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A HYBRID GLOBAL NUMERICAL OPTIMIZATION WITH COMBINATION OF EXTREMAL OPTIMIZATION AND SEQUENTIAL QUADRATIC PROGRAMMING

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ABSTRACT

In recent years, many efforts have focused on cooperative (or hybrid) optimization approaches for their robustness and efficiency to solve decision and optimization problems. This paper proposes a novel hybrid solution with the integration of bio-inspired computational intelligence extremal optimization (EO) and deterministic sequential quadratic programming (SQP) for numerical optimization, which combines the unique features of self-organized criticality (SOC), non-equilibrium dynamics and global search capability in EO with local search efficiency of SQP. The performance of proposed EO-SQP algorithm is tested on twelve benchmark numerical optimization problems and compared with some other state-of-the-art approaches. The experimental results show the EO-SQP method is capable of finding the optimal or near optimal solutions for nonlinear programming problems effectively and efficiently.

Keywords: *Extremal optimization (EO), Sequential quadratic programming (SQP), Memetic algorithms (MA), Nonlinear programming (NLP), Numerical optimization*

1. INTRODUCTION

With the high demand in decision and optimization for many real-world problems and the progress in computer science, the research on novel global optimization solutions has been a challenge to academic and industrial societies. During past few decades, various optimization techniques have been intensively studied; those techniques follow different approaches and can be divided roughly into three main categories, namely, the deterministic methods [1], stochastic methods [2] and bio-inspired computational intelligence [3].

In general, most global optimization problems are intractable, especially when the optimization problem has complex landscape and the feasible region is concave and covers a very small part of the whole search space [4]. Solution accuracy and global convergence are two important factors in the development of optimization techniques. Deterministic search methods are known to be very efficient with high accuracy. Unfortunately, they are easily trapped in local minima [5]. On the other hand, the methods of computational intelligence [3] are much more effective for traversing these complex surfaces and inherently better suited for avoiding local minima. However, computational intelligence has its weakness in slow convergence and providing a precise enough solution because of the failure to exploit local information [6]. Moreover, for constrained optimization problems involving a number of constraints with which the optimal solution must satisfy, computational intelligence methods often lack an explicit mechanism to bias the search in feasible regions [4][7][8].

During the last decades, a particular class of global-local search hybrids named "memetic algorithms" (MAs) are proposed [9], which are motivated by Richard Dawkins's theory [10]. MAs are a class of stochastic heuristics for global optimization which combine the global search nature of EA with local search to improve individual solution [11]. They have been successfully applied to hundreds of real-world problems such as optimization of combinatorial optimization [12], multi-objective optimization [13], bioinformatics [14], etc.

This paper proposed a novel hybrid EO-SQP method with the combination of recently proposed

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extremal optimization (EO) and the popular deterministic sequential quadratic programming (SQP) under the conceptual umbrella of MA. EO is a general-purpose heuristic algorithm, with the superior features of self-organized criticality (SOC), non-equilibrium dynamics, co-evolutions in statistical mechanics and ecosystems respectively [15] [16]. SQP has been one of the most popular methods for nonlinear optimization because of its efficiency of solving medium and small size nonlinear programming problems [1][17]. It guarantees local optima as it follows a gradient search direction from the starting point towards the optimum point and has special advantages in dealing with various constraints [18]. This will be particularly helpful for the hybrid EO-SQP algorithm when solving constrained optimization problems: the SQP can also serve as a means of "repairing" infeasible solutions during EO evolution. The proposed method balances both aspects through the hybridization of heuristic EO as the global search scheme and deterministic SQP as the local search scheme.

The rest of this paper is organized as follows: In section 2, the nonlinear optimization problem with/without constraints is described in a general formulation. Section 3 presents the EO-SQP fundamental and algorithm in detail. In section 4, the proposed approach is used to solve twelve benchmark test functions, and the results are quantitatively compared with genetic algorithm (GA), particle swarm optimization (PSO), SQP and some other popular methods such as Stochastic Ranking (SR) [8], Simple Multimembered Evolution Strategy (SMES) [19] and Auxiliary Function Method (AFM) [20]. Finally, the concluding remarks are addressed in Section 5.

