

DISTURBANCE REJECTION FOR A CLASS OF NONLINEAR SYSTEMS WITH INPUT TIME-DELAY*

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ABSTRACT

Disturbance-observer-based control (DOBC) has been considered as a robust control scheme against the various unknown disturbances in many applications. The existing DOBC methods were only applicable for the case where disturbances satisfy time matching condition, i.e., they enter the system in the same time as the control inputs. By appropriately designing an auxiliary observer, the disturbance of future time is predicted after appropriate coordinate transformation, new disturbance estimation and rejection law are designed in this paper. One novel feature of the proposed method is that the merits of DOBC can be inherited for a class of nonlinear systems with input time-delay. By integrating the disturbance observers with conventional robust control laws, the disturbances can be rejected and the desired dynamic performances can be guaranteed. Simulations demonstrate the advantages of the proposed scheme.

Keywords: *Disturbance observer, Time delay, Disturbance rejection, Sinusoidal disturbance*

1. INTRODUCTION

Disturbance observer based control (DOBC) is a prevalent anti-disturbance control strategy, compensation for modeling errors and exogenous disturbance has been considered as a robust control scheme when the errors or disturbance can be measured. DOBC was established in the late of 1980s for linear frequency-domain systems, has a simple structure and is easily implemental in engineering (see surveys [1] and references therein).

In another case, if the priori property of disturbance to be estimated is known, DOBC can be implemented where the disturbance compensation dynamic property within a composite system can be analyzed ([2]-[6]). Originating from [6], a hierarchical control strategy is established in ([13]) aiming at multiple disturbances in multi-input multi-output (MIMO) nonlinear system, the outcome shows that the strategy has high precision together with strong robustness. The literatures mentioned above show that the model-based DOBC is feasible for more complex structure and can avoid heavy computation, such as resolve of partial

differential equations (PDEs) compared with output regulation theory ([8]).

However, one of the limitations of the classical DOBC is that the disturbance and input must satisfy time matching condition. The question arises how the DOBC can be extended to the general case in which the system model has input time-delay. In classic DOBC ([5, 7, 9, 10, 12, 15, 16]), the disturbance is seen as extended state, correspondingly an extended state observer, i.e. disturbance observer (DO) can be constructed to estimate the disturbance. Once the disturbance enters system in advance of control input, no effective disturbance estimation method can be utilized directly to reject the disturbance as the time matching condition ([14]) is not satisfied. It is difficult to can not estimate the disturbance and predict the future disturbance influence simultaneously using present technique. To the best of our knowledge, very few literatures contribute on this problem, and it is still a challenging work. Originated from the analysis discussed above, the aim of this note is to provide a novel approach to estimate and reject the disturbances with input time delay, such that the merits of DOBC can be

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inherited. We first construct an auxiliary observer, and then derive the mapping relationship between the disturbance and compensation controller by appropriate transformation coordination.

The organization of the problem is given below. Section 2 gives the problem formulation. In Section 3, the formulation for the disturbance estimation is proposed for a class of nonlinear systems. In Section 4, the compensation controller is designed in DOBC frame with input time-delay. In Section 5, the proposed method is applied to a nonlinear system, simulations demonstrate the advantages of the proposed scheme. Section 6 provides conclusions.

2. FORMULATION OF THE PROBLEM

The following nonlinear continuous MIMO system with input time delay and disturbances is considered:

$$\dot{x}(t) = Ax(t) + F_1 f_1(x(t), t) + B[u(t - \tau) + d(t)] \quad (1)$$

where $x(t) \in R^n$, $u(t) \in R^m$ ($m \leq n$) is the state, the control input and the measurement output, respectively. $A \in R^{n \times n}$ is the coefficient matrices, $B \in R^{n \times m}$ satisfies $rank(B) = m$, F_1 is the corresponding weighting matrices, $f_1(x(t), t)$ is nonlinear functions, $d(t) = [d_1(t), d_2(t), \dots, d_m(t)]^T$ is vector of sinusoidal disturbance and $\tau < \infty$ is the delay time. Such a model can also represent a wider class of time-delay system compared with paper [5,10,12].

