wang Baojun, "Research on One Type of Saturated Nonlinear Stabilization Control Method of X-Z Inverted Pendulum", *Acta Automatica Sinica*, Vol. 38, No. 8, pp.1-5,2012.

[13] GRAICHEN K, TREUER M, ZEITZ M, "Swing-up of The Double Pendulum on A Cart by Feed Forward and Feedback Control with Experimental Validation", *Automatica*, Vol. 43, No. 1, pp. 63–71, 2007.

[14] GRAICHEN K, HAGENMEYER V, ZEITZ M, "A New Approach to Inversion-based Feed Forward Control Design for Nonlinear Systems", *Automatica*, Vol. 41, No. 12, pp. 2033 – 2041, 2005.

[15] C.W. TaoJ. Taur, J.H. Chang, F. Su, "Adaptive Fuzzy Switched Swing-up and Sliding Control for the Double-Pendulum-and-Cart System", *IEEE Transactions on Systems, Man, and Cybernetics (Part B: Cybernetics)*, Vol. 40, No.1, pp. 241-252, 2010.

[16] Shamsudin, M.A., "An Adaptive Control of Two Wheel Inverted Pendulum Robot Based on Particle Swarm Optimization", *Automation, International Conference on Robotics and Applications (ICARA)*, pp. 151 – 156, 2011.

[17] Song Qingkun, Jiang Lei, Hao Min, "Fuzzy Network Controller Based on Genetic Algorithm", *Electric Machines and Control*, vol. 19, no.16, pp. 617-620, 2005.

© 2005 - 2013 JATIT & LLS. All rights reserved·
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[18] Zhang Minglian, Sun Changling, Yang Yawei, "Human-imitating Control for 2-D Inverted Pendulum", *Control and Decision*, vol. 17, No. 1, pp39-44, 2002.