2. PROBLEM FORMULATION

Many real-world optimization problems can be mathematically modeled in terms of a desired objective function subject to a set of constraints as follows:

$$\text{Minimize } f(X), \quad X = [x_1, x_2, ..., x_n] \tag{1}$$

subject to

$$g_t(X) \le 0; \quad t = 1,2,..., p \tag{2}$$

$$h_u(X) = 0; \quad u = 1,2,..., q \tag{3}$$

$$\underline{x}_v \le x_v \le \overline{x}_v; \quad v = 1,2,..., n \tag{4}$$

where $X \in R^n$ is an n-dimensional vector representing the solution of the problem (1) - (4), $f(X)$ is the objective function, which needs to satisfy p-inequality constraints $g_t(X)$, q-equality constraints $h_u(X)$, $\underline{x}_v$ and $\overline{x}_v$ are the lower and upper bounds of the variable x_v.

The above formulation is an instance of the well-known nonlinear programming (NLP) problem. In general, the global optimization of NLP is one of the toughest NP-hard problems. Solving this type of problems has become a challenge to computer science and operations research.

3. EO-SQP OPTIMIZATION INSPIRED BY MEMETIC ALGORITHM

3.1 Extremal Optimization

The Extremal Optimization (EO) proposed by Boettcher and Percus [15][16] is derived from the fundamentals of statistical physics and self-organized criticality (SOC) [21] based on Bak-Sneppen (BS) model [22] which simulates far-from equilibrium dynamics in statistical physics and co-evolution[23]. Generally speaking, EO is particularly applicable in dealing with large complex problems with rough landscape, phase transitions passing "easy-hard-easy" boundaries or multiple local optima. It is less likely to be trapped in local minima than traditional gradient-based search algorithms. The research results by Chen and Lu show EO and its derivatives can be effectively applied in solving multi-objective combinatorial hard benchmarks and real-world optimization problems [24] [25] [26] [27].

3.2 Sequential quadratic programming (SQP)

After its initial proposal by Wilson in 1963 [28], the sequential quadratic programming (SQP) method was popularized in the 1970's by Han [29] and Powell [30]. SQP proves itself as the most successful method and outperforms other nonlinear programming methods in terms of efficiency and accuracy to solve nonlinear optimization problems. The solution procedure is on the basis of formulating and solving a quadratic sub-problem with iterative search.

3.3 MA based Hybrid EO-SQP algorithm

As mentioned above, conventional optimization techniques based on deterministic rules often fail or get trapped in local optimum when solving complex problems. In contrast to deterministic optimization techniques, many computational intelligence based optimization methods are good at global search, but relatively poor in fine-tuned local search when the solutions approach to a local region near the global optimum. According to so-called "No-Free-Lunch"

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Theorem by Wolpert and Macready [31], a search algorithm strictly performs in accordance with the amount and quality of the problem knowledge they incorporate. This fact clearly underpins the exploitation of problem knowledge intrinsic to MAs [32]. Under the framework of MAs, the stochastic global search heuristics work together with problem-specific solvers, in which Neo-Darwinian's natural evolution principles are combined with Dawkins' concept of a meme [10] defined as a unit of cultural evolution that is capable of performing individual learning (local refinement). The global character of the search is given by the evolutionary nature of computational intelligence approaches while the local search is usually performed by means of constructive methods, intelligent local search heuristics or other search techniques [11]. The hybrid algorithms can combine the global explorative power of computational intelligence methods with the local exploitation behavior of conventional optimization techniques, complement their individual weak points, and thus outperform either one used alone.

