MAINTENANCE POLICY OPTIMIZATION FOR A DETERIORATING SYSTEM WITH MULTI-FAILURES

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ABSTRACT

For complex systems in modern industry system, optimal maintenance policy is difficult to determine concerned balance between risks and costs because numerical elements have effectives to each others. To solve this critical problem, this paper presents an optimal maintenance model for a multi-state deteriorating system subject to multi-failures. The system process is modeled as a semi-Markov process. The model has considered both multi-condition states and multi-failure states of the system. The system deterioration process is described from "as good as new" to the state "completely failed". In each state, the probability of the system transferred to certain failure is different. Using different maintenance policy will lead to the different state transition probability. The certain aim of the study is to find the best maintenance policy to minimize the long-run expected average cost per unit time. Condition monitoring and state prediction techniques are considered in the model. Finally, a numerical example and an industry example are presented to illustrate the implementation of the computational approach.

Keywords: *Optimal Maintenance Policy; Semi-markov Process; Condition Monitoring; Long Run Cost*

1. INTRODUCTION

In modern industry systems, the study on complicate large scale equipment optimal maintenance policy has become a hot point in recent years. Proper maintenance policy should make the system run in high safety level and low cost which both are critical issues concerned in the modern industry. Low cost will increase market competitiveness. For instance, during natural gas transportation process, the investment on compressor station takes 1/4 of the whole invests. The station operation cost takes 1/2 of the whole system. The compressor operation cost takes 70% of the whole station. If the compressor maintenance cost be controlled well while not increase the risk level. It is undoubtedly will increase the gas production efficiency. However, the maintenance policy usually made by equipment producer, who mainly concerned the rated condition, ignoring the fact that most equipment is used in non-full rated operating state. That makes the maintenance policy adopted badly in real world condition. □

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In modern petroleum industry systems, because a lot of elements have affections to each others, optimal maintenance policy is difficult to determine concerned balance between risks and costs. Jacobinical maintenance policy will raise the risk level, while conservative maintenance policy may cause unnecessary investment. To solve this critical problem, our study tries to present an optimal maintenance model for a multi-state deteriorating system subject to multi-failures.

A multi-state system (MSS) is capable of assuming a range of performance levels, varying from full functioning to complete failure. Because it can be used in many industrial fields, the optimal maintenance policy of deteriorating MSS has been concerned in many literatures. Extensive reviews of maintenance policies on a deteriorating system research can be found in papers [1]-[3].

This paper mainly focused on maintenance optimization of multi-state systems. The optimization of PM (preventive maintenance) problem is initially addressed by Levitin and Lisnianski [4]. They find an optimal sequence of maintenance actions which minimizes maintenance cost while assuring the desired system reliability level. Nabil and Abdelhakim [5] improve Levitin's work by using the universal generating function technique and extended great deluge algorithm. Chiang and Yuan [6] proposed a state-dependent maintenance policy for a multi-state continuous-time Markovian deteriorating system with brand new state to failure state subject to aging and fatal shocks. Maxstaley, Leonardo and Carlos [7] combine optimization model and input parameters estimation from empirical data to propose condition-based maintenance policies. They use the Hidden Markov Model theory to adequate the model inputs to the empirical data available. Wu and Wang [8] presented a three states and two repair levels model to find an optimal sampling policy for the inspection process. Michael and Viliam [9] present a semi-Markov decision process with the optimality criterion being the minimization of the long-run expected average cost per unit time by a modified the embedded technique. The MSS model has been widely used and adopted well in many complex industry systems [10, 11].

Section 2 presents the model and basic concepts. In Section 3, the detailed computational approach is proposed. In section 4, the influence of condition monitoring is discussed. Section 5 gives the modified model considering risks. Section 6 and Section 7 presents a numerical example and a real industry example.

2. MODEL AND CONCEPTS

In this section, we formulate a degradation system subject to multi-failures. The process is described by a semi-Markov model. The subsections below sequentially defined state space, maintenance action space, transition probabilities, expected cost and time between epochs associated with the model.

2.1. State Space Description

The system operation state is described as $S = \{1, 2 \cdots N\}$. State 1 means the system is "as good as new". Each state gets closer to state N means higher deterioration degree. Meanwhile, the failure state is described as $F = \{F_{ij}, |i \in (1, 2, \cdots, K), j \in S\}$. Different state has different probabilities transfer to failure states. Thus the total state space is $O = S \cup F$. Fig1 shows the deterioration process if no repair takes place of the model. De Leve [12] defined it as a natural process. If seperate major failure as an independent state, the state space of the model is same as Kim described [9].