To precede our following discussion, rewrite sinusoidal d_i as

$$\begin{cases} \dot{w}_i = \Gamma_i w_i \\ d_i = V_i w_i \end{cases} \quad (2)$$

where $w_i \in R^2$, and the linear uncertain matrix $\Gamma_i \in R^{2 \times 2}$, (Γ_i, V_i) is uniformly observable. Without loss of generality, (Γ_i, V_i) has observable canonical form, which can be expressed as:

$$\Gamma_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & 0 \end{bmatrix}, V_i = [1 \quad 0], i = 1, 2, \dots, n \quad (3)$$

from (2-3) we can calculate that $w_i = \phi_i(t)w_i^0$ with initial value w_i^0 and

$$\phi_i(t) = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) / \omega \\ -\omega \sin(\omega t) & \cos(\omega t) \end{bmatrix}, \quad (4)$$

Similar to the output regulation theory, each unknown external disturbance d_i is supposed to be generated by an exogenous system, model (2) we define as in form (3) can represents sinusoidal disturbance with unknown phase and magnitude. In application, many kinds of disturbances encountered in engineering can be described by this sinusoidal model, for example, the control of aircraft control [6], magnetic bearing control [10], robotic systems [3], etc.

Assumption 1: For system (1), there exists a stabilizing law $u = u_0$ so that $x = 0$ is a uniform globally asymptotically stable equilibrium of :

$$\dot{x}(t) = Ax(t) + F_1 f_1(x(t), t) + B_0 u_0(t - \tau),$$

Moreover, there is a function $\Omega_0(x) : R_+ \times R^n$ satisfying

$$\begin{aligned} \gamma_1(\|x\|) \leq \Omega_0(x) \leq \gamma_2(\|x\|), \left\| \frac{\partial^T \Omega_0(x)}{\partial x} \right\| \leq \gamma_3 \|x\|, \\ \frac{\partial^T \Omega_0(x)}{\partial x} (Ax(t) + F_1 f_1(x(t), t) + B_0 u_0(t - \tau)) \\ \leq -\gamma_4(\|x\|) \end{aligned} \quad (5)$$

where $\gamma_1(\cdot), \gamma_2(\cdot)$ are of K_∞ -class and $\gamma_4(\cdot)$ of K -class

As ω_i^0 in (2) is unavailable value of disturbance can not to be obtained directly, a conventional method is to identify ω_i^0 with adaptive technique, but the strategy is sensitive to un-modeled dynamic and lead to semi-negative system ([11]). This paper, we will give the relationship between the parameters Γ_i ($i = 1, 2, \dots, m$) and $d(t)$, such that the exogenous disturbance may be compensated by a phase advance controller. The control problem considered will be solved in DOBC frame instead of adaptive control.

3. NONLINEAR DISTURBANCE OBSERVER

In this section, we suppose that $f_1(x(t), t)$ are given and Assumptions 1 hold. When all states of the system are available, it is unnecessary to estimate the states, then the estimation of the disturbance need to be concerned. According to (2), d in (1) can be expressed as follows:

$$\begin{cases} \dot{w} = \Gamma w \\ d = V w \end{cases}, \quad (6)$$

where $w \in R^{2m}$, $\Gamma \in R^{2m \times 2m}$ and

$$\Gamma = \text{diag}\{\Gamma_1, \Gamma_2 \dots \Gamma_m\}, V = \text{diag}\{V_1, V_2 \dots V_m\} \quad (7)$$

Construct an auxiliary MIMO nonlinear system as follows:

$$\begin{cases} \dot{\xi} = G(v + \psi) - \bar{L}\bar{B}f_1(x) - \bar{L}\bar{B}Ax - \bar{L}\bar{B}Bu(t - \tau) \\ \psi = \bar{L}\bar{B}x \end{cases} \quad (8)$$

where

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_m \end{bmatrix}, \xi_i = \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \end{bmatrix}$$

G and L are given constant matrices that satisfy

$$G = \text{diag}\{G_1, G_2 \dots G_m\}, L = \text{diag}\{L_1, L_2 \dots L_m\} \quad (9)$$

where

$$G_i = \begin{bmatrix} 0 & 1 \\ -g_{i1} & -g_{i2} \end{bmatrix}, L_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (10)$$

and G_i is Hurwitz by properly selection of g_{i1} and g_{i2} . Considering $\text{rank}(B) = m$, there exist pseudo inverse \bar{B} such that $\bar{B}B = 1$, so system (8) can be transformed as

$$\dot{\xi} = G\xi + Ld \quad (11)$$

Comparing (2) and (8) with (11) yields

$$\begin{bmatrix} \dot{\xi} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} G & LV \\ 0 & \Gamma \end{bmatrix} \begin{bmatrix} \xi \\ w \end{bmatrix} \quad (12)$$