In this study, a MA based hybrid EO-SQP algorithm is developed and applied to nonlinear programming problems. The proposed algorithm is a hybridization of EO and SQP. We intend to make use of the capacity of both algorithms: the ability of EO to find a solution close to the global optimum and effectively dealing with phase transition; the ability of SQP to fine-tune a solution quickly by means of local search and repair infeasible solutions. To implement EO-SQP optimization, the following practical issues need to be addressed.

3.4 Fitness function definition

The fitness function measures how fit an individual (i.e., solution) is, and the "fittest" one has more chance to be inherited into the next generation. A "global fitness" must be defined to evaluate a solution in the proposed EO-SQP algorithm. To solve the NLP optimization problems, the global fitness is defined as the object function value in Eq. (1) for unconstrained benchmark problems:

$$Fitness_{global}(S) = f(S) \qquad (5)$$

For constrained NLP optimization problems, a popular penalization strategy is used in EO-SQP evolution in order to transform the constrained problem to unconstrained one. If a solution is infeasible, its fitness value is penalized according to the violations of constraints defined in Eq. (2) and Eq. (3):

$$Fitness_{global}(S) = f(S) + \underset{\prod g_i, \prod h_u}{Penalty}(S) \qquad (6)$$

Unlike GA, which works with a population of candidate solutions, EO depends on a single individual (i.e. chromosome) based evolution. Through always performing mutation on the worst component and its neighbors successively, the individual in EO can evolve itself toward the global optimal solution generation by generation. This requires a suitable representation which permits each component to be assigned with a quality measure (i.e. fitness) called "local fitness". In this paper, the local fitness λ_k is defined as an improvement in global fitness $Fitness_{global}$ made by the mutation imposed on the *k*th component of best-so-far chromosome S:

$$\begin{aligned}\lambda_k &= Fitness_{local}(k) = \Delta Fitness_{global}(k) \\ &= Fitness_{global}(S) - Fitness_{global}(S'_k)\end{aligned} \qquad (7)$$

3.5 Termination criteria

Termination criteria are used for the detection of an appropriate time to stop the optimization run. In this paper, the termination criteria are designed based on two widely used rules. If the predefined maximum generation is exceeded; or an error measure in dependence on the known optimum is satisfied (we can assume that the algorithm has managed to discover the global minimum), the algorithm should terminate. Denote S_{best} as the best-so-far solution found by the algorithms and S^* as the optimum solution of the functions. The search is considered successful, or in other words, the near-optimal solution is found, if S_{best} satisfies that $|(F^* - F)/F^*| < 1e-3$ (for the case optimum value $F^* \neq 0$) or $|(F^* - F)| < 1e-3$ (for the case optimum value $F^* = 0$). These criteria are perfectly suitable for comparing the performance of different algorithms.

3.6 Workflow and Algorithm

The hybrid algorithm proposed in this study combines EO and SQP method. The structure of the hybrid EO-SQP is based on the standard EO with which the characteristic of gradient search is added by propagating individual solution with SQP algorithm during the EO evolution. In this section, we illustrate the workflow of the EO-SQP algorithm and introduce three mutation operators adopted in this paper: the standard EO mutation, SQP mutation and Multi-start Gaussian mutation; To utilize the advantages of each mutation operator, one or more phases of local search (mutation

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operator) are applied to the best-so-far solution S based on a probability parameter p_m in each generation. In contrast to the standard EO mutation, when SQP mutation or Multi-start Gaussian mutation is adopted, we use the "GEO_{var}" [33] strategy to evolve the current solution by improving all variables simultaneously, as an attempt to speed up the process of local search. The flowchart of the proposed EO-SQP algorithm is shown in Figure. 1.