2.2 Maintenance Action Space

Consider the system is in state i. Then the maintenance action "a" repairs the system from state i to state i-a. a=0 means no repair actions are taken, while a=i-1 means the action repairs the system to state 1 which means replacement actions are taken. Then the action space associate to operational space i is $A(i) = \{ 0 \leq a \leq i-1, a \in Z\}$. For any failure state F_{ij}, a repair action R_{ij} will be taken to repair the system back to state j. Then, $A(F_{ij}) = \{R_{ij}\}$. For state i, after repair action a has been carried out, the repaired state will be:

$$r_i(a) = \begin{cases} i-a, & i \in S \\ j, & i \in \{F_{ij} \mid 1 \leq i < k\} \\ 1, & i = F_{kj} \end{cases} \quad (1)$$
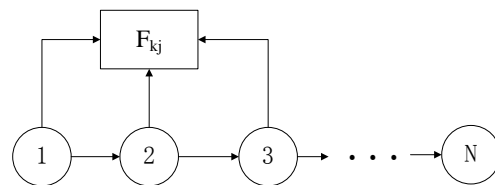

Figure 1. The System Deterioration Process Without Maintenance Action

2.3 Transition Probabilities

Based on the definitions above, if the system is in state $i \in O$, then action $a \in A(i)$ is taken, the system will be repaired to state $r_i(a)$. We define $p_{ij}(a)$ as the probability that the system will be in state j while the current state is i and an action a is taken. When no actions are taken, a=0, $p_{ij}(a) = p_{ij}(0)$ represents the transition probabilities from state i to j. When applied the model in real systems, the system states should be clearly defined. Then the transition probabilities can be estimated by history records or lab experiments. For example, Arun and Mahesh [11] defined 4 states in nuclear pipeline system and estimate the transition probabilities by investigating operation records.

2.4 Expected Cost and Time between Epochs

Define u(i) as the time system will stay in state i if no action is taken. Define d_i as the cost per unit time when system stays in state i. Define $b_i(a)$ as the cost of state i with action a and $t_i(a)$ as time cost of state i take action a. Obviously, if no action is taken, $b_i(a)=b_i(0)=t_i(0)=0$. Define m as the system cost with general repair. Then we can get the total cost of system in state i take action a is:

$$c_i(a) = b_i(a) + mt_i(a) + d_{r_i(a)}u_{r_i(a)} \qquad (2)$$

The total time stay in the interval is:

$$T_i(n) = t_i(a) + u_{r_i(a)} \qquad (3)$$

3. COMPUTATIONAL APPROACH

The object of this model is to minimize the long run cost of the system per unit time. In some level, the cost of maintenance and system reliability is a pair of contradictions. Design more redundant equipment and backup system, full scale condition monitoring, frequent inspection, choose replacement rather than repair. All will highly increase systems reliability. However, in most cases, resources and time for maintenance are limited while as higher reliability as possible is requested. Thus, how to get optimal maintenance policy to use resources (manpower, time, money) efficiently becomes an important work.

Generally, there are several kinds of optimal maintenance works depend on the system kinds. A) The system is inspected periodically. Then the optimal time between inspections is an interesting work. The objection of the work is to find out the longest interval between inspections, while keep the system on a certain safety and reliability level. B) The system is under continuous monitoring. Then the determination of detailed maintenance action is an interesting and critical issue. When the system achieves a certain need-action state, repairing or replacement may both be able to bring the system to a better state. Repair cost less but can't bring the system to the best state. And after more repairs, the system won't be able to repair back to the ideal situation. Replacement cost more and makes the system brand new. It will restore the theoretically longest system remaining life. For long time consideration, what maintenance policy is the best is a considerable issue.

Define Z as policies set. Z includes all alternative policies. Define z∈Z as a function associated with state i. Generally, the optimization process of maintenance policy followed next steps [9]:

1) Choose a policy z∈Z.

2) Compute g, v_i, i∈O as solution to linear system equations:

$$v_i = c_i(z(i)) - gT_i(z(i)) + \sum_{j \in O} p_{ij}(z(i))v_j \quad, i \in O, (4)$$

$$v_o = 0, \text{ for some } v \in O$$

3) For each state i∈O, using the values g and v_i calculated in step 2), determine the action $a_i \in A(i)$ minimizes the expression

$$c_i(a_i) - gT_i(a_i) + \sum_{j \in O} p_{ij}(a_i)v_j \qquad (5)$$

The new policy z' ∈ Z is obtained by taking z'(i)=a_i for each i∈O.