Lemma 1: For system (8), if $G_i (i = 1, 2 \dots m)$ and $L_i (i = 1, 2 \dots m)$ have form of (10), there exists an invertible matrix Π_i such that

$$\xi_i + \Pi_i w_i = G_i(\xi_i + \Pi_i w_i) \quad (13)$$

Proof: Considering an invertible matrix

$$P_i^{-1} = \begin{bmatrix} I & \Pi_i \\ 0 & I \end{bmatrix}, \quad (14)$$

where

$$\Pi_i = \frac{\begin{bmatrix} -(g_{i1} - \omega_i^2) & g_{i2} \\ -\omega_i^2 g_{i2} & -(g_{i1} - \omega_i^2) \end{bmatrix}}{(g_{i1} - \omega_i^2)^2 + g_{i2}^2 \omega_i^2} \quad (15)$$

it is obvious that Π_i is invertible in global region of ω_i^2 , furthermore it can be derived that

$$\Pi_i^{-1} = \begin{bmatrix} \omega_i^2 - g_{i1} & -g_{i2} \\ \omega_i^2 g_{i2} & -g_{i1} + \omega_i^2 \end{bmatrix} \quad (16)$$

According to (12), notice that

$$\begin{bmatrix} \dot{\xi}_i \\ \dot{w}_i \end{bmatrix} = \begin{bmatrix} G_i & V_i L_i \\ 0 & \Gamma_i \end{bmatrix} \begin{bmatrix} \xi_i \\ w_i \end{bmatrix}$$

Then define the following coordinate transformation

$$\begin{bmatrix} \bar{\xi}_i \\ \bar{w}_i \end{bmatrix} = P_i^{-1} \begin{bmatrix} \xi_i \\ w_i \end{bmatrix} \quad (17)$$

Combining (13) with (15) it is clear that

$$\begin{bmatrix} \dot{\bar{\xi}}_i \\ \dot{\bar{w}}_i \end{bmatrix} = P_i^{-1} \begin{bmatrix} G_i & V_i L_i \\ 0 & \Gamma_i \end{bmatrix} P_i \begin{bmatrix} \bar{\xi}_i \\ \bar{w}_i \end{bmatrix} \quad (18)$$

After calculation it can be verified that

$$P_i^{-1} \begin{bmatrix} G_i & V_i L_i \\ 0 & \Gamma_i \end{bmatrix} P_i = \begin{bmatrix} G_i & 0 \\ 0 & \Gamma_i \end{bmatrix}$$

and

$$\begin{bmatrix} \dot{\bar{\xi}}_i \\ \dot{\bar{w}}_i \end{bmatrix} = \begin{bmatrix} G_i & 0 \\ 0 & \Gamma_i \end{bmatrix} \begin{bmatrix} \bar{\xi}_i \\ \bar{w}_i \end{bmatrix} \quad (19)$$

Finally (13) can be got directly following (18) and (19).

Q.E.D

Based on Lammal we can give another form of d_i as

$$d_i(t) = V_i w_i(t) = -\bar{V}_i \xi_i(t) + \bar{V}_i \bar{\xi}_i(t) \quad (20)$$

where

$$w_i(t) = -\Pi_i^{-1} \xi_i(t) + \Pi_i^{-1} \bar{\xi}_i(t), \bar{V}_i = V_i \Pi_i^{-1} \quad (21)$$

with

$$\bar{\xi}_i(t) = \xi_i(t) + \Pi_i w_i(t) \quad (22)$$

and satisfies

$$\dot{\bar{\xi}}_i = G_i \bar{\xi}_i \quad (23)$$

Up to now, we have given the another form of d_i by constructing auxiliary observer ξ , whereas (20) can not be implanted to reject the disturbance directly for reason that there exist time delay in control input, this is a major hurdle impede the further research and application in DOBC and other disturbance rejection research.

The following will introduce a novel technique to estimate the phase advance of disturbance.