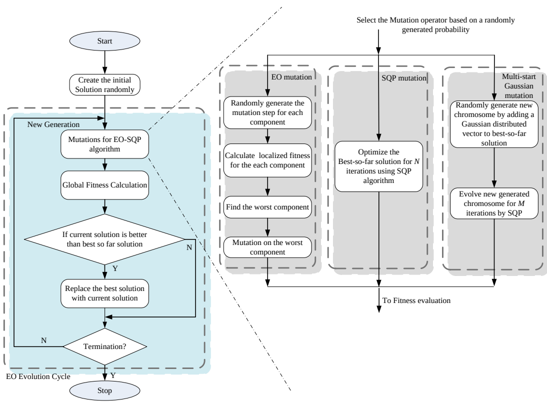

Figure. 1 Flowchart of the EO-SQP algorithm

4. EXPERIMENTAL TEST ON BENCHMARK FUNCTIONS

In this section, twelve widely used NLP benchmark problems are introduced, which have been already extensively discussed in many published literatures. These benchmark functions make it possible to study the proposed EO-SQP algorithm in comparison with other state-of-the-art methods and some well-known results published recently.

4.1 Unconstrained problems

First, the performance of the proposed algorithm is tested on six well-known unconstrained problems with the detailed description in appendix. For close comparison of solution accuracy, the Runtime, success rate, best, average, worst and standard deviation values obtained from total 10 independent runs of proposed EO-SQP, standard GA, standard PSO and standard SQP on the six test functions are presented in Table 1. The best results among the four approaches are shown in bold. "Success" represents the success rate (percentage of success to discover the global minimum), and "Runtime" is the average runtime when the algorithm stops according to the termination criteria defined in section 3.5. In our experiments, the population size of GA and PSO are set to 100 and 50, respectively.

As shown in Table 1, the EO-SQP algorithm proposed in this study is able to find the global optima consistently for all six unconstrained benchmark functions. The GA, PSO, SQP have a very low successful rate for most benchmark problems (Michalewicz, Schwefel, Rastrigin and Rosenbrock functions). In contrast, the proposed EO-SQP algorithm achieved a success rate of 100% for all the six problems. Moreover, EO-SQP is a quite efficient method; the computational time is significantly reduced in comparison with GA and

Table 1. Comparison results for six benchmark unconstrained functions

PROBLEMS (*OPTIMUM)	ALGORITHM	RUNTIME (S)	SUCCESS (%)	WORST	MEAN	BEST	STD. DEV
MICHALEWICZ (-9.66)	**EO-SQP**	25.6757	**100**	**-9.66**	**-9.66**	**-9.66**	**1.39E-07**
	GA	179.2503	0	-8.7817	-9.2020	-9.4489	0.1989
	PSO	93.7919	0	-8.6083	-9.0733	-9.4796	0.2264
	SQP	0.1497	0	-3.0995	-4.7615	-6.4559	1.0656
SCHWEFEL (-12569.5)	**EO-SQP**	0.8257	**100**	**-1.2569E+04**	**-1.2569E+04**	**-1.2569E+04**	**6.2674E-08**
	GA	1233.5	0	-1.014E+04	-1.0719E+04	-1.1227E+04	321.5311
	PSO	218.7393	0	-7.150E+03	-8.8250E+03	-1.0037E+04	832.7256
	SQP	0.1046	0	-5.5547E+03	-6.8011E+03	-7.7497E+03	720.3097
GRIEWANK (0)	**EO-SQP**	0.0962	**100**	**0**	**0**	**0**	**0**
	GA	64.1398	80	0.1346	0.0172	7.8931E-04	0.0422
	PSO	102.9561	0	5.1443	0.7026	0.0074	1.5786
	SQP	0.0167	**100**	**0**	**0**	**0**	**0**
RASTRIGIN (0)	**EO-SQP**	13.2099	**100**	**0**	**0**	**0**	**0**
	GA	137.1764	30	2.9849	0.9999	7.7225E-004	1.1942
	PSO	102.3654	0	109.8554	70.4078	44.4531	22.6459
	SQP	0.7289	0	267.6421	188.2451	92.5310	62.4574
ACKLEY (0)	**EO-SQP**	3.8563	**100**	**0**	**0**	**0**	**0**
	GA	16.8079	90	1.5017	0.1511	9.3102E-04	0.4619
	PSO	103.9701	0	11.7419	7.0320	3.8700	2.7755
	SQP	0.0404	0	19.8725	19.5720	19.1787	0.2086
ROSENBROCK (0)	**EO-SQP**	2.6241	**100**	**4.3065E-04**	**4.3200E-05**	**1.5547E-08**	**1.3614E-04**
	GA	1195.6	20	22.1819	3.5821	9.9942E-04	6.6396
	PSO	94.3400	0	853.3601	150.7588	27.1222	242.7105
	SQP	1.6760	50	3.9866	1.9933	1.1961E-007	2.1011