4) If z'=z, then the optimal stationary policy is z. Otherwise, return step 1) and replace z with z'.

As shown above, the general method is based on the idea that the minimum cost makes the best maintenance policy. This idea adopted well in many conditions. However, building up the model is different from using it in practical processes. For example, for a newly established system, the deterioration process, the transition probability, and failure transportation regularity are all unknown. There are several solutions to solve the problem:

1. Design rational experiments and trial operation of the system.

2. Analyze the detail deterioration process of the system.

3. Use condition monitoring and risk estimation techniques to manage the system life cycle.

In the next section, how the solution 3 influence the model will be further discussed.

4. CONDITION MONITORING TECHNIQUE

The system condition monitoring technology has been widely used in modern industry systems. It is initially used in automation systems. The system's state is monitored by using sensors to collect noise, vibration, voltage information. Then information is fused and analyzed to describe the current system condition. Condition monitoring is the base of CBM (condition-based maintenance). The CBM approach is growing in popularity since the 1990s which highlight the importance of maintenance policies that rely on the conditions (past and present) of systems. In fact, the CBM denotes monitoring

for the purpose of determining the current "health status" of a system's internal components and predicting its remaining operating life. With the development of signal prediction technology and computer technology, continuous system state monitoring gradually steps from theory to practice. But how does it influence the traditional Markov deterioration model and how to count the condition monitoring and state prediction technology into optimal maintenance policy computation has not been properly discussed before.

The state space, maintenance action space, and transition probabilities wouldn't change by the conditional monitoring and state prediction technology.

The model described in section 2 is already a model with continuous condition monitoring technology because the system states are precisely determined. If the model is based on interval inspection, the time of the decision making epoch would include interval between inspections.

5. RISK ESTIMATION

In MSS system, the estimation of system reliability and risk level can be considered in two ways:

1. Limited the system states to certain safety level. For each state, the system reliability and risk level can be settled. Then according to the required safety level, we can get the certain critical state. When the system state is worse than it, the action will have to be replacement or general repair. When the system state is better than it, the system will be surely above the required safety level. Thus, more consideration will put into calculate the optimal maintenance policy of the system states above the required safety level. The states space of the model would include states above the certain risk level.

2. For some complicate systems, analysis safety level and consequence of certain actions is a feasible way to describe system reliability and risk level. For computing the total cost of maintenance policy, risk and consequences should be finally calculated into the cost. Li and Gao's [13] method to analyze risk and maintenance cost is a referential approach. Firstly, FMECA (Failure Mode, Effects and Criticality Analysis) are used to analyze important equipment's failure mode analysis, failure cause analysis and failure consequence analysis. Secondly, high, medium criticality failures would be analyzed by FTA (Fault Tree analysis) to find the root causes of failures. Then use the matrix to evaluate the criticality in four aspects: safety,

environment, production loss, and maintenance cost. The expected cost of the model should include safety, environment and production loss.

The method 2 is adopted in this paper. Define L_{ij} as loss cost by F_{ij}. Then equation in step 2 and 3 of the computational approach would be

$$v_i = c_i(z(i)) - gT_i(z(i)) + \sum_{j \in O} p_{ij}(z(i))v_j + \sum_{j \in F} p_{ij}(z(i))L_j \quad (6)$$

$$c_i(a_i) - gT_i(a_i) + \sum_{j \in O} p_{ij}(a_i)v_j + \sum_{j \in F} p_{ij}(a_i)L_j \quad (7)$$

The introduction of the last element of formular connected risk evaluation to total cost. With the participation of condition monitoring and state prediction, the real time dynamic optimal maintenance policy can apply to real production systems.