From (13) we can predict that

$$\begin{aligned} \xi(t + \tau) &= -\Pi w(t + \tau) + \xi(t + \tau) + \Pi w(t + \tau) \\ &= -\Pi \phi(t + \tau) w_0 + \xi(t + \tau) + \Pi w(t + \tau) \\ &= -\Pi \psi(\omega, \tau) \phi(t) w_0 + (\xi + \Pi w)(t + \tau) \\ &= -\Pi \psi(\omega, \tau) w + \xi(t + \tau) + \Pi w(t + \tau) \end{aligned} \quad (24)$$

where

$$\begin{aligned} \Pi &= \text{diag}\{\Pi_1, \Pi_2, \dots, \Pi_m\}, \\ \psi(\omega, \tau) &= \text{diag}\{\psi(\omega_1, \tau), \psi(\omega_2, \tau), \dots, \psi(\omega_m, \tau)\} \\ \phi(t) &= \text{diag}\{\phi_1(t), \phi_2(t), \dots, \phi_m(t)\} \\ \psi(\omega, \tau) &= \begin{bmatrix} \cos(\omega\tau) & \sin(\omega\tau)/\omega \\ -\omega \sin(\omega\tau) & \cos(\omega\tau) \end{bmatrix} \end{aligned} \quad (25)$$

Following (24) we can get

$$\begin{aligned} \xi_i(t+\tau) &= -\Pi_i \psi(\omega, \tau) (-\Pi_i^{-1} \xi_i(t) + \Pi_i^{-1} (\xi_i(t) \\ &+ \Pi w_i(t))) + \xi_i(t+\tau) + \Pi_i w_i(t+\tau) \end{aligned} \quad (26)$$

Combining with (20) shows that

$$\begin{aligned} d_i(t+\tau) &= -V_i \Pi_i^{-1} \xi(t+\tau) + \bar{V}_i \bar{\xi}_i(t+\tau) \\ &= -V_i \psi_i(\omega, \tau) \Pi_i^{-1} \xi(t) + V_i \psi_i(\omega, \tau) \Pi_i^{-1} \bar{\xi}(t) \\ &+ 2\bar{V}_i \bar{\xi}_i(t+\tau) \\ &= -V_i \psi_i(\omega, \tau) \Pi_i^{-1} \xi_i(t) + \bar{V}_i \delta_i(t+\tau) \end{aligned} \quad (27)$$

where

$$\bar{V}_i = \begin{bmatrix} V_i \psi(\omega_i, \tau) \Pi_i^{-1} & 2\bar{V}_i \end{bmatrix} \delta_i(t+\tau) = \begin{bmatrix} \bar{\xi}_i(t) \\ \bar{\xi}_i(t+\tau) \end{bmatrix}$$

A notable property of (27) is that $d_i(t+\tau)$ can be expressed by auxiliary vector $\xi_i(t)$ and a decay vector $\delta_i(t+\tau)$. So, we can predict the future disturbance at time $t+\tau$ as:

$$\hat{d}_i(t+\tau) = -V_i \psi_i(\omega, \tau) \Pi_i^{-1} \xi_i(t) \quad (28)$$

Thus the proposed method will exhibit classic DOBC property.

4. NONLINEAR DISTURBANCE OBSERVER

The structure of the DOBC controller is formulated as by using of the separation principle.

$$u(t) = -\hat{d}(t+\tau) + u_0 \quad (29)$$

where

$$\hat{d}(t+\tau) = [\hat{d}_1(t+\tau) \quad \hat{d}_2(t+\tau) \quad \dots \quad \hat{d}_m(t+\tau)]^T$$

Thus, the dynamic system (1) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + F_1 f_1(x(t), t) \\ &+ B[u_0(t-\tau) + d(t) - \hat{d}(t)] \end{aligned} \quad (30)$$

Theorem 2 For the auxiliary system (8), if (10) is satisfied and under assumption 1 then the close-loop system (30) is asymptotically stable under DOBC law (29).