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PSO. Although the deterministic SQP is the fastest method among the four, it is easily trapped in local minima as shown in simulation results (Michalewicz, Schwefel, Rastrigin, Ackley and Rosenbrock functions). The proposed EO-SQP method can successfully prevent solutions from falling into the deep local minimal which is far from the global optimum, reduce evolution process significantly with efficiency, and converge to the global optimum or its close vicinity.

4.2 Constrained problems

In this study, we selected six (g04, g05, g07, g09, g10 and g12) out of thirteen benchmark functions published in [8][19][20] as constrained test problems, since the characteristics of those functions contain the "difficulties" in having global optimization problems by using an evolutionary algorithm. Table 2 shows the performance comparisons among Stochastic Ranking (SR) [8], Simple Multimembered Evolution Strategy (SMES) [19], Auxiliary Function Method (AFM) [20] and proposed hybrid EO-SQP. The best results among the four approaches are shown in bold.

Among these four methods (see Table 2), the EO-SQP appears to be more promising. It provided three better "best" results (*g05*, *g07* and *g10*) among six functions, and two similar "best" results (*g04* and *g12*). Moreover, the EO-SQP provided better "mean" results for three problems (*g05*, *g07* and *g10*), and similar "mean" results in other two (*g04* and *g12*). Finally, the EO-SQP obtained better "worst" results in two problems (*g05* and *g07*), and it reached similar "worst" solutions in other two problems (*g04* and *g12*). The proposed EO-SQP can produce the better, if not optimal, solutions for most of the six benchmark problems with the exception of test function *g09*. With respect to test function *g09*, although the EO-SQP fails to provide superior results, the performance of the four methods are very close actually. Generally, constrained optimization problems with equality constraints are very difficult to solve. It should be noted that for the three test functions with equality constraints (*g05*, *g07*, and *g10*), EO-SQP can provide better performance than other three methods, the optimum solutions are found by EO-SQP for all the three problems with equality constraints; while the SR, SMES and AFM fail to find the global optimums. This is due to the hybrid mechanism that the EO-SQP can benefit from the strong capability of SQP to deal with constraints during the EO evolution.

4.3 Dynamics analysis of the hybrid EO-SQP

The convergence and the dynamics during the optimization of EO and its derivatives remain up to now challenging open problems. In this section, we use a typical optimization run of Ackley function ($F^* = 0$) as an example to analyze evolution dynamics of the proposed EO-SQP and show the mechanism strength of proposed algorithm.

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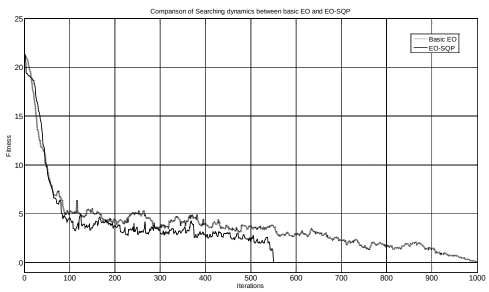

Figure. 2 Comparison of current solution fitness generation by generation between EO and EO-SQP on Ackley Function

Figure. 2 shows the search dynamics of basic EO and the hybrid EO-SQP, which demonstrates the fitness evolution of the current solution in a typical run on Ackley function. The figure shows that both basic EO and hybrid EO-SQP descend sufficiently fast to a near-optimal solution with enough fluctuations to escape from local optima and explore new regions of configuration space. It can be seen quite clearly that with the help of SQP, the hybrid EO-SQP can execute a more deeply search in comparison with basic EO. The search efficiency of basic EO can be improved significantly by incorporating local search method SQP into the evolution, when EO-SQP converges to the close vicinity of the global optimum, the SQP mutation will help to find the global optimum point in just a few runs, as shown in Figure. 2.