6. EXAMPLE AND DISCUSSION

In this section, a similar example as Kim [9] presented in 2009 is used. Define the system has 3 states and subject to 7 minor/major failures. Three kinds of maintenance policies are considered: do-nothing, repair and replacement. The total state space O= { 1, 2, 3, F_{11}, F_{21}, F_{12}, F_{22}, F_{13}, F_{23}, F_{33} }. This example considers 3 operational states (1-3), 7 failure states (F_{11}-F_{33}) to describe multi-kinds failure. Table1 provides the mean transition costs and times of the process. The natural evolution of the system is described by transition probability matrix (7)

$$P = \begin{bmatrix} 0 & 0.1 & 0.2 & 0.1 & 0.2 & 0 & 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0.1 & 0 & 0 & 0.1 & 0.2 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.2 & 0.7 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

The start initial policy is z_0={0, 0, 0, R_{11}, R_{21}, R_{12}, R_{22}, R_{13}, R_{23}, R} which no action is taken in operational states. Using Eqs.described in section 3, the valve is obtained: v_1=-8562.2, v_2=-4159.4, v_3=1587.5, v_{F11}=-7218.8, v_{F21}=-6946.9, v_{F12}=-6275.0, v_{F22}=-4803.1, v_{F13}=-2831.2, v_{F23}=-1359.4, v_{F33}=0, and g=106.6. After carrying out the policy-improvement step described in Eq. (5), a new policy is obtained z_1= {0, 1, 2, R_{11}, R_{21}, R_{12}, R_{22}, R_{13}, R_{23}, R} $\neq z_0$. Thus, return to step1 and define z1 as the initial policy. Policies obtained after each iteration are summarized in Table 2. The final

optimal maintenance policy is z= {0, 1, 2, R_{11}, R_{21},   g=100.4.
R_{12}, R_{22}, R_{13}, R_{23}, R} with the optimal cost rate

Table 1. Expected Costs and Times

State	1	2	3	F_{11}	F_{21}	F_{12}	F_{22}	F_{13}	F_{23}	F_{33}
u_i	100	80	60	-	-	-	-	-	-	-
d_i	10	15	20	-	-	-	-	-	-	-
$t_i(1)$	-	20	30	-	-	-	-	-	-	-
$b_i(1)$	-	4000	4500	-	-	-	-	-	-	-
$t_i(2)$	-	-	50	-	-	-	-	-	-	-
$b_i(2)$	-	-	8000	-	-	-	-	-	-	-
$t_i(R_{ij})$	-	-	-	10	15	20	25	30	35	-
$b_i(R_{ij})$	-	-	-	3000	3500	4000	4500	5000	5500	-
$t_i(R)$	-	-	-	-	-	-	-	-	-	60
$b_i(R)$	-	-	-	-	-	-	-	-	-	9000
L_{ij}	-	-	-	500	800	1500	3000	5000	6500	8000

Tbale 2 Summary of Values and Policies Determined after Each Iterations

	1	2	3	F_{11}	F_{21}	F_{12}	F_{22}	F_{13}	F_{23}	F_{34}	g
z_0	0	0	0	R_{11}	R_{21}	R_{12}	R_{22}	R_{13}	R_{23}	R	
v_i	-8562	-4159	1587	-7219	-6947	-6275	-4803	-2831	-1359	0	106.6
z_1	0	1	2	R_{11}	R_{21}	R_{12}	R_{22}	R_{13}	R_{23}	R	
v_i	-7695	-3299	1292	-6529	-6225	-5522	-4019	-2016	-513	0	100.4
z_2	0	1	2	R11	R21	R12	R22	R13	R23	R	

Table 3. Expected Costs (10^2RMB) and Times (10^2h) for Main Pump

State	1	2	3	4	F_{11}	F_{21}	F_{12}	F_{22}	F_{13}	F_{23}	F_{24}	F_{34}
u_i	110	82	65	37	-	-	-	-	-	-	-	-
d_i	8	25	20	30	-	-	-	-	-	-	-	-
$t_i(1)$	-	0.1	0.2	0.4	-	-	-	-	-	-	-	-
$b_i(1)$	-	200	350	500	-	-	-	-	-	-	-	-
$t_i(2)$	-	-	0.5	0.8	-	-	-	-	-	-	-	-
$b_i(2)$	-	-	700	1000	-	-	-	-	-	-	-	-
$t_i(3)$	-	-	-	1.5								
$b_i(3)$	-	-	-	12000								
$t_i(R_{ij})$	-	-	-	-	0.02	0.03	0.05	0.08	0.5	0.6	0.8	-
$b_i(R_{ij})$	-	-	-	-	150	210	300	500	700	720	810	-
$t_i(R)$	-	-	-	-	-	-	-	-	-	-	-	1.6
$b_i(R)$	-	-	-	-	-	-	-	-	-	-	-	18000
L_{ij}	-	-	-	-	80	180	350	620	1200	2100	3000	10000