Proof:

Based on (27) it can be shown that

$$\begin{aligned} d(t+\tau) &= -V \Pi^{-1} \xi(t+\tau) + \bar{V} \bar{\xi}(t+\tau) \\ &= -V \psi(\omega, \tau) \Pi^{-1} \xi(t) + V \psi(\omega, \tau) \Pi^{-1} \bar{\xi}(t) \\ &+ 2\bar{V} \bar{\xi}(t+\tau) \\ &= -V \psi(\omega, \tau) \Pi^{-1} \xi(t) + \bar{V} \delta(t+\tau) \end{aligned} \quad (31)$$

where

$$\begin{aligned} V &= \text{diag}\{V_1, V_2, \dots, V_n\}, \\ \bar{V} &= \text{diag}\{V_1 \Pi_1^{-1}, V_2 \Pi_2^{-1}, \dots, V_n \Pi_n^{-1}\} \\ \bar{V} &= \begin{bmatrix} V \psi(\omega, \tau) \Pi^{-1} & 2\bar{V} \end{bmatrix} \delta(t+\tau) = \begin{bmatrix} \bar{\xi}(t) \\ \bar{\xi}(t+\tau) \end{bmatrix} \end{aligned} \quad (32)$$

Compared with (28), the estimation error satisfies

$$d(t) - \hat{d}(t) = \bar{V} \delta(t) \quad (33)$$

and

$$\dot{\delta}(t) = \bar{G} \delta(t)$$

where

$$\bar{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix} \quad (30)$$

Notice that \bar{G} also satisfy Hurwitz condition according to (10) and (30), We can find $P > 0$ such that

$$GP + PG = -I \quad (31)$$

Define a Lyapunov function candidate as

$$\Omega(t) = \Omega_0(x, t) + \gamma \delta^T(t) P \delta(t)$$

Take the derivative of $\Omega(t)$ along the composite closed-loop system leads to

$$\begin{aligned} \dot{\Omega}(t) &= \frac{\partial^T \Omega_0(x)}{\partial x} (Ax(t) + F_1 f_1(x(t), t) + B_0 u_0(t-\tau)) \\ &+ \frac{\partial^T \Omega_0(x)}{\partial x} \bar{V} \delta(t) + \gamma \delta^T(t) (\bar{G}^T P + P \bar{G}) \delta(t) \\ &\leq -\gamma_4 (\|x\|) + \gamma_3 \left\| \bar{V} \right\| \|x\| \|\delta(t)\| - \gamma \|\delta(t)\|^2 \end{aligned}$$

Based on Assumption 1, one can find corresponding constant $\gamma_5 > 0$ such that

$$\dot{\Omega} \leq -\gamma_5 \begin{bmatrix} x \\ \delta(t) \end{bmatrix},$$

where γ_5 is also of k -class with respect to x and $\delta(t)$.

Thus the proof is completed.

Q.E.D

It is noticed that as G can be chosen arbitrarily, we can determine the value of P as desired, which influences the convergence property of $\delta(t)$ according to (23). The disturbance observer based control design procedure can be summarized as

follows:

Step 1) Select weighting matrices G and L with form of (9) and (10), apply G and L into (8) to calculate auxiliary vector ξ .

Step 2) According to (28), give another form of disturbance represented by auxiliary vector.

Step 3) Apply $\hat{d}(t+\tau)$ and feedback controller u_0 into (1), DOBC control can be realized.

5. SIMULATION

To show the efficiency of the proposed scheme, Let us consider the system (1) with coefficient matrices

$$A = \begin{bmatrix} -1.0 & 0.5 \\ 0 & -2.0 \end{bmatrix}, B = \begin{bmatrix} -1.5 \\ 2.0 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.2 \end{bmatrix}.$$

Nonlinear function $f_1(x(t), t) = x(t) \cos(t)$, suppose exogenous disturbance $d = \sin(t)$ and delay time $\tau = 2$. In assumption 1, we choose $u_0 = Kx$, and $K = [-0.2213 \quad 0.2506]$ in simulation.

Set

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

As input with time-delay, the disturbance can not be rejected at real-time. We need to predict the future disturbance which is thought as a stuff task to be completed. Fig.1-2 demonstrates the system performance using the proposed DOBC schemes considering input time-delay, and obviously show that the disturbance estimation error at time $t+\tau$ and system output converges to zero with sufficiently small steady error. The results show that although there exists time-delay in the system, the disturbance rejection performance is enhanced and satisfaction system responses can be achieved. Fig.3 plots the estimation error of the sinusoidal disturbances with traditional DOBC method. The results show that if system input suffers time-delay, it may bring larger disturbance estimation error and the control performance is deteriorated (Fig.4).