Evolutions of best solution fitness as a function of time for EO-SQP, GA, PSO and SQP on Ackley function are also shown in Figure. 3. The convergence rate of the proposed EO-SQP algorithm is a little slower than GA and PSO at the early stage, due to better solution diversity of population based methods (GA and PSO); however, when the solution approach to a near region of the global optimum, the convergence rate of EO-SQP increases rapidly and reaches the global minimum very fast due to the efficiency of gradient based SQP local search. On the other hand, the conventional SQP converges to a local minimal far from the global optimum with high efficiency and can't escape from it during the rest runs due to the weakness of gradient search.

As a general remark on the comparisons above, EO-SQP shows better performance with respect to state-of-the-art approaches in terms of the quality, the robustness, and the efficiency of search. The results show that the proposed EO-SQP finds optimal or near-optimal solutions quickly, and has more statistical soundness and faster convergence rate than the compared algorithms. It should be noted that the factors contributing to the performance of the proposed EO-SQP method are the global search capability of EO and the capability of the gradient-based SQP method to search local optimum efficiently with high accuracy and deal with various constraints.

Table 2. Comparison results for six benchmark functions with constraints

Function and optimum	Statistical features	SR[8]	SMES[19]	AFM[20]	EO-SQP
		Approaches for constrained optimization			
g04	Best	**-30665.539**	**-30665.539**	-30665.50	**-30665.539**
	Mean	**-30665.539**	**-30665.539**	-30665.32	**-30665.539**
-30665.539	Worst	**-30665.539**	**-30665.539**	-30665.23	**-30665.539**
	Std. Dev	2.0e-05	**0**	0.063547	4.4238e-07
g05	Best	*5126.497	5126.599	5126.5	**5126.498**
	Mean	5128.881	5174.492	5126.65	**5126.498**
5126.498	Worst	5142.472	5304.167	5126.96	**5126.498**
	Std. Dev	3.5	5.006e+01	0.145896	**7.4260e-13**
g07	Best	24.307	24.327	24.30694	**24.306**
	Mean	24.374	24.475	24.30789	**24.306**
24.306	Worst	24.642	24.843	24.30863	**24.306**
	Std. Dev	6.6e-02	1.32e-01	4.9999e-04	**3.0169e-14**
g09	Best	**680.630**	680.632	680.6376	680.6387
	Mean	680.656	**680.643**	680.67833	680.8047
680.630	Worst	680.763	680.719	**680.6980**	680.9844
	Std. Dev	3.4e-02	**1.55e-02**	0.016262	0.1145
g10	Best	7054.316	7051.903	7049.333	**7049.248**
	Mean	7559.192	7253.047	7049.545	**7049.312**
7049.248	Worst	8835.655	7638.366	**7049.603**	7049.891
	Std. Dev	5.3e+02	1.3602e+02	**0.071513**	0.2034
g12	Best	**1**	**1**	**1**	**1**
	Mean	**1**	**1**	0.999988	**1**
1	Worst	**1**	**1**	0.999935	**1**
	Std. Dev	**0**	**0**	1.7e-05	**0**

* The best result of problem *g05* by SR is even better than the optimal solution of 5126.498. This is the consequence of transforming equality constraints into inequality constraints by a relaxed parameter ε [8].

5. CONCLUDING REMARKS

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In this paper, a novel MA based Hybrid EO-SQP algorithm is proposed for global optimization of NLP problems which are typically quite difficult to solve exactly. Traditional deterministic methods are more vulnerable to getting trapped in the local minima; while most computational intelligence based optimization methods with global search capability tend to suffer from high computation cost. Therefore, under the framework of MA, the general-purpose heuristic EO with deterministic local search method SQP are combined together in order to develop a robust and fast optimization technique with global search capability and mechanism to deal with constraints. The hybrid method avoids the possibility of local minimum by providing the gradient search method with the exploration ability of EO. Those advantages have been clearly demonstrated by the comparison with some other state-of-the-art approaches over 12 widely used benchmark functions.

The future studies involve more fundamental research on evolution dynamics of the proposed method and the application of EO-SQP for more benchmark and real-world problems.

APPENDIX:

Function	Function expression	Search space	Global minimum
Michalewicz	$f_1(X)=-\sum_{i=1}^{n}\sin(x_i)\sin^{2m}(\frac{i-x_i^2}{\pi}),\ m=10$	$(0,\pi)^n$	-9.66
Schwefel	$f_2(X)=-\sum_{i=1}^{n}(x_i\sin(\sqrt{\lvert x_i\rvert}))$	$(-500,500)^n$	-12569.5
Griewank	$f_3(X)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	$(-600,600)^n$	0
Rastrigin	$f_4(X)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	$(-5.12,5.12)^n$	0
Ackley	$f_5(X)=20+e-20e^{\left[-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right]}-e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)}$	$(-32.768,32.768)^n$	0
Rosenbrock	$f_6(X)=\sum_{i=1}^{n-1}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	$(-30,30)^n$	0

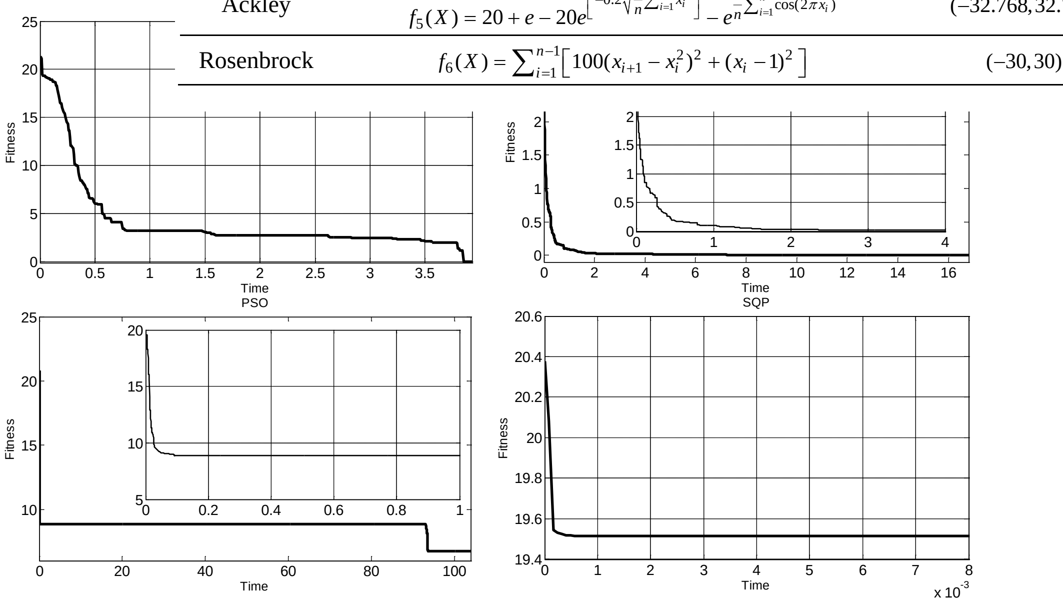

Figure. 3 Comparison of best evolved solution fitness generation by generation between EO, GA, PSO and SQP on Ackley Function

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