Tbale 4 Summary of Values and Policies Determined after Each Iterations for Main Pump

	1 (× 10^4)	2 (× 10^4)	3 (× 10^4)	4 (× 10^4)	F_{11} (× 10^4)	F_{21} (× 10^4)	F_{12} (× 10^4)	F_{22} (× 10^4)	F_{13} (× 10^4)	F_{23} (× 10^4)	F_{24} (× 10^4)	F_{34} (× 10^4)	g
z_0	0	0	0	0	R_{11}	R_{21}	R_{12}	R_{22}	R_{13}	R_{23}	R_{24}	R	
v_i	-11.08	-7.10	-5.63	-2.67	-11.06	-11.04	-7.04	-6.99	-5.48	-5.39	-2.35	0	805.3
z_1	0	1	2	3	R_{11}	R_{21}	R_{12}	R_{22}	R_{13}	R_{23}	R_{24}	R	
v_i	-3.95	-3.93	-4.22	-2.76	-3.93	-3.91	-3.87	-3.82	-4.04	-3.95	-2.40	0	245.3
z_2	0	1	1	3	R_{11}	R_{21}	R_{12}	R_{22}	R_{13}	R_{23}	R_{24}	R	
	-3.98	-3.96	-2.09	-2.78	-3.96	-3.94	-3.89	-3.85	-2.07	-2.06	-2.41	0	96.0
z_3	0	1	1	3	R_{11}	R_{21}	R_{12}	R_{22}	R_{13}	R_{23}	R_{24}	R	

7. INDUSTRY EXAMPLE

In oil long distance transportation process, main pump is the heart of the system. Impeller of the pump suffered to corrosion, aging and fatigue etc. multi failure models.

Based on field experience and historical records, the main pump has 4 states and subject to 2 minor failures and 1 major failure. Four kinds of maintenance policies are considered: do-nothing, minor repair, major repair and replacement. The total state space $O' = \{1, 2, 3, 4, F_{11}, F_{21}, F_{12}, F_{22}, F_{13}, F_{23}, F_{24}, F_{34}\}$. Table 3 provides the mean transition costs and times of the process. The natural evolution of the system is described by transition probability matrix (8)

$$P = \begin{bmatrix} 0 & 0.15 & 0.1 & 0.05 & 0.03 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0.005 \\ 0 & 0 & 0.05 & 0.03 & 0 & 0 & 0.03 & 0.02 & 0 & 0 & 0 & 0.008 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0.03 & 0.02 & 0 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0.3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The start initial policy is $z_0 = \{0, 0, 0, 0, R_{11}, R_{21}, R_{12}, R_{22}, R_{13}, R_{23}, R_{24}, R_{34}, R\}$ which no action is taken in operational states. Using Eqs.described in section 3, the valve is obtained: $v_1 = -11.08 \times 10^4$, $v_2 = -7.10 \times 10^4$, $v_3 = -5.63 \times 10^4$, $v_{F11} = -2.67 \times 10^4$, $v_{F21} = -11.06 \times 10^4$, $v_{F12} = -11.04 \times 10^4$, $v_{F22} = -7.04 \times 10^4$, $v_{F13} = -6.99 \times 10^4$, $v_{F23} = -5.48 \times 10^4$, $v_{F24} = -5.39 \times 10^4$, $v_{F34} = -2.35 \times 10^4$, $v_{F33} = 0$, and $g = 805.3$. After carrying out the policy-improvement step described in Eq. (5), a new policy is obtained $z_1 = \{0, 1, 2, 3, R_{11}, R_{21}, R_{12}, R_{22}, R_{13}, R_{23}, R_{24}, R_{34}, R\} \neq z_0$. Thus, return to step1 and define z1 as the initial policy. Policies obtained after each iteration are summarized in Table 2. After two steps of optimization, the final optimal maintenance policy is $z = \{0, 1, 1, 3, R_{11}, R_{21}, R_{12}, R_{22}, R_{13}, R_{23}, R_{24}, R_{34}, R\}$ with the optimal cost rate $g = 96.0$.

8. CONCLUSIONS

This paper presented a mult-state model for a system subject to multi-failures. The deteriorating system was described by a semi-markov process. The objection was to find out the minimum long-run expected average cost per unit time. The model considered influence of condition monitoring system and risk evaluation. A numerical example was presented to certify the effect of the algorithm. The suggestion for future research is to consider the influence of migration of failure states in the production system.

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