One can observe that to achieve a good tracking the actual sinusoidal disturbance, need not estimate their true values $w_i (i = 1, 2, \dots, m)$. By appropriately coordinate transformation, the disturbance can be approached by a function of auxiliary observer. Furthermore, when suffers system input time-delay,

the future disturbance can be predicted by this type of observer.

6. CONCLUSION

It has been well known that DOBC are effective

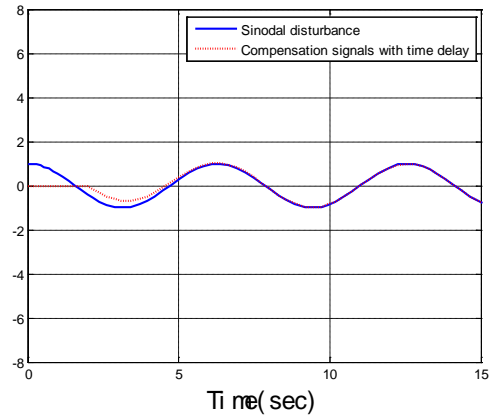


Fig. 1: Disturbance tracking property using proposed auxiliary observer

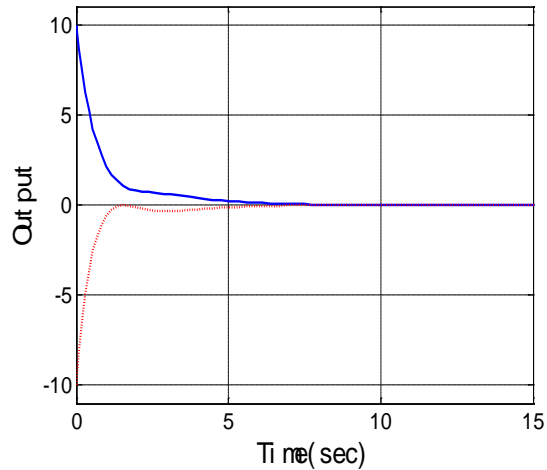


Fig. 2 : Output using proposed DOBC

robust control schemes against the external disturbances and the modelling perturbations. Whereas how to reject the disturbance with input time-delay remains to be resolved up to now. A class of MIMO continue systems with nonlinearity and input time-delay are considered in this paper, the sinusoidal disturbance is described by an exogenous system. An auxiliary observer is structured for the estimation of the disturbance and predicts its future influence simultaneously. Simulation examples are given to show the effectiveness of the proposed results compared with the pure DOBC method.

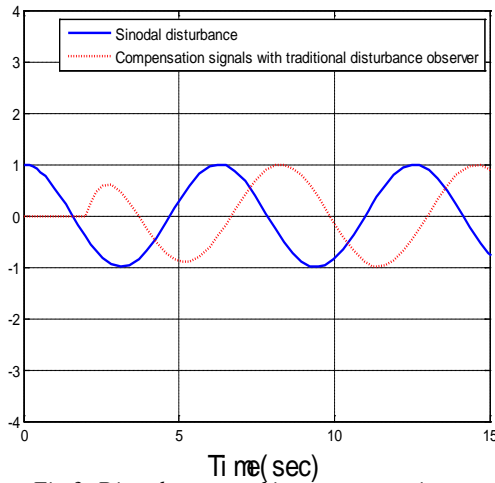


Fig.3: Disturbance tracking property using traditional observer

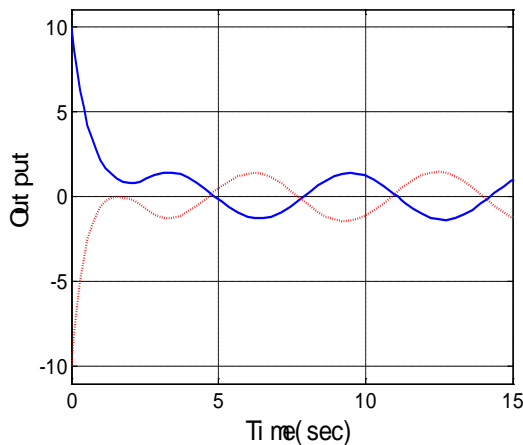


Fig. 4 : Output using traditional DOBC

In this paper, we utilize the frequencies of the sinusoidal disturbance. With the proposed framework we can extend the problem to the case that frequencies is unknown, which remains our further research